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Which demands affect optimal international portfolio choices?

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ABSTRACT

This study analyzes the asset allocations of simple international portfolios that include domestic risky assets, foreign risky assets, and domestic risk-free bonds, through a theoretical analysis. A close-form solution for the optimal holding rates is derived, and can be further sub-divided into three categories of demand: speculative demand, diversified demand, and hedging demands. We carefully explore the essential problem of identifying the underlying reasons for asset allocations, which in turn allows us to answer the question of *which* of these demands are critical in influencing holding changes.

1. Introduction

Nowadays, with financial trading activities having gradually become globalized, investors are concerned not only with risks and returns of risky assets, but also with risks of the exchange rate. Due to international financial markets being globalized and integrated, domestic investors can easily hold foreign financial assets denominated in foreign currency. Therefore, asset allocations of international portfolios have become an important issue for both academics and market participants. Of particular importance is the possibility of exchange rates varying greatly over a period of time, especially since many nations have deregulated currency transfers and exchange rate variations.

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Investing in foreign financial assets benefits the wealth allocations for a domestic individual because the return-risk features of foreign assets differ from those of domestic assets. Although adding a foreign asset into a domestic portfolio can enhance the mean-variance efficiency, it also brings with it the risk of the exchange rate because foreign assets are denominated in foreign currencies. Therefore, balancing the benefits and disadvantages of investing in foreign assets is a challenge for these investors.

There are numerous reasons for investing foreign assets. First, the holding of foreign assets has a speculative demand due to the relative performances of various risky assets. Investors would like to allocate more wealth on assets that have better performance. Second, holding foreign assets also has a hedging demand that stems from the desire to avoid the risk of the exchange rate. Third, diversified risks are also considered in investors' diversified demands. That is to say, holding foreign assets presents a risk-return trade-off in international portfolios. One question that naturally arises is *whether* the speculative demand, diversified demand, or the hedging demand is more critical in terms of affecting an individual's international portfolio choices. This study discusses possible demands of holding risky assets, and looks at what roles these demands play in determining the total holding rates in a market with a stochastic setting of the exchange rate.

This study attempts to analyze the asset allocations of international portfolios for a representative individual who is exposed to the risk of the exchange rate. We derive the optimal decision-making for a simple international portfolio including domestic risk free assets, domestic risky assets, and foreign risky assets in a stochastic environment in terms of the exchange rate. Framing a continuous-time decision model, we concern ourselves with how individuals form their speculative demand, diversified demand, and hedging demand for both domestic and foreign assets in response to the volatility risk and the jump risk of the exchange rate. Thus, we can answer the question of *whether* the individual should increase his holding rates of domestic stock or foreign stock if the volatility of the exchange rate increases. Specifically, this study identifies *which* the reason for increasing one's holdings of domestic assets or foreign assets stems from speculative demand, diversified demand, or hedging demands. This study also explains *why* several determinants can change one of demands, without changing the other demands.

Although previous studies have provided some observations on international portfolios, none of them has gone so far as to derive closed-form solutions for optimal fractions of wealth between domestic and foreign assets (see e.g., Biger, 1979; Das and Uppal, 2004; Smedts, 2004; Lioui and Poncet, 2003; Veraart, 2010; Topaloglou et al., 2008; Martinez and Nava, 2009). Topaloglou et al. (2008) developed a sequence of investment decisions at discrete points in time for international portfolio management using a multi-stage stochastic programming model. They confirmed that an appropriate use of currency forward contracts could reduce risks of international portfolios. Larsen (2010) analyzed how investors choose their optimal strategies in an international market, and showed that the observed investment gain came from speculative investment only. In our study, a simple continuous-time, three-asset model can yield a closed-form solution, and can analyze which demands matter most in terms of affecting the holding rates of various assets.

In addition, in his pioneering studies, Merton (1971, 1990) documented the speculative demand and hedging demand for risky assets. That is to say, he analyzed the reasons for holding risky assets as coming from these two demands. Our study further analyzes the financial implications of asset holdings by breaking demand down into several components. Specifically, in addition to speculative demand and hedging demand against the volatility risk, we observe other reasons for holding risky assets, namely, diversified demand and hedging demand against the jump risk of the exchange rate. That is, although previous numerous studies have mentioned the demand categories of holding risky assets, they lost the contrasts of which demands are important for determining the asset holdings.

In short, our contributions are follows. First, compared to the numerical solutions of optimal holdings in numerous previous studies, we provide a closed-form solution for the problem of optimal weights of international portfolios. Second, this study answers the question of *how* investors allocate the assets in their wealth portfolio in response to the volatility risk and jump risk of the exchange rate. Third, speculative demand, diversified demand, and hedging demands for these assets will be clearly analyzed as the individual's portfolio exposes him to the risks of the exchange rate. That is to say, this study clearly identifies *which* demands act to drive changes in terms of the total holdings of risky assets.

The remainder of this paper proceeds as follows. The following section reviews the literature on international portfolio selections. Section 3 constructs a continuous time decision model using a stochastic dynamic programming methodology for analyzing the optimal international portfolio choices. Section 4 details numerous examples for the purpose of examining the impacts of asset features and exchange rate features on optimal holdings. Section 5 contains the conclusion.

2. Decision-makings in international portfolio choices

Many previous studies have analyzed the potential benefits of international diversification (see e.g., Grubel, 1968; Glen and Jorion, 1993; Shawky et al., 1997; Flavin and Panopoulou, 2009). Although the correlations between international assets have gradually increased in recent years because of market integration and globalization, foreign assets are still popular investment choices for domestic individuals who want to diversify their portfolios. Real-world market participants frequently concern themselves with how to dynamically allocate their wealth between domestic and foreign assets in financial markets. Due to international portfolios exposing investors to the risk of the exchange rate, the portfolio choices of such individuals should be dynamically adjusted in response to with changes in the currency risk and asset risks.

Martinez and Nava (2009) extended Merton's (1969, 1971) model by including sudden and unexpected jumps in the stochastic dynamics of the exchange rate and interest rate to analyze the jump effects on portfolios, consumption, and wealth changes in the expectations regarding the exchange rate's depreciation. Discussing the optimal international portfolio choices with a time-varying investment opportunity set, Ang and Bekaert (2002) found that international diversification was still feasible, although correlations between international financial market returns tended to increase in highly volatile bear markets. Basu et al. (2010) considered the effects of return predictability on international portfolio choices. They documented that the separation of bear and bull markets benefited the making of successful portfolio choices. Das and Uppal (2004) concerned themselves with the tendency of jumps in international equities to occur at the same time across countries, leading to systemic risk, and they examined how systemic risk affected international portfolio choices.

