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Mental arithmetic calculation in the addition and subtraction of two-digit numbers: The case of third and fourth grade elementary school pupils

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Abstract

The objective of the present study is to investigate pupils' ability to add and subtract two digit numbers mentally, to observe the strategies they use, to investigate the effect of teaching based on the new mathematics curriculum and to ascertain whether there is any notable difference in the performance between pupils in different year groups. One hundred and twenty pupils of grade 3 and 4 took part in this research. The results showed that the pupils encounter difficulties in operations with a digit carried over, that subtraction is more difficult than addition and finally, that both the positive effect of contemporary approaches to teaching (when the pupils had followed the new maths curriculum from the first grade), and the negative influence of the teaching of written operations on pupils' spontaneous mental strategies, are clear.

1. Introduction

Mental calculation is a process in which a numerical calculation can be made quickly and accurately without the aid of external means, for example, manipulatives, pencil and paper, etc, and be done in a conscious way, using some strategy (Maclellan, 2001, 145-146).

According to the international literature, mental calculation holds an important place in the teaching and learning of mathematics. In particular, it develops problem solving skills, provides opportunities for making calculated estimates and contributes to the understanding of the concept of number (McIntosh et al., 1995; Threlfall, 2002). In addition, it encourages children to manipulate numbers with ease, it is the foundation for the development of calculating skills and it aids the understanding and development of written methods of calculation (Varol and Farran, 2007). Finally, Heirdsfield and Cooper (2004) point out that mental calculation helps children understand how numbers 'work', and what their structure is, as well as helping them to discover strategies for problem solving while developing skills for making hypothesis and generalizations.

It is known that mental calculations involve a wide range of strategies, unlike traditional problem solving techniques. According to the international bibliography, extensive research has shown the variety of strategies that pupils use when they perform mental calculations involving two-digit numbers, for addition and subtraction (Maclellan, 2001; Threlfall, 2002; Lucangeli et al., 2003; Macintyre and Forrester, 2003; Heirdsfield and Cooper, 2004; Heirdsfield and Lamb, 2005; Lemonidis, 2003; Karantzis & Tollou, 2009).

From all of this research, the strategies which pupils employ in mental calculations for the addition and subtraction of two-digit numbers have been identified and codified, and are presented below:

1.1 Strategies for addition and subtraction

1) *Instant recall (AUTO)*: instant recall of result.
2) *Separation of tens and units or 1010*: With this strategy, first the tens and then the units are calculated (e.g. 54+25: 50+20=70, 4+5=9, 70+9=79, and 68-26: 60-20=40, 8-6=2, 40+2=42), or first the units and then the tens, known as *u-1010* (Heirdsfield and Lamb, 2005) (e.g. 54+25: 4+5=9, 50+20=70, 70+9=59 and 68-26: 8-6=2, 60-20=40, 40+2=42). This category also includes the case which, according to Varol and Farran (2007) is called *10s*, where, for addition, the units of the first addend are added to the sum of the tens, and then the units of the second addend (e.g. 54+25: 50+20=70, 70+4=74, 74+5=79), while for subtraction without carry digit the numbers are analysed in their tens, and then, from the difference between the tens,

the units of the subtrahend are subtracted, and then the units of the minuend are added, (e.g. 68-26: 60-20=40, 40-6=34, 34+8=42). Finally, in subtraction with a carry digit, the tens are dealt with first and then the units of the minuend are added to their difference, and from the result, the units of the subtrahend are subtracted (e.g. 84-59: 80-50=30, 30+4=34, 34-9=25). It is worth mentioning that the strategy 1010,u-1010 is characterised as a 'separation' strategy (Heirdsfield and Lamb, 2005).

