Πίνακας στοιχειωδών αόριστων ολοκληρωμάτων

1. $\int dx = x + c$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + c$

4. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + c$

5. $\int e^{ax}\, dx = \frac{1}{a}e^{ax} + c$

6. $\int a^x\, dx = \frac{a^x}{\ln a} + c$

7. $\int x\, e^{ax}\, dx = \frac{e^{ax}}{a^2}(ax-1) + c$

8. $\int x^2 e^{ax}\, dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2) + c$

9. $\int \frac{dx}{1+e^{ax}} = -\frac{1}{a}\ln\left(1+e^{-ax}\right) + c = \frac{1}{a}\ln\frac{e^{ax}}{1+e^{ax}} + c$

10. $\int \ln x\, dx = x\ln x - x + c$

11. $\int \sin kx\, dx = -\frac{1}{k}\cos kx + c$

12. $\int \cos kx\, dx = \frac{1}{k}\sin kx + c$

13. $\int \tan kx\, dx = -\frac{1}{k}\ln\cos kx + c$

14. $\int \cot kx\, dx = \frac{1}{k}\ln\sin kx + c$

15. $\int \arcsin\frac{x}{a}\, dx = x\arcsin\frac{x}{a} + \sqrt{a^2 - x^2} + c$

16. $\int \arccos\frac{x}{a}\, dx = x\arccos\frac{x}{a} - \sqrt{a^2 - x^2} + c$

17. $\int \arctan\frac{x}{a}\, dx = x\arctan\frac{x}{a} - \frac{a}{2}\ln\left(a^2 + x^2\right) + c$

18. $\int \operatorname{arctan}\frac{x}{a}\, dx = x\operatorname{arctan}\frac{x}{a} - \frac{a}{2}\ln\left(a^2 + x^2\right) + c$

19. $\int \operatorname{arc\,cot}\frac{x}{a}\,dx = x\operatorname{arc\,cot}\frac{x}{a} + \frac{a}{2}\ln\left(a^2 + x^2\right) + c$

20. $\int \frac{dx}{\sin kx} = \int \operatorname{cosec} kx\,dx = \frac{1}{k}\ln\tan\frac{kx}{2} + c = \frac{1}{k}\ln(\operatorname{cosec} kx - \cot kx) + c$

21. $\int \frac{dx}{\cos kx} = \int \sec kx\,dx = \frac{1}{k}\ln\tan\left(\frac{kx}{2} + \frac{\pi}{4}\right) + c = \frac{1}{k}\ln(\sec kx + \tan kx) + c$

22. $\int e^{ax}\sin kx\,dx = \frac{e^{ax}}{a^2 + k^2}(a\sin kx - k\cos kx) + c$

23. $\int e^{ax}\cos kx\,dx = \frac{e^{ax}}{a^2 + k^2}(a\cos kx + k\sin kx) + c$

24. $\int \sin^2 kx\,dx = \frac{1}{2}x - \frac{1}{4k}\sin 2kx + c$

25. $\int \cos^2 kx\,dx = \frac{1}{2}x + \frac{1}{4k}\sin 2kx + c$

26. $\int \sin mx\sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + c \qquad |m| \neq |n|$

27. $\int \cos mx\cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + c \qquad |m| \neq |n|$

28. $\int \sin mx\cos nx\,dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + c \qquad |m| \neq |n|$

29. $\int \frac{dx}{\sin^2 kx} = -\frac{1}{k}\cot kx + c$

30. $\int \frac{dx}{\cos^2 kx} = \frac{1}{k}\tan kx + c$

31. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\operatorname{arctan}\frac{x}{a} + c$

32. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\frac{x-a}{x+a} + c$

33. $\int \sqrt{ax+b}\,dx = \frac{2}{3a}(ax+b)^{3/2} + c$

34. $\int \frac{dx}{\sqrt{ax+b}} = \frac{2}{a}\sqrt{ax+b} + c$

35. $\int \frac{\sqrt{x}}{\sqrt{a+x}}dx = \sqrt{x(a+x)} - a\ln\left(\sqrt{x} + \sqrt{a+x}\right) + c$

36. $\int \frac{\sqrt{x}}{\sqrt{a-x}}dx = a\operatorname{arcsin}\sqrt{\frac{x}{a}} - \sqrt{x(a-x)} + c$

37. $\int \sqrt{x^2 + a}\,dx = \frac{1}{2}\left[x\sqrt{x^2+a} + a\ln\left(x + \sqrt{x^2+a}\right)\right] + c$

38. $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \arcsin\frac{x}{a} \right) + c$

39. $\int \frac{dx}{\sqrt{x^2 + a}} = \ln\left(x + \sqrt{x^2 + a} \right) + c$

40. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c$

41. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\left|\frac{x}{a}\right| + c$

42. $\int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{x^2 + a^2}}{x} \right) + c$

43. $\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 - x^2}}{x} \right) + c$

Πίνακας στοιχειωδών ορισμένων ολοκληρωμάτων

1. $\int_0^\infty \frac{dx}{a + x^2} = \frac{\pi}{2\sqrt{a}} \qquad (a > 0)$

2. $\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$

3. $\int_0^{\pi/2} \sin x\, dx = \int_0^{\pi/2} \cos x\, dx = 1$

4. $\int_0^{\pi/2} \sin^2 x\, dx = \int_0^{\pi/2} \cos^2 x\, dx = \frac{\pi}{4}$

5. $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (n-3)(n-1)}{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (n-2)\, n} \qquad (n = \text{περιττό})$

6. $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (n-3)(n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (n-2)\, n} \frac{\pi}{2} \qquad (n = \text{άρτιο})$

7. $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \qquad (a > 0, n > -1)$

8. $\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \qquad (a > 0)$

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Πίνακες ολοκληρωμάτων υπάρχουν επίσης σε μαθηματικά προγράμματα ηλεκτρονικών υπολογιστών, όπως το MATHEMATICA και άλλα.