The original works of Merton (1969, 1971) analyzed continuous-time optimal decision-making in terms of consumption and asset allocations in stochastic environments. Many subsequent studies have further considered asset classes and risk sources in stochastic settings. For example, the studies of Ang and Bekaert (2002) and Das and Uppal (2004) focused on the jump risks in the prices of foreign stocks. Fischer (1975) analyzed the index bond under an inflation risk using a stochastic dynamic programming methodology. Larsen (2010) analyzed the optimal investment decisions in an international economy with stochastic interest rates.

Building on the theoretical and empirical results of previous studies, this study develops a continuous-time decision model for analyzing the optimal international portfolio choices. Compared to the previous literature, the present study focuses on the speculative demand, diversified demand, and hedging demands of holding assets. Specifically, we consider whether an increase in domestic stock holdings comes from the relative performances of risky assets, comes from the diversified benefits, or rather comes from a desire to avoid currency risk. Previous studies have stressed how the currency risk affects the holdings of risky assets; however, the present study carefully analyzes the essential changes of the reasons for choosing particular asset holdings. Thus, we can identify *which* demand has a more important influence on holding changes, in which this issue has not been discussed in previous studies.

3. International portfolio choices

This section establishes a continuous time decision model for analyzing international asset allocations reacting to the risk of the exchange rate. We first state several assumptions, and then examine the asset allocations of a representative individual who is exposed to the risk of the exchange rate.

3.1. Assumptions

We consider three representative assets in the financial market: the domestic risk free bond, domestic stock, and foreign stock. The representative domestic individuals usually hold foreign stocks rather than foreign risk free assets if they want to invest international assets. Thus, we ignore foreign risk free bonds in individuals' international portfolios in this study. The price dynamic ($S(t)$) of the domestic stock follows a geometric Brownian motion:

$$dS = \alpha_S S\ dt + \sigma_S S\, dZ_S, \quad S(t=0) = S_0 \tag{1}$$

where α_S and σ_S denote the instantaneous expected growth rate and volatility rate of domestic stock return, respectively. We assume that both α_S and σ_S are constant. dZ_S follows a standard Wiener process.

The price ($B(t)$) of domestic risk free bond is as follows:

$$dB = rBdt, \quad B(t=0) = B_0 \tag{2}$$

where $B(T) = B(t)e^{r(T-t)}$, r denotes the riskless interest rate. The third asset is represented by the foreign stock whose price dynamic ($P(t)$) is as follows:

$$\frac{dP}{P} = \alpha_P\ dt + \sigma_P\, dZ_P, \quad P(t=0) = P_0 \tag{3}$$

where α_P and σ_P are the instantaneous growth rate and volatility rate of domestic stock, and are assumed to be constant. dZ_P also follows a Wiener process. Specifically, because the foreign assets can be valued by a foreign currency, we transfer the values of foreign risky assets denominated by domestic currency. Due to the frequent variance of the exchange rate recently, this study assumes that the exchange rate (E) follows a mixed geometric Brownian motion-Poisson process as follows:

$$dE = (\alpha_E - \lambda\eta)E\ dt + \sigma_E E\, dZ_E + X(t)E(t)\, dN(t), \quad E(t=0) = E_0 \tag{4}$$

where α_E and σ_E denote the instantaneous expected growth rate and volatility rate of exchange rate, respectively. $N(t)$ follows a Poisson process with intensity λ. η denotes the risk premium of a Poisson jump. X denotes the jump size in the exchange rate.

We transfer the price of foreign stock denominated in a foreign currency into the domestic price unit denominated in a domestic currency.

$$Q = P \times E \tag{5}$$

Because both P and E are stochastic processes, through transformation using Ito's Lemma yields the price dynamic (Q) of foreign stock denominated in domestic currency, as follows:

$$dQ = d(P \times E) \tag{6}$$

Further, the return process of foreign stock denominated in domestic currency is rewritten as:

$$\begin{aligned} \frac{dQ}{Q} &= (\alpha_P + \alpha_E - \lambda\eta + \sigma_P\sigma_E\rho_{PE}) \times dt + (\sigma_P\, dZ_P + \sigma_E\, dZ_E) + X(t)\, dN(t) \\ &= (\alpha_Q) \times dt + (\sigma_Q\, dZ_Q) + X(t)\, dN(t) \end{aligned} \tag{7}$$

where $\alpha_Q = \alpha_P + \alpha_E - \lambda\eta + \sigma_P\sigma_E\rho_{PE}$, and $\sigma_Q\, dZ_Q = \sigma_P\, dZ_P + \sigma_E\, dZ_E$.

We assume that the exchange rate risk correlates with the risk of foreign stock returns with the instantaneous coefficient $\rho_{PE}(t)\, dt = E(dZ_E(t)\, dZ_P(t))$, and correlates with the domestic stock risk with the coefficient $\rho_{SE}(t)\, dt = E(dZ_E(t)\, dZ_S(t))$.

Following the assumptions of Merton (1971, 1973), and Copeland et al. (2005), we assume that a perfect market satisfies the following conditions. First, the financial market is frictionless, transparent, and efficient; that is the trading in the financial market has no transaction costs and taxes, and the market information is conveniently available. All assets are allowed to be infinitely divisible and short trading is allowed. Second, a representative individual is a price taker in a competitive market. Third, this rational individual pursues his or her expected utility by trading financial assets under a continuous-time framework.

3.2. International portfolio choices

The financial decisions to be made by the individual considered in this study include the holding weights of domestic stock (w_S), foreign stock (w_Q), and domestic risk free bonds ($w_B = 1 - w_S - w_Q$), respectively. Given the lifetime period $0 \le t \le T$, the dynamic process of wealth capital ($W(t)$) is expressed as follows:

$$\begin{aligned} dW(t) &= [rW + w_S W(\alpha_S - r) + w_Q W(\alpha_Q - r) - uW] \times dt + w_S W \sigma_S \, dZ_S \\ &\quad + (w_Q W \sigma_Q \, dZ_Q + w_Q W X \, dN) + \sigma_W W \, dZ_W, \quad W(t=0) = W_0 \end{aligned} \tag{8}$$

where u denotes an extra payment of asset management and is assumed as a constant percentage of the wealth. Observing Eq. (8), we find that the risk sources come from the risky domestic stock, the foreign stock, and the wealth itself.

