

Γνώση του A όταν οι B, C δεν γνωρίζουν το χρώμα τους

Ορισμός Έστω M_A ένα μοντέλο Kripke που αναπαριστά την γνώση του A:
Ο παίκτης A γνωρίζει ότι αληθεύει θ , αν $M_A \models \theta$.

Take any cluster S of possible worlds that A can not distinguish between.

Partition this cluster into sub-clusters, say T_1, T_2 , etc., that are *indistinguishable for B*.

That is, B can not distinguish between members of T_1 ; B can not distinguish between members of T_2 , and so on.

1) By [B sees A], A knows that either B knows $AisWh$ or B knows $\neg AisWh$.

This means that in T_1 it cannot happen that $AisWh$ is true at some worlds and false at others; similarly for T_2 , and so on. Briefly, for each k , either $AisWh$ is true at all the possible worlds of T_k , or else $AisWh$ is false at all the possible worlds of T_k .

2) By [B hears C], A knows that B knows that $AisWh \vee BisWh$.

Then $AisWh \vee BisWh$ must be true throughout each T_k .

Combining with (1) it follows that, for each k :

either $AisWh$ is true at every possible world of T_k ,
or $BisWh$ is true at every possible world of T_k .

Άρα, $AisWh \vee K_B BisWh$ is true at every possible world of T_k .

Since the T_k are a partition of S , $AisWh \vee K_B BisWh$ must be true at every world of S .

Since $AisWh \vee K_B BisWh$ is true at all the worlds that A thinks are possible,

$M_A \models AisWh \vee K_B BisWh$.

3) By [B doesn't know], A knows that B doesn't know $BisWh$.

Then $M_A \models \neg K_B BisWh$,

and $M_A \models AisWh$.

ΕΡΩΤΗΜΑ Τι συμβαίνει όταν ο A είναι μαύρος;

Tableau proof for: $1, 2 \models A \text{ knows } (A \text{isWh} \vee K_B \text{ BisWh})$

- 1 (s1) A knows that (B knows AisWh or B knows \neg AisWh)
- 2 (s1) A knows that B knows that (AisWh \vee BisWh)
- 3 (s1) \neg A knows (AisWh \vee K_B BisWh)
- 4 (s1 A s2) \neg (AisWh \vee K_B BisWh) from 3
- 5 (s1 A s2) \neg AisWh from 4
- 6 (s1 A s2) \neg K_B BisWh from 4
- 7 (s1 A s2 B s3) \neg BisWh from 6
- 8 (s1 A s2) B knows that AisWh \vee BisWh from 2
- 9 (s1 A s2 B s3) AisWh \vee BisWh from 8
- 10 (s1 A s2 B s3) AisWh from 9 , 7
- 11 (s1 A s2) B knows AisWh **or** B knows \neg AisWh from 1

BRANCH 1

- 11,1 (s1 A s2) B knows \neg AisWh
- 12,1 (s1 A s2 B s3) \neg AisWh

X with 10

BRANCH 2

- 11,2 (s1 A s2) B knows AisWh
- 12,1 (s1 A s2) AisWh

X with 5

