FURTHER O.R. TECHNIQUES

EXERCISES

UNIT 2: Queuing Theory

1. A local cinema has only one ticket desk that can serve on average 6 customers per minute. Customers arrive at an average rate of 4 per minute during busy hours. Assume that arrivals are distributed according to the Poisson distribution and service times according to the exponential distribution.

(a) What is the average number of customers waiting in the queue?

(b) What is the average waiting time?

(c) What is the average number of customers in the system?

(b) What is the average time in the system?

(e) What is the probability that a new customer, arriving at the cinema, will have to wait?

(f) Suppose that the cost of waiting in the queue or being served is 50 monetary units per minute and the service cost is 200 monetary units per minute. Calculate the total operating cost of the system.

2. A small travel bureau employs one agent and has 3 chairs for waiting customers. Customers arrive at the bureau according to the Poisson process at a rate of 5 customers per hour. The agent can serve on average 10 customers per hour. Calculate the following:

(a) The average number of customers in the system and in the waiting area.

(b) The average waiting time.

(c) The percentage of lost customers who are forced to leave because the bureau is full.

3. A factory has 25 machines operating in its production line. Whenever a machine breaks down, it is repaired by one of the 6 technicians employed in the factory. Every machine breaks down on average after one hour of operation. The repair time is on average 12 minutes.

(a) What is the probability that all 25 machines are operating?

(b) What is the average number of machines that are out of order and the average number of the ones waiting to be repaired?

(c) If a machine that is not operating costs 25 Euros per hour, is it worth hiring an extra technician at 7 Euros per hour?

4. Consider a service system, where customers arrive according to the Poisson process at a rate of 25 per hour. In order to serve the customers, we can either employ two servers, each serving each customer on average in 4 minutes, or only one server serving each customer on average in 2 minutes. Which of these two options is preferable if the objective is to minimize:

(a) The average waiting time in the queue?

(b) The average waiting time in the system?

5. At the call center of company ABC, phone calls arrive according to the Poisson process at an average rate of 40 calls per hour. The call center operates two automatic switch centers (call them Y1 and Y2) with incoming calls assigned to either of them at equal probability. The time it takes a switch-center to process each phone call is on average 2.5 minutes, exponentially distributed. The operating cost of each switch-center is 3 Euros per hour and the cost of ABC for each incoming call (either in waiting or being processed is 0.5 Euros.

(a) What is the average number of calls waiting in the queue and what is the average waiting time?

(b) What is the total cost of the system per hour?

(c) Assume that calls are not divided into two separate queues, but form one queue before the two switch centers, that are now operating in parallel. The rest of the data is the same. What is the average number of calls in the queue and what is the average waiting time under this configuration?

(d) What is the new cost of the system?

(e) Compare the total system cost for the two different configurations and advise the company as to which configuration is better.

6. A canteen is open from 10 a.m. until 10 p.m. and employs two attendants. Customers arrive at an average rate of 20 per hour, according to the Poisson process. They line up in one queue, waiting their turn for the first available attendant to serve them. The time it takes to process an order (take order + preparation + payment + delivery) is distributed exponentially with a mean of 5 minutes. Assume that the waiting area in front of the canteen is practically limitless and that customers arrive from an infinite population.

(a) What is the average number of customers in the queue?

(b) What is the probability that a new customer will not have to wait?

(c) What is the total cost of the system if the waiting time of a customer in the queue or during service is 10 Euros per hour and labor cost is 5 Euros per hour?

(d) What is the average time that the canteen must remain open after closing time in order to finish serving the customers that are still in the queue at that time?

7. In the ER department of a hospital patients arrive following the Poisson process at a rate of 1 patient every half hour. It takes on average 20 minutes for the only doctor working in the ER department to examine each patient.

(a) The management of the hospital considers hiring a second doctor. What do you suggest?

(β) It has since been observed that the time it takes to examine a patient depends on the number of patients already in the ER department. More specifically, a doctor examines a patient in 24 minutes when there are no other patients in the department and in 12 minutes when there are five more patients waiting to be examined. What is the average waiting time in this case?