The decision-making problem for the individual is how to maximize the lifetime expected utility of his wealth subject to the wealth constraint and exchange rate dynamic. That is to say, the individual allocates his wealth among the various assets to earn revenues for maximizing his objective of expected utility.

$$\underset{\{w_S, w_Q\}}{Max} \; E_0 \left[\int_0^T U(W(t)) \, dt \right] \tag{9}$$

where E_0 denotes the conditional expectation at time 0. $U(\cdot)$ denotes the direct utility function, which is twice continuously differentiable, strictly increasing, and concave. The objective function satisfies the *Inada* conditions of wealth and time as follows:

$$\lim_{W \to 0} U_W(W(t), t) = \lim_{t \to 0} U_t(W(t), t) = \infty, \text{ and}$$

$$\lim_{W \to \infty} U_W(W(t), t) = \lim_{t \to \infty} U_t(W(t), t) = 0$$

The individual pursues the maximum expected utility subject to the dynamics of wealth capital and exchange rate. Thus, the decision-making problem for the individual can be expressed as an indirect utility function ($J(W(t), E, t)$). This function also satisfies the features of being twice continuously differentiable, strictly increasing, and concave.

$$J(W(t), E, t) = \underset{\{w_S, w_Q\}}{Max} \; E_t \left[\int_t^T U(W(t)) \, dt \right] \tag{10}$$

In this problem, the state equations include the exchange rate process and the individual's wealth budget constraint, and the control variables are the holding ratios of the risk free bonds, domestic stock, and foreign stock. Following Bellman's principle of optimality, we can derive a Hamilton–Jacobi–Bellman (henceforth, HJB, see, Kamien and Schwartz, 1991) equation in a continuous-time dynamic model using a stochastic dynamic programming methodology:

$$\begin{aligned} 0 = \; & \underset{w}{Max} \; J_W[rW + w_S W(\alpha_S - r) + w_Q W(\alpha_Q - r) - uW] \\ & + \frac{1}{2} J_{WW}[w_S^2 W^2 \sigma_S^2 + w_Q^2 W^2 \sigma_Q^2 + \sigma_W^2 W^2 + 2 w_S w_Q W^2 \sigma_S \sigma_Q \rho_{SQ} \\ & + 2 w_S W^2 \sigma_S \sigma_W \rho_{SW} + 2 w_Q W^2 \sigma_Q \sigma_W \rho_{QW}] + J_E(\alpha_E - \lambda \times \eta) E + \frac{1}{2} J_{EE}(\sigma_E^2 E^2) \\ & + J_{WE}[\sigma_E E w_S W \sigma_S \rho_{SE} + \sigma_E E w_Q W \sigma_Q \rho_{EQ} + \sigma_E E \sigma_W W \rho_{EQ}] \\ & + J_t + \lambda \times E[J(W \times (1 + w_Q X), E(1 + X), t) - J(W, E, t)] \end{aligned} \tag{11}$$

Next, we differentiate the HJB equation with respect to the holding ratios of risky domestic stock and foreign stock to yield the following Corollary:

Corollary 1. Given that the exchange rate is a mixed geometric Brownian motion-Poisson process, the individual's optimal decision-making regarding allocations in his international portfolio for the purpose of maximizing his expected lifetime utility (10) subject to the wealth budget constraint (8) and the process of exchange rate (4) is as follows:

Optimal weight of risky domestic stock (w_S^*):

$$w_S^* = \frac{1}{1-\rho_{SQ}^2}\left[\frac{-J_W}{J_{WW}W}\left(\frac{(\alpha_S - r)}{\sigma_S^2} - \frac{(\alpha_Q - r)\rho_{SQ}}{\sigma_Q\sigma_S}\right) - \frac{\sigma_W}{\sigma_S}(\rho_{SW} - \rho_{SQ}\rho_{QW}) - \frac{J_{WE}\sigma_E E(\rho_{SE} - \rho_{EQ}\rho_{SQ})}{J_{WW}W\sigma_S} + \frac{\lambda X\rho_{SQ}[J_W(W(1+w_Q^*X), E(1+X), t)]}{J_{WW}W\sigma_Q\sigma_S}\right] \tag{12}$$

Optimal weight of risky foreign stock (w_Q^*):

$$w_Q^* = \left(\frac{1}{1-\rho_{SQ}^2}\right)\left[\frac{-J_W}{J_{WW}W}\left(\frac{\alpha_Q - r}{\sigma_Q^2} - \frac{(\alpha_S - r)}{\sigma_S\sigma_Q}\rho_{SQ}\right) - \frac{\sigma_W}{\sigma_Q}(\rho_{QW} - \rho_{SQ}\rho_{SW}) - \frac{J_{WE}\sigma_E E}{J_{WW}W\sigma_Q}(\rho_{EQ} - \rho_{SQ}\rho_{SE}) - \frac{\lambda X[J_W W(1+w_Q^*X), E(1+X), t)]}{J_{WW}W\sigma_Q^2}\right] \tag{13}$$

Proof: the derivations of optimal weights are listed in Appendix A.

Observing Eq. (12), we find that the optimal holding ratio of domestic stock is composed of several terms. The first term, $((\alpha_S - r)/\sigma_S^2) - ((\alpha_Q - r)\rho_{SQ}/\sigma_Q\sigma_S)$, is the speculative demand for the domestic stock. In the speculative demand, the term $((\alpha_S - r)/\sigma_S^2)$ is the investment performance of domestic stock, and the term $((\alpha_Q - r)\rho_{SQ}/\sigma_Q\sigma_S)$ denotes the investment performance of foreign stock. Therefore, the individual will allocate more wealth on the domestic stock if the performance of the domestic stock is superior to that of the foreign stock. In addition, the speculative demand is correlated with the inverse of the Arrow–Pratt relative risk aversion coefficient $-J_W/J_{WW}W$. The higher the coefficient, the more likely an individual with a low risk aversion attitude toward asset risk will hold domestic stock.

The second term, $(\sigma_W/\sigma_S)(\rho_{SW} - \rho_{SQ}\rho_{QW})$, denotes the diversified demand of an international portfolio. The impacts of asset diversification through the holding of various assets on the stock holdings depend on the direction and size of the correlation coefficients.

