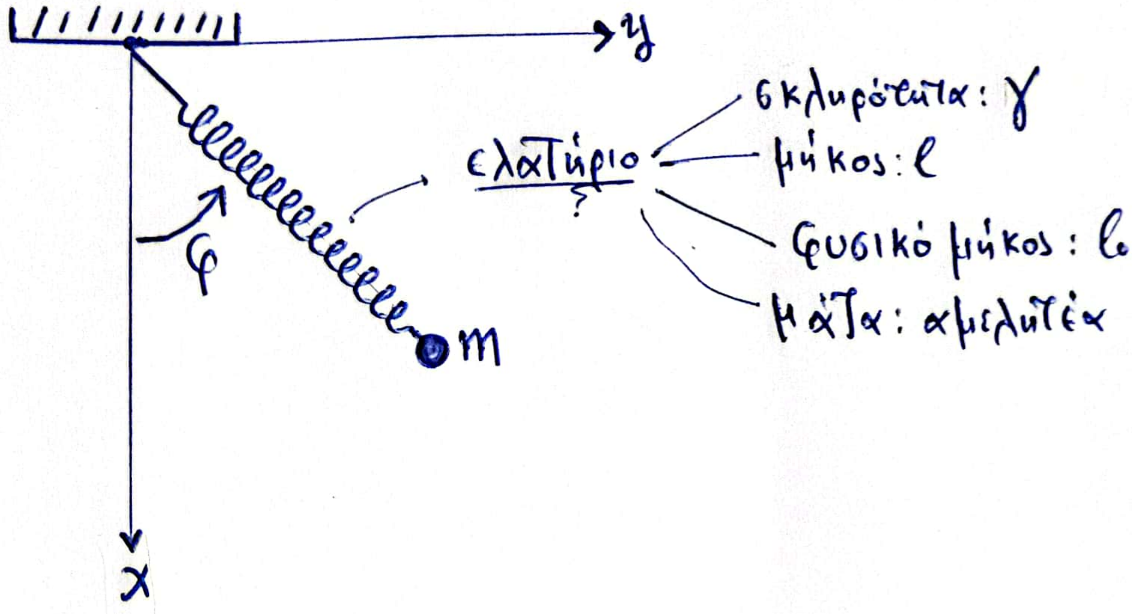
Ελαστικό εκκρεμές



- $\left.\begin{aligned} x &= \ell\,\text{συν}\varphi \\ y &= \ell\,\text{ημ}\varphi \end{aligned}\right\}$ (1) $\Rightarrow$ $\left.\begin{aligned} \dot{x} &= \dot{\ell}\,\text{συν}\varphi - \ell\,\text{ημ}\varphi\cdot\dot{\varphi} \\ \dot{y} &= \dot{\ell}\,\text{ημ}\varphi + \ell\,\text{συν}\varphi\cdot\dot{\varphi} \end{aligned}\right\}$ (2)

- $\upsilon^2 = (\dot{x})^2 + (\dot{y})^2 \overset{(2)}{=} \dots = (\dot{\ell})^2 + \ell^2(\dot{\varphi})^2$ (3) $\Rightarrow$

  $\Rightarrow K = \frac{1}{2} m\left[(\dot{\ell})^2 + \ell^2(\dot{\varphi})^2\right]$ (4)

- $U = -m\cdot g\cdot x + \frac{1}{2}\gamma(\ell-\ell_0)^2$ (5)

- Έστω το σύστημα μονάδων $\boxed{m = g = \ell_0 = 1}$ (6)

- $L \equiv K - U \xRightarrow[(6)]{(4)(5)} L = \frac{1}{2}\dot{\ell}^2 + \ell^2\dot{\varphi}^2 + x - \frac{1}{2}\gamma(\ell-1)^2$ (7)

- Παρατήρηση: $\left\{\begin{array}{l} \text{Πλήθος σωματιδίων} : N = 1 \\ \text{γενικευμένες συντεταγμένες} : \ell \ \text{κ'}\ \varphi \\ \text{βαθμοί ελευθερίας} : \kappa = 2 \end{array}\right.$