3) *Calculation based on the first number, or N10*: For addition using this strategy, first the units and then the tens of the second *addend* are added to the first (e.g. 54+25: 54+5=59, 59+20=79), strategy *u-N10* (Heirdsfield and Lamb, 2005), or first the tens and then the units (e.g. 54+25: 54+20=74, 74+5=79). In addition, Lucangeli et al., (2003), separate the second addend in another way, for example: 54+25 = (54+10+10)+5=79. For subtraction, first the units and then the tens of the subtrahend are subtracted from the minuend, strategy *u-N10* (Heirdsfield and Lamb, 2005) (e.g. 68-26: 68-6=62, 62-20=42), or first the tens and then the units (e.g. 68-26: 68-20=48, 48-6=42). This strategy N10, u-N10 is characterized as an 'aggregation' strategy. (Heirdsfield and Lamb, 2005).

4) *Rounding up/rounding down, or N10C*: Here the pupil, in order to calculate the sum or the difference rounds up or down one of the two numbers (e.g. 54+25: 55+25=80, 80-1=79, and 68-26: 70-26=44, 44-2=42 or 34+25:40+19=59, and 48-26: 50-28 = 22). This is known as a 'holistic' strategy.

5) *Mental counting from a specific point (counting on, CON)*: Here, for addition, the units of the second addend are added to the first, one at a time (e.g. 54+25: 54+1+1…= 79) or the tens of the second addend ten by ten, followed by the units (e.g. 54+25: 54+10=64, 64+10=74, 74+5=79). For subtraction, the units of the subtrahend are subtracted one at a time from the minuend (e.g. 68-26: 68-1-1…= 42), or first the tens are subtracted ten by ten and then the units (e.g. 68-26: 68-10=58, 58-10=48, 48-6=42).

6) *Counting:* The counting starts with the units of the one number and continues with the units of the other. The units of the second addend may also be

added to the first addend one at a time, using either the fingers, or sometimes a rhythmic movement of the head (Counting on fingers, COF). Similarly, in subtraction, the counting begins with the objects that the minuend expresses, and then the objects that the subtrahend expresses are subtracted, and finally whatever is left is counted. The units of the subtrahend may also be subtracted from the minuend one at a time, using either the fingers, or sometimes a rhythmic movement of the head. (Counting on fingers, COF).

7) *Strategy C10*: In order to make addition or subtraction easier, rounding up to 10 is used. This strategy is characterized by Varol and Farran (2007) as strategy A10 (e.g. 54+25: 54+6=60, {25-6=19}, 60+19=79 and 74-69: 74-4=70 {69-4=65}, 70-65=5).

8) *Mental-traditional algorithm (MA)*: The traditional algorithm is performed mentally following the algorithm of the vertical addition or subtraction (e.g. 54+25: 5+4=9, 5+2=7,. Sum: 79, and 36-25: 5 from 6 leaves 1, 2 from 3 leaves 1, so the answer is 11).

9) *Completion of the subtrahend*: In this strategy the pupil increases the units of the subtrahend until he reaches the minuend. The number that represents the increase is the correct answer. This strategy is divided into two categories. In the first: when the subtrahend is a small distance from the minuend the pupil adds the units one by one (or all together) to the subtrahend until he reaches the minuend. (e.g. 72-69: 69+3=72, so the correct answer is 3). On the other hand, when the subtrahend is a greater distance from the minuend then the answer is found in stages (making up the tens first, etc). (e.g. 73-36: 36+4=40, 40+30=70, 70+3=73, 4+30+3=37).

10) *Addition of tens until the number is surpassed*: (Van de Walle, 2005). (e.g. 73-46: 46+30=76, 76-3=73, 30-3=27) or *adding on to the minuend* (e.g. 73-46:73+3=76, 76-46=30, 30-3=27).

In conclusion, counting, counting on fingers and mental traditional algorithms are characterized as low level strategies since they are not effective and do not promote mathematical thought in children. On the other hand, high level strategies which contribute to a better understanding of numbers are the following: separation of tens-units (1010), calculation based on the first term (N10), rounding up (N10c), C10, completion of the subtrahend, addition of tens until the amount is surpassed, adding on