The third term, $J_{WE}\sigma_E E(\rho_{SE} - \rho_{EQ}\rho_{SQ})/J_{WW}W\sigma_S$, is the hedging demand for the individual against the volatility risk of the exchange rate. The holding ratios of the stock investment increase with the hedging demand. The fourth term, $\lambda X\rho_{SQ}E[J_W(W(1+w_QX), E(1+X), t)]/J_{WW}W\sigma_Q\sigma_S$, denotes the hedging demand for the individual against the jump risk of the exchange rate. We further analyze these factors to see how they affect the holding ratios of domestic stock with an analytical form later.

Similarity, the holding ratio of foreign stock correlates to several demands, as do the holding weights of domestic stock investments. In Eq. (13), the first term denotes the speculative demand for foreign stock investment, in which the speculative demand depends on the relative performances between the various assets. In addition, the holdings of foreign stock are also positively correlated with the inverse of the Arrow–Pratt relative risk aversion coefficient. The individuals prefer foreign stock if they have a low risk aversion attitude. Specifically, the investors can also adjust the holding ratios of foreign stock depending on the diversified demand, as well as the hedging demands against the risks of the exchange rate. These results imply that investors may change their holdings of foreign stock because of different demands.

To yield an explicit solution in terms of optimal holdings, this study assumes that the individual's utility presents a constant relative risk aversion (CRRA) form as follows:

$$J(W, E, t) = \frac{1}{1-\gamma}\left(\frac{W}{E}\right)^{1-\gamma}\phi(t) \tag{14}$$

where, γ denotes the risk aversion coefficient, and $1-\gamma$ denotes the Arrow-Pratt relative risk aversion measure, while $\phi(t)$ denotes a risk premium changed over time. Taking the partial derivations of the indirect utility function (14) into Corollary 1, we obtain Corollary 2:

Corollary 2. Given that the exchange rate following a mixed geometric Brownian motion-Poisson process, the optimal decision-making of an international investor with a CRRA utility for maximizing expected lifetime utility (10) subject to the wealth budget constraint (8) and the exchange rate dynamic (4) is as follows:

Optimal holding ratio of domestic stock (w_S^*):

$$w_S^* = \left(\frac{1}{1-\rho_{SQ}^2}\right)\left[\frac{1}{\gamma}\left(\frac{\alpha_S - r}{\sigma_S^2} - \frac{\alpha_Q - r}{\sigma_Q\sigma_S}\rho_{SQ}\right) - \frac{\sigma_W}{\sigma_S}(\rho_{SW} - \rho_{SQ}\rho_{QW}) + \frac{\gamma-1}{\gamma} \times \frac{\sigma_E}{\sigma_S}(\rho_{SE} - \rho_{EQ}\rho_{SQ}) - \frac{\lambda\rho_{SQ}X(1+w_Q^*X)^{-\gamma}(1+X)^{\gamma-1}}{\gamma\sigma_Q\sigma_S}\right] \quad (15)$$

Optimal holding ratios of foreign stock (w_Q^*):

$$w_Q^* = \left(\frac{1}{1-\rho_{SQ}^2}\right)\left[\frac{1}{\gamma}\left(\frac{\alpha_Q - r}{\sigma_Q^2} - \frac{\alpha_S - r}{\sigma_S\sigma_Q}\rho_{SQ}\right) - \frac{\sigma_W}{\sigma_Q}(\rho_{QW} - \rho_{SQ}\rho_{SW}) + \frac{\gamma-1}{\gamma} \times \frac{\sigma_E}{\sigma_Q}(\rho_{EQ} - \rho_{SQ}\rho_{SE}) + \frac{\lambda X(1+w_Q^*X)^{-\lambda}(1+X)^{\gamma-1}}{\gamma\sigma_Q^2}\right] \quad (16)$$

Proof: Please see Appendix B.

Corollary 2 suggests that the optimal holding of risky assets is correlated with the relative performances of risky assets, assets' correlations, investor's risk preferences, and the risks of the exchange rate. Moreover, the instantaneous volatility and jump risks can affect the holdings. The holding of domestic stock correlates to the risk of the exchange rate due to fact that the individual's portfolio is adversely affected by the risk of the exchange rate; thus, the individual adjusts the optimal weights of these assets as a way of reacting to these risks. Specifically, the exchange rate is not in and of itself the matter in the international portfolio choices; however, it is the risks associated with the exchange rate (both volatility risk and jump risk) that are critical.

The holding rates of risky assets can further be divided into several components, as in Eqs. (15) and (16). These components are the speculative demand, diversified demand, hedging demand against the volatility risk of the exchange rate, and hedging demand against the jump risk of the exchange rate, in that order. The first term involves the relative performances of various assets, which the investors hold more of those assets with higher performance. The second term involves the diversification effect of portfolios among these assets with different correlation coefficients. That is to say, investors allocate assets according to the correlation degree (size and direction) of risky assets. The third term denotes the holding demand for investors avoiding the adverse effect of volatility risk of the exchange rate. The fourth term denotes the holding demand for investors considering the jump risk of the exchange rate. Each demand has its own different features, and each demand is affected by different factors. For example, the jump probability of the exchange rate does not affect the speculative demand, but affects the hedging demand against the jump risk.

Next, we observe the relationship between the optimal holdings of risky assets as follows:

$$w_S^* = \frac{-J_W(\alpha_S - r)}{J_{WW}W\sigma_S^2} - \left(\frac{\sigma_Q\rho_{SQ}}{\sigma_S}\right)w_Q^* - \frac{\sigma_W\rho_{SQ}}{\sigma_S} - \frac{J_{WE}\sigma_E E\rho_{SE}}{J_{WW}W\sigma_S} \quad (17)$$

Given other factors, the holding weight of domestic stock is negatively correlated with the holding weight of foreign stock if the correlation between domestic and foreign risky assets is positive. That is to say, if two risky assets are substitutes, a decrease in the holding of one asset causes an increase

Table 1
Optimal holdings and expected return and volatility of domestic stock return.

