Σύνθεση ν συγγραμμικών α.α.τ. ίδιου πλάτους, ίδιας συχνότητας κ' ίσων διαδοχικών διαφορών φάσεων

Δ	$x_1 = \alpha \cos \omega t$ $x_2 = \alpha \cos(\omega t + \delta)$ $x_3 = \alpha \cos(\omega t + 2\delta)$ $\vdots$ $x_\nu = \alpha \cos[\omega t + (\nu-1)\delta]$
Z	$\tilde{x} \equiv x_1 + x_2 + \cdots + x_\nu = ;$

Λύση: Παρατηρώ ότι $x_1 = Re(z_1)$ όπου $z_1 \equiv \alpha e^{i\omega t} =$
$= \alpha \cos\omega t + i\alpha \sin\omega t.$

Ομοίως παρατηρώ ότι
$x_2 = Re(z_2)$ όπου $z_2 \equiv \alpha e^{i\omega t + i\delta}$
$x_3 = Re(z_3)$ όπου $z_3 \equiv \alpha e^{i(\omega t + 2\delta)}$ (1)
$\vdots$
$x_\nu = Re(z_\nu)$ όπου $z_\nu \equiv \alpha e^{i[\omega t + (\nu-1)\delta]}$

Συνεπώς: $\tilde{x} = x_1 + x_2 + \cdots + x_\nu \overset{(1)}{=} Re(z_1) + Re(z_2) + \cdots + Re(z_\nu) =$
$= Re(z_1 + z_2 + \cdots + z_\nu) = Re(\tilde{z})$ (2) όπου έθεσα

όπου έθεσα $\tilde{z} \equiv z_1 + z_2 + \cdots + z_\nu \overset{(1)}{=} \alpha \cdot e^{i\omega t}[1 + e^{i\delta} + \cdots + e^{i(\nu-1)\delta}]$

$$\Rightarrow \frac{\tilde{z}}{\alpha} e^{-i\omega t} = 1 + e^{i\delta} + e^{i\cdot 2\delta} + \cdots + e^{i(\nu-1)\delta}$$

και $$\frac{\tilde{z}}{\alpha} e^{-i\omega t} e^{i\delta} = e^{i\delta} + e^{i\cdot 2\delta} + e^{i\cdot 3\delta} + \cdots + e^{i\cdot\nu\delta}$$

$\Rightarrow$ αφαιρώ κατά μέλη

$$\Rightarrow \frac{\tilde{z}}{\alpha} e^{-i\omega t}(1 - e^{i\delta}) = 1 - e^{i\cdot\nu\delta} \Rightarrow$$

$$\Rightarrow \frac{\tilde{z}}{\alpha} e^{-i\omega t} = \frac{1 - e^{i\nu\delta}}{1 - e^{i\delta}} = \frac{e^{i\nu\delta/2}}{e^{i\delta/2}} \cdot \frac{e^{-i\nu\delta/2} - e^{i\nu\delta/2}}{e^{-i\delta/2} - e^{i\delta/2}} = \cdots$$

$$\ldots = e^{i(v-1)\delta/2} \frac{(-2i)\sin(v\delta/2)}{(-2i)\sin(\delta/2)} \Rightarrow **$$

$$e^{-i\varphi} - e^{i\varphi} = e^{i(-\varphi)} - e^{i\varphi} = \cos(-\varphi) + i\sin(-\varphi) - \cos\varphi - i\sin\varphi =$$
$$= \cos(\varphi) - i\sin\varphi - \cos\varphi - i\sin\varphi =$$
$$= -2i\sin\varphi$$

$$** \Rightarrow \tilde{z} = \alpha e^{i\omega t} e^{i(v-1)\delta/2} \frac{\sin(v\delta/2)}{\sin(\delta/2)} \overset{2}{\Rightarrow}$$

$$\overset{2}{\Rightarrow} \boxed{\tilde{x} = Re(\tilde{z}) = \alpha \cos\left[\omega t + i(v-1)\frac{\delta}{2}\right] \frac{\sin(v\delta/2)}{\sin(\delta/2)}}$$

$$Re(e^{i\alpha} e^{i\beta}) = Re\left[e^{i(\alpha+\beta)}\right] = Re[\cos(\alpha+\beta) + i\sin(\alpha+\beta)] =$$
$$= \cos(\alpha+\beta)$$