## Bioinformatics

## Applications Aho-Corasick(1)

## Exact pattern matching with do not cares ('*') character

Let T be a string with n characters and P be a string with $\mathrm{k}-1$ do not care ('*') characters of total length $m$.

## Algorithm

0 . Let C be an array of integers of length n initialized in zeros.

1. Let $P=\{p 1, p 2, \ldots, p k)$ be the (multi-)set of substrings of $P$ that do not contain wildcard characters. Let $11,12, \ldots, \mathrm{lk}$ be the initial positions in P of each of these substrings $(11=1)$.
2. Using the Aho-Corasick algorithm find for each string Pi in P , all the initial positions of Pi in T . For each position j of Pi in T , increase the number in cell $\mathrm{j}-\mathrm{li}+1$ of C by one.
3. Scan array $C$ to find cells with value $k$. There is an appearance of $P$ in $T$ starting from position p if and only if $\mathrm{C}(\mathrm{p})=\mathrm{k}$.

## Applications Aho-Corasick(2)

## Two dimensional Pattern Matching

Let T be a two-dimensional text with $\mathrm{n}=\mathrm{n} 1 \times \mathrm{n} 2$ cells and P a two-dimensional pattern of $\mathrm{m}=\mathrm{m} 1 \times \mathrm{m} 2$ cells. We want to identify all occurrences of P in T .

The method is divided into two phases.
In the first phase, look for all occurrences of each of the rows of P among the rows of T. To do this, add an end-of-line marker (a character that does not exist in the alphabet) to each line of $T$, and concatenate these lines into a text string of $T^{\prime}$ length $O(n)$.

Then, treating each line of P as a separate pattern, use the Aho-Corasick algorithm to search for all occurrences in $\mathrm{T}^{\prime}$ of any row of P .

Therefore, the first phase identifies all occurrences of P and takes time $\mathrm{O}(\mathrm{n}+\mathrm{m})$.

## Applications Aho-Corasick(3)

Whenever an occurrence of line i of $P$ starting with position ( $p, q$ ) of $T$ is detected, write the number $i$ in position ( $\mathrm{p}, \mathrm{q}$ ) of another array M with the same dimensions as T . Because each line of $P$ is considered distinct, and because $P$ is rectangular, at most one number will be written in any cell of $M$.
in the second phase scan each column of $m$ looking for an occurrence of the string 1.2 .
$\ldots \mathrm{m} 1$ in consecutive single-column cells.
This gives an $\mathrm{O}(\mathrm{n}+\mathrm{m})$ solution if a linear time pattern matching algorithm is used in each column, regardless of the alphabet (Z-algorithm, Knuth Morris Pratt)

Now suppose that the rows of P are not all discrete. It is enough to identify all identical rows of P and give them a common label.

## Techniques for Analysis and Comparison of Biological Data Sequences

- Suffix Tree
- Generalized Suffix Tree
- Applications in Molecular Biology Problems


## Definitions

- String: $x=x[1] x[2] \ldots . . x[n], x[i] \in \Sigma \&|x|=n$ $x=\operatorname{acgttaaaca},|x|=10 \& \Sigma=\{a, c, g, t\}$
- Empty string: $\varepsilon$
- Substring w: $\mathrm{x}=\mathrm{uwv}$
- Prefix w: $x=w u$
- Suffix w: x=uw
- Each string $S$, length $|S|=m$, has $m$ non-empty suffixes which are the following: $\mathrm{S}[1 . . . \mathrm{m}], \mathrm{S}[2 \ldots \mathrm{~m}], \ldots . \mathrm{S}[\mathrm{m}-1 . . . \mathrm{m}]$ kaı S[m].
- Example "sequence" : sequence, equence, quence, uence, ence, nce, ce, e.


## Suffix Tree

Definition: "stores all possible suffixes of a string".

$$
\mathrm{p}=\text { xabxac }
$$



Definition A suffix tree T for a string of n characters is a rooted directed tree with exactly $n$ leaves numbered from 1 to n . Each inner node, except the root, has at least two children, and each edge is marked with a nonempty substring. It is not possible for a node to have two edges starting from the same character. The key feature of the suffix tree is that for any leaf $i$, the concatenation of edge-labels in the path from root to leaf $i$ is equal to the suffix of the string starting at position $i$.

Dan Gusfield Algorithms on Strings, Trees and Sequences, Cambridge University Press,

## Suffix Tree

Definition: The suffix tree of a string $S[1 \ldots n]$ is a compact trie that contains as keys, all suffixes S[i...n], $1 \leq i \leq n$.


## Trie Definition:

Let universe $U=\Sigma^{0} \cup \ldots \cup \Sigma^{\prime}$ for alphabet $\Sigma$ and $I>0$.

$$
\begin{aligned}
& \left(x \in U: x=d_{1} d_{2} \cdots d_{l}\right) S \subseteq U: \\
& \quad \sigma_{\sigma_{1}} \sigma_{\sigma_{2} \ldots} \\
& \quad\left(\sigma_{i} \in \Sigma\right)
\end{aligned}
$$

Trie (uncompressed)

## Trie - example

$$
S=\{102,120,121,212,211,120\}, \quad \Sigma=\{0,1,2\}
$$


digit 1
Time ins/del/search :
O(I)
digit 2 Space: O(nlk)
digit 3

## Compressed Trie



## Compressed Trie - example



## Preprocessing time to build Suffix tree

- Weiner's algorithm [FOCS, 1973]

Knuth "The algorithm of 1973"

- McCreight's algorithm [JACM, 1976]

Linear time and space

- Ukkonnen's algorithm [Algorithmica, 1995]

Linear time and less space

- Farach's algorithm [FOCS 1997],
gave the first linear time alphabet indepedent


# Implementation (https://en.wikipedia.org/wiki/Suffix tree) 

|  | Lookup | Insertion | Traversal |
| ---: | :--- | :--- | :--- |
| Sibling lists / unsorted arrays | $O(\sigma)$ | $\Theta(1)$ | $\Theta(1)$ |
| Bitwise sibling trees | $O(\log \sigma)$ | $\Theta(1)$ | $\Theta(1)$ |
| Hash maps | $\Theta(1)$ | $\Theta(1)$ | $O(\sigma)$ |
| Balanced search tree | $O(\log \sigma)$ | $O(\log \sigma)$ | $O(1)$ |
| Sorted arrays | $O(\log \sigma)$ | $O(\sigma)$ | $O(1)$ |
| Hash maps + sibling lists | $O(1)$ | $O(1)$ | $O(1)$ |