Panel A: w_S	α_S				
σ_S	0.02	0.04	0.06	0.08	0.10
0.15	0.1129	0.5656	1.0183	1.4709	1.9236
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0314	−0.0358	−0.0407	−0.0465	−0.0529
	−0.0296	0.4187	0.8664	1.3134	1.7596
0.20	0.0422	0.2962	0.5515	0.8061	1.0608
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0229	−0.0246	−0.0265	−0.0285	−0.0307
	−0.0640	0.1889	0.4417	0.6943	0.9467
0.25	0.0134	0.1764	0.3394	0.5023	0.6653
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0181	−0.0189	−0.0198	−0.0268	−0.0218
	−0.0713	0.0908	0.2528	0.4149	0.5768
0.30	0.0001	0.1130	0.2262	0.3394	0.4525
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0150	−0.0155	−0.0160	−0.0165	−0.0170
	−0.0766	0.0420	0.1547	0.2673	0.3800
Panel B: w_Q					
0.15	0.3948	0.3338	0.2728	0.2117	0.1507
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1178	0.1341	0.1528	0.1742	0.1984
	0.6793	0.6345	0.5922	0.5526	0.5157
0.20	0.4082	0.3738	0.3395	0.3052	0.2708
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1145	0.1231	0.1324	0.1425	0.1534
	0.6893	0.6636	0.6386	0.6144	0.5909
0.25	0.4144	0.3924	0.3704	0.3484	0.3265
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1130	0.1184	0.1240	0.1299	0.1362
	0.6941	0.6774	0.6611	0.6450	0.6293
0.30	0.4177	0.4025	0.3872	0.3719	0.3567
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1122	0.1159	0.1197	0.1236	0.1277
	0.6966	0.6850	0.6735	0.6622	0.6510

Notes: The table lists the speculative demand, diversified demand, hedging demand against to the volatility risk of the exchange rate, hedging demand against the jump risk, and total holding rate, in that order. The parameters are set as follows: $\alpha_P = 0.04$, $\alpha_P = 0.2$, $\alpha_E = 0.04$, $\alpha_E = 0.2$, $\gamma = 2$, $\lambda = 0.01$, $\rho_{SP} = 0.1$, $\rho_{SE} = 0.1$, $\eta = 0.05$, $r = 0.01$, $\sigma_w = 0.2$, $\rho_{SW} = 0.1$, $\rho_{PE} = 0.1$, $\rho_{EW} = 0.1$, $\rho_{PW} = 0.1$, $X = 30$.

in the holding of another asset. Moreover, the individual may hold the domestic stock rather than the foreign stock if he is exposed to the risk of the exchange rate because the individual reduces his holdings of foreign risky assets as the risk of the exchange rate increases.

4. Numerical examples

To discuss how these several parameters affect an individual's optimal holdings of domestic stock and foreign stock, we use numerical examples. Thus, we can find out the impacts (i.e., the direction and the sensitivity) of the features of risky assets and the exchange rate on portfolio choices. Moreover, we can examine which demands are more important in the determination of asset allocation.

Table 2
Optimal holdings, expected return and volatility of foreign stock return.

Panel A: w_s	α_P				
σ_P	0.02	0.04	0.06	0.08	0.10
0.15	0.3135	0.2875	0.2615	0.2355	0.2095
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0249	−0.0187	−0.0142	−0.0111	−0.0088
	0.2001	0.1802	0.1587	0.1358	0.1121
0.20	0.3200	0.2969	0.2737	0.2506	0.2274
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0315	−0.0246	−0.0193	−0.0154	−0.0124
	0.2052	0.1889	0.1710	0.1519	0.1317
0.25	0.3264	0.3060	0.2856	0.2653	0.2449
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0382	−0.0309	−0.0251	−0.0204	−0.0168
	0.2098	0.1966	0.1821	0.1664	0.1497
0.30	0.3320	0.3141	0.2961	0.2782	0.2603
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0445	−0.0372	−0.0311	−0.0260	−0.0218
	0.2133	0.2027	0.1909	0.1781	0.1643
Panel B: w_Q					
0.15	0.3391	0.4878	0.6364	0.7851	0.9337
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1421	0.1067	0.0813	0.0632	0.0501
	0.7019	0.8152	0.9850	1.0690	1.2046
0.20	0.2581	0.3738	0.4816	0.6053	0.7211
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1577	0.1231	0.0967	0.0768	0.0619
	0.5825	0.6636	0.7530	0.8488	0.9496
0.25	0.1954	0.2859	0.3765	0.4670	0.5575
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1698	0.1375	0.1115	0.0909	0.0746
	0.4915	0.5498	0.6143	0.6841	0.7584
0.30	0.1483	0.2200	0.2916	0.3633	0.4350
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1779	0.1488	0.1242	0.1038	0.0871
	0.4229	0.4655	0.5127	0.5639	0.6189

Notes: The table lists the speculative demand, diversified demand, hedging demand against the volatility risk of the exchange rate, hedging demand against the jump risk, and total holding rate, in that order. The parameters are set as follows: $\alpha_S = 0.04$, $\alpha_S = 0.2$, $\alpha_E = 0.04$, $\alpha_E = 0.2$, $\gamma = 2$, $\lambda = 0.01$, $\rho_{SP} = 0.1$, $\rho_{SE} = 0.1$, $\eta = 0.05$, $r = 0.01$, $\sigma_w = 0.2$, $\rho_{SW} = 0.1$, $\rho_{PE} = 0.1$, $\rho_{EW} = 0.1$, $\rho_{PW} = 0.1$, $X = 30$.

4.1. Domestic stock

First, we analyze how the risk-return features of domestic stock affect the holding ratios of risky assets. Table 1 presents the extent to which optimal weights of domestic and foreign stocks vary by the expected return rate and volatility rate of domestic stock in Panels A and B, respectively. As we have analyzed in the previous section, the holdings of domestic stock increase with the expected return rate (α_S) of domestic stock in Panel A, and the holdings of foreign stock decrease with the expected return rate of domestic stock in Panel B. The reason is that the individuals are attracted to invest in domestic stock due to its relative performance.

Table 1 also shows that the volatility rate (σ_S) of domestic stock has an adverse effect on the holdings of domestic stock, and has a favorable effect on the holdings of foreign stock. The individual tends to avoid an investment with a high risk as the volatility rate of a domestic asset increases. Thus, the volatility rate of domestic stock benefits the holding weight of foreign stock.