- Εφεξής: $\int \dots \equiv \int_{t=t_A}^{t=t_B} dt\,\{\dots\}$ (8)

▶ $\delta I = \delta \int_{t=t_A}^{t=t_B} dt\{L(\varphi,\ell,\dot\varphi,\dot\ell)\} \overset{(8)}{\equiv} \delta \int L(\varphi,\ell,\dot\varphi,\dot\ell) \Rightarrow$

$$\Rightarrow \delta I = \delta \int L = \int \delta L \overset{(1)}{\underset{(7)}{=}} \int \delta \left[\frac{\dot\ell^2}{2} + \frac{\ell^2\dot\varphi^2}{2} + \ell\,\sigma\upsilon\nu\varphi - \frac{\gamma}{2}(\ell-1)^2 \right]$$

$$\boxed{\delta L(\varphi,\ell,\dot\varphi,\dot\ell) = \frac{\partial L}{\partial \varphi}\delta\varphi + \frac{\partial L}{\partial \ell}\delta\ell + \frac{\partial L}{\partial \dot\varphi}\delta\dot\varphi + \frac{\partial L}{\partial \dot\ell}\delta\dot\ell}$$

$$= \int \dot\ell\,\delta\dot\ell + \ell\dot\varphi^2\delta\ell + \dot\varphi\ell^2\delta\dot\varphi + \sigma\upsilon\nu\varphi\,\delta\ell - \ell\,\eta\mu\varphi\,\delta\varphi - \gamma(\ell-1)\delta\ell$$

$$\boxed{\begin{array}{l} \dot\ell\,\delta\dot\ell = \dot\ell\,(\delta\ell)^{\cdot} = (\dot\ell\,\delta\ell)^{\cdot} - (\dot\ell)^{\cdot}\delta\ell = (\dot\ell\,\delta\ell)^{\cdot} - \ddot\ell\,\delta\ell \\ \kappa' \text{ ομοίως} \quad \dot\varphi\ell^2\delta\dot\varphi = (\dot\varphi\ell^2\delta\varphi)^{\cdot} - (\ddot\varphi\ell^2 + 2\dot\varphi\ell\dot\ell)\delta\varphi \end{array}}$$

$$\overset{(8)}{=} \cancelto{0}{\left[\dot\ell\,\delta\ell\right]_{t=t_A}^{t=t_B}} + \cancelto{0}{\left[\dot\varphi\ell^2\delta\varphi\right]_{t=t_A}^{t=t_B}} +$$

$$+ \int_{t=t_A}^{t=t_B} dt \left\{ \left[-\ddot\ell + \ell\dot\varphi^2 + \sigma\upsilon\nu\varphi - \gamma(\ell-1)\right]\delta\ell + \right.$$

$$\left. + \left[-\ddot\varphi\ell^2 - 2\dot\varphi\dot\ell\ell - \ell\,\eta\mu\varphi\right]\delta\varphi \right\} \quad (9)$$

▶ Η απαλοιφή των όρων $[-//-]_{t_A}^{t_B}$ έγινε επειδή $\delta\varphi(t_A) = \delta\varphi(t_B) = \delta\ell(t_A) = \delta\ell(t_B) = 0$.

▶ Σύμφωνα με την αρχή του Χάμιλτον $\delta I = 0$ για τυχαία $\delta\varphi$ κ' $\delta\ell$ άρα (9) $\Rightarrow$

$$\left\{ \begin{array}{ll} \ddot\ell - \ell\dot\varphi^2 - \sigma\upsilon\nu\varphi + \gamma(\ell-1) = 0 & (10) \\ \ddot\varphi + 2\dfrac{\dot\ell}{\ell}\dot\varphi + \dfrac{1}{\ell}\eta\mu\varphi = 0 & (11) \end{array} \right.$$