Specifically, we observe the holding components. The speculative demands vary obviously with both the expected return and the volatility rate of domestic assets, yet the other three demands exhibit almost no change whatsoever in response to the return and risk of domestic assets. That is to say, the changes in total holding rates of risky assets come from the changes of speculative demands, and do not come from the changes associated with the other three demands. This is because the increase in the expected return rate or the decrease in the volatility rate of domestic asset mainly benefits the speculative demand of domestic stock, and inhibits the speculative demand of foreign stock. The risk-return features of domestic stock change the relative performances of various assets in international portfolios.

4.2. Foreign stock

Next, we are interested in examining how the expected return and volatility rate of foreign stock change the portfolio selections in Table 2. As we expected, the expected return (α_P) of foreign stock can benefit the holding weights of foreign stock, and can decrease the holdings of domestic stock. In addition, the volatility rate (σ_P) of the foreign stock reduces the holding weight of foreign stock, and increases the holding ratio of domestic stock. That is, the individual transfers foreign stock investments to domestic stock investments as the foreign stock's risk increases. In other word, the individual would prefer to hold domestic stock as the risk of foreign stock increases. Specifically, the sensitively of holding rates of foreign stock is more obvious than that of domestic stock. In short, considering both foreign stock factors (expected return and volatility rate), we find that the holdings of foreign stock increase with the investment performance of foreign stock, and vice versa.

Moreover, we find that the changes of holding rates of both risky assets correlate to the changes of speculative demands, because the expected return rate and volatility rate of foreign stock closely affect the speculative demands. That is to say, if the asset features of foreign stock change, the reason for portfolio selection changes as a result of the speculative demands, not as a result of other demands.

4.3. Correlations

Next, this study analyzes how the correlations affect portfolio selections. Table 3 shows that the holding rates of both types of assets vary with the correlations (ρ_{SP} and ρ_{SE}). Whatever the holdings of foreign stock or domestic stock are, both holding rates are negatively related with the correlations. If the correlation coefficient (ρ_{SP}) between domestic stock and foreign stock tends to be negative, the holding rates of domestic stock and foreign stock increase gradually. In addition, the two holdings rates also present a negative relation with the correlation coefficient (ρ_{SE}) as between domestic stock and exchange rate. A negative and strong correlation coefficient can promote the diversification effect, and a positive and strong correlation coefficient can depress the diversification effect. Specifically, all holding components in the holding rates vary with correlations because the correlations change each demand.

4.4. Exchange rate

Turning to the exchange rate, we first analyze how the risk-return features affect the holding rates. A higher expected return of the exchange rate increases the holdings of foreign stock, but decreases the holding weights of domestic stock, as shown in Table 4. This is because the values of foreign stock are denominated in the foreign currency. Consequently, an increase in the expected return of the exchange rate will appreciate the value of foreign stock. Otherwise, the volatility rate of the exchange rate is adverse for holding foreign stock; thus, the individual prefers domestic stock.

Table 3
Optimal holdings and correlations.

Panel A: w_S	ρ_{SE}				
ρ_{SP}	−0.2	−0.1	0.0	0.1	0.2
−0.2	0.5846	0.5216	0.4670	0.4187	0.3750
	−0.1471	−0.1327	−0.1204	−0.1096	−0.1000
	0.0000	0.0261	0.0509	0.0753	0.1000
	0.0377	0.0294	0.0204	0.0107	0.0000
	0.4752	0.4444	0.4180	0.3951	0.3750
−0.1	0.5216	0.4670	0.4187	0.3750	0.3348
	−0.1327	−0.1204	−0.1096	−0.1000	−0.0913
	−0.0261	0.0000	0.0251	0.0500	0.0753
	0.0298	0.0206	0.0107	0.0000	−0.0118
	0.3927	0.3673	0.3449	0.3250	0.3069
0.0	0.4670	0.4187	0.3750	0.3348	0.2969
	−0.1204	−0.1096	−0.1000	−0.0913	−0.0833
	−0.0509	0.0108	0.0000	0.0251	0.5059
	0.0208	0.0108	0.0000	0.0118	−0.0249
	0.3165	0.2948	0.2750	0.2568	0.2396
0.1	0.4187	0.3750	0.3348	0.2969	0.2604
	−0.1096	−0.1000	−0.0913	0.0833	−0.0758
	−0.0753	−0.0500	−0.0251	0.0000	0.0261
	0.0108	0.0000	−0.0117	−0.0246	−0.0391
	0.2446	0.2250	0.2066	0.1889	0.1715
Panel B: w_Q					
−0.2	0.5594	0.5145	0.4768	0.4449	0.4176
	−0.1176	−0.1090	−0.1019	−0.0959	−0.0909
	0.2500	0.2536	0.2546	0.2534	0.2500
	0.0943	0.0980	0.1022	0.1069	0.1125
	0.7861	0.7571	0.7318	0.7094	0.6892
−0.1	0.5145	0.4768	0.4449	0.4176	0.3941
	−0.1090	−0.1019	−0.0959	−0.0909	−0.0868
	0.2464	0.2500	0.2511	0.2500	0.2466
	0.0994	0.1031	0.1074	0.1125	0.1184
	0.7514	0.7281	0.7076	0.6892	0.6727
0.0	0.4768	0.4449	0.4176	0.3941	0.3738
	−0.1019	−0.0959	−0.0909	−0.0868	−0.0833
	0.2454	0.2489	0.2500	0.2489	0.2454
	0.1041	0.1079	0.1125	0.1179	0.1243
	0.7245	0.7058	0.6892	0.6741	0.6602
0.1	0.4449	0.4176	0.3941	0.3738	0.3563
	−0.0959	−0.0909	−0.0868	−0.0833	−0.0806
	0.2466	0.2500	0.2511	0.2500	0.2464
	0.1085	0.1125	0.1173	0.1231	0.1302
	0.7041	0.6892	0.6758	0.6636	0.6524

Notes: The table lists the speculative demand, diversified demand, hedging demand against the volatility risk of the exchange rate, hedging demand against the jump risk, and total holding rate, in that in order. The parameters are set as follows: $\alpha_S = 0.04$, $\alpha_S = 0.2$, $\alpha_P = 0.04$, $\alpha_P = 0.2$, $\alpha_E = 0.04$, $\alpha_E = 0.2$, $\gamma = 2$, $\lambda = 0.01$, $\eta = 0.05$, $r = 0.01$, $\sigma_w = 0.2$, $\rho_{SW} = 0.1$, $\rho_{PE} = 0.1$, $\rho_{EW} = 0.1$, $\rho_{PW} = 0.1$, $X = 30$.

Moreover, this study further examines the change trends of the various demands of asset holdings through exchange rate changes. First, the speculative demands of foreign stock and domestic stock change obviously with the expected return and volatility rate of the exchange rate. Both risk-return factors of the exchange rate directly change the relative performances of risky assets in the component of speculative demand. Second, the diversified demands do not vary with the expected return of the exchange rate, but vary with the volatility rate of the exchange rate, as shown in Table 4, because the volatility risk can change the risk diversification effect in a portfolio. Third, the expected return does also not change the hedging demand against the volatility risk of the exchange rate; however, the hedging demand of foreign stocks against the volatility risk increases with the volatility rate of the exchange rate. The reason is that we assume the investor is a risk-lover in this numerical example

Table 4
Optimal holdings, expected return and volatility of exchange rate.

Panel A: w_S	α_E				
σ_E	0.02	0.04	0.06	0.08	0.10
0.15	0.3135	0.2875	0.2615	0.2355	0.2095
	−0.0836	−0.0836	−0.0836	−0.0836	−0.0836
	0.0050	0.0050	0.0050	0.0050	0.0050
	−0.0320	−0.0239	−0.0180	−0.0137	−0.0107
	0.2029	0.1850	0.1649	0.1432	0.1202
0.20	0.3200	0.2969	0.2737	0.2506	0.2274
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.0000	0.0000	0.0000	0.0000	0.0000
	−0.0315	−0.0246	−0.0193	−0.0154	−0.0124
	0.2052	0.1889	0.1710	0.1519	0.1317
0.25	0.3264	0.3060	0.2856	0.2653	0.2449
	−0.0835	−0.0835	−0.0835	−0.0835	−0.0835
	−0.0021	−0.0021	−0.0021	−0.0021	−0.0021
	−0.0302	−0.0245	−0.0200	−0.0164	−0.0136
	0.2076	0.1929	0.1771	0.1603	0.1427
0.30	0.3320	0.3141	0.2961	0.2782	0.2603
	−0.0839	−0.0839	−0.0839	−0.0839	−0.0839
	−0.0097	−0.0097	−0.0097	−0.0097	−0.0097
	−0.0285	−0.0239	−0.0201	−0.0170	−0.0145
	0.2099	0.1967	0.1825	0.1677	0.1523
Panel B: w_Q					
0.15	0.3391	0.4878	0.6364	0.7851	0.9337
	−0.0936	−0.0936	−0.0936	−0.0936	−0.0936
	0.1856	0.1856	0.1856	0.1856	0.1856
	0.1831	0.1366	0.1028	0.0785	0.0612
	0.6142	0.7164	0.8312	0.9556	1.0869
0.20	0.2581	0.3738	0.4896	0.6053	0.7211
	−0.0833	−0.0833	−0.0833	−0.0833	−0.0833
	0.2500	0.2500	0.2500	0.2500	0.2500
	0.1577	0.1231	0.0967	0.0768	0.0619
	0.5825	0.6636	0.7530	0.8488	0.9496
0.25	0.1954	0.2859	0.3765	0.4670	0.5575
	−0.0733	−0.0733	−0.0733	−0.0733	−0.0733
	0.3004	0.3004	0.3004	0.3004	0.3004
	0.1343	0.1089	0.0888	0.0730	0.0606
	0.5568	0.6219	0.6924	0.7671	0.8452
0.30	0.1483	0.2200	0.2916	0.3633	0.4350
	−0.0645	−0.0645	−0.0645	−0.0645	−0.0645
	0.3387	0.3387	0.3387	0.3387	0.3387
	0.1141	0.0954	0.0802	0.0679	0.0579
	0.5365	0.5896	0.6461	0.7054	0.7670

Notes: The table lists the speculative demand, diversified demand, hedging demand against the volatility risk of the exchange rate, hedging demand against the jump risk, and total holding rate, in that order. The parameters are set as follows: $\alpha_S = 0.04$, $\alpha_S = 0.2$, $\alpha_P = 0.04$, $\alpha_P = 0.2$, $\gamma = 2$, $\lambda = 0.01$, $\rho_{SP} = 0.1$, $\rho_{SE} = 0.1$, $\eta = 0.05$, $r = 0.01$, $\sigma_w = 0.2$, $\rho_{SW} = 0.1$, $\rho_{PE} = 0.1$, $\rho_{EW} = 0.1$, $\rho_{PW} = 0.1$, $X = 30$.

($\gamma = 2$). If the investor is risk-averse, the holding weight of foreign stock decreases, as shown in Table 5, which we will discuss in more detail later. Fourth, both the expected return and the volatility rate of the exchange rate affect the hedging demand against the jump risk slightly. Therefore, a change in the total demands is mainly caused by the speculative demands if the expected return rate and volatility rate of the exchange rate change.

Next, we discuss how the jump intensity (λ) of the Poisson process affects the holding rates. Table 5 shows the holding rates of a representative risk-loving individual and a representative risk-averse individual. As the jump intensity increases, the holdings of foreign stock increase for the risk-loving individual, and decreases for the risk-averse individual. That is to say, the risk-loving individual would like to take more jump risk of the exchange rate by investing in foreign stock. In addition,

Table 5
Optimal holdings, risk aversion coefficient and jump probability.

λ	$\gamma = 2$ (risk-lover)	$\gamma = -2$ (risk-averse)
Panel A: w_S		
0.001	0.2964	−0.2964
	−0.0833	−0.0833
	0.0000	0.0000
	−0.0034	0.0001
	0.2096	−0.3796
0.005	0.2966	−0.2966
	−0.0833	−0.0833
	0.0000	0.0000
	−0.0143	0.0005
	0.1990	−0.3794
0.010	0.2969	−0.2969
	−0.0833	−0.0833
	0.0000	0.0000
	−0.0246	0.0011
	0.1889	−0.3791
0.050	0.2992	−0.2992
	−0.0833	−0.0833
	0.0000	0.0000
	−0.0710	0.0051
	0.1448	−0.3774
Panel B: w_Q		
0.001	0.3765	−0.3765
	−0.0833	−0.0833
	0.2500	0.2500
	0.0170	−0.0005
	0.5601	0.2897
0.005	0.3753	−0.3753
	−0.0833	−0.0833
	0.2500	0.2500
	0.0715	−0.0027
	0.6134	0.2887
0.010	0.3738	−0.3738
	−0.0833	−0.0833
	0.2500	0.2500
	0.1231	−0.0054
	0.6636	0.2874
0.050	0.3623	−0.3623
	−0.0833	−0.0833
	0.2500	0.2500
	0.3552	−0.0256
	0.8841	0.2788

Notes: The table lists the speculative demand, diversified demand, hedging demand against the volatility risk of the exchange rate, hedging demand against the jump risk, and total holding rate, in that order. The parameters are set as follows: $\alpha_S = 0.04$, $\alpha_S = 0.2$, $\alpha_P = 0.04$, $\alpha_P = 0.2$, $\alpha_E = 0.04$, $\alpha_E = 0.2$, $\rho_{SP} = 0.1$, $\rho_{SE} = 0.1$, $\eta = 0.05$, $r = 0.01$, $\sigma_w = 0.2$, $\rho_{SW} = 0.1$, $\rho_{PE} = 0.1$, $\rho_{EW} = 0.1$, $\rho_{PW} = 0.1$, $X = 30$.

the speculative demand is insensitive to the jump intensity. The hedging demands against the jump risk, on the other hand, are sensitive to the jump intensity. The changes of holding rates originate from the hedging demand against the jump risk if the jump intensity changes. Specifically, the speculative demands of the risk-loving individual are the same, in terms of amount, as those of the risk-averse individual, but they have a different direction. The risk-loving individual holds a positive weight for the risky assets while the risk-averse individual takes a short-selling strategy. The diversified demands keep constant, whatever the changes of risk preference or jump risk. This is because of the diversified strategy only being related with the volatility risk and the correlations among assets.

5. Conclusions

This study solves the optimal decision-making problem of international portfolio choices framed in a stochastic decision model. We carefully examine the holding rates of risky assets by observing the holding components of speculative demand, diversified demand, and hedging demands. Thus, we can derive a closed-form solution of the optimal holding rates of domestic and foreign stocks. Furthermore, this study can answer the question of *which* demands cause the changes in the holding rates of risky assets, and identify *which* factors can affect these specific demands.

The main implication of this study is as follows. We document several demands that affect an investor's holdings of foreign stock or domestic stock in his portfolio. The speculative demand for one asset comes from the investments of various assets with relatively better performances. The investor forms his diversified demand due to the diversification effect among the various assets of the portfolio. The third term is the hedging demand for investors to avoid the adverse effect of the volatility risk of the exchange rate. The fourth term is the hedging demand for investors considering the presence of the jump risk of the exchange rate. The changes of the total holding rates of risky assets can originate from one of the above demands. Specifically, some factors can affect one of the above demands, without changing the others.

Compared to previous research, although several studies have documented the differences between speculative demand and hedging demand in portfolio choices, we are the first to propose the various demands of holding assets with a multiple-component form and with a closed-form solution form. Thus, we are the first to analyze *which* demands have an influence on the changes of holding weights. Furthermore, this study also specifically identifies *which* factors can affect *which* demands of holding risky assets. This study therefore advances a new point of view in the field of portfolio selection literature, since we can now answer several important questions that previous studies have not examined.

Appendix A. Proof of Corollary 1

Using Bellman's principle of optimality, we differentiate the HJB equation (11) with respect to the domestic stock holding and foreign stock holding, respectively. Thus, the first-order conditions of optimality are as follows:

$$J_W[(\alpha_S - r)W] + J_{WW}[w_S^* W^2 \sigma_S^2 + w_Q^* W^2 \sigma_S \sigma_Q \rho_{SQ} + W^2 \sigma_S \sigma_W \rho_{SW}] + J_{WE}[\sigma_E E W \sigma_S \rho_{SE}] = 0 \tag{A1}$$

$$\begin{aligned} J_W[W(\alpha_Q - r)] &+ J_{WW}[w_Q^* W^2 \sigma_Q^2 + w_S^* W^2 \sigma_S \sigma_Q \rho_{SQ} + W^2 \sigma_Q \sigma_W \rho_{WQ}] + J_{WE}[\sigma_E E W \sigma_Q \rho_{EQ}] \\ &+ \lambda W X (J_W(W(1 + w_Q^* X), E(1 + X), t)) = 0 \end{aligned} \tag{A2}$$

Solving the above two equations yields the following equations:

$$w_S^* = \frac{-J_W(\alpha_S - r)}{J_{WW} W \sigma_S^2} - \frac{w_Q^* \sigma_Q \rho_{SQ}}{\sigma_S} - \frac{\sigma_W \rho_{SQ}}{\sigma_S} - \frac{J_{WE} \sigma_E E \rho_{SE}}{J_{WW} W \sigma_S} \tag{A3}$$

$$w_Q^* = \frac{-J_W(\alpha_Q - r)}{J_{WW} W \sigma_Q^2} - \frac{w_S^* \sigma_S \rho_{SQ}}{\sigma_Q} - \frac{\sigma_W \rho_{WQ}}{\sigma_Q} - \frac{J_{WE} \sigma_E E \rho_{EQ}}{J_{WW} W \sigma_Q} - \frac{\lambda X [J_W(W(1 + w_Q^* X), E(1 + X), t)]}{J_{WW} W \sigma_Q^2} \tag{A4}$$

We further simplify Eqs. (A3) and (A4), and then yield Eqs. (12) and (13).

Appendix B. Proof of Corollary 2

Using the CRRA utility function (14), we differentiate the indirect utility function with respect to wealth capital and exchange rate, yielding the following partial derivations:

$$J_W = W^{-\gamma}E^{\gamma-1}\phi \tag{B1}$$

$$J_{WW} = -\gamma W^{-\gamma-1}E^{\gamma-1}\phi \tag{B2}$$

$$J_E = -W^{1-\gamma}E^{\gamma-2}\phi \tag{B3}$$

$$J_{EE} = (\gamma-2)W^{1-\gamma}E^{\gamma-3}\phi \tag{B4}$$

$$J_{WE} = (\gamma-1)W^{-\gamma}E^{\gamma-2}\phi \tag{B5}$$

$$J_{WE} = \frac{1}{1-\gamma}W^{1-\gamma}E^{\gamma-1}\phi' \tag{B6}$$

Taking above equations into Corollary 1, we obtain the optimal holdings of domestic stock (w_S^*) and foreign stock (w_Q^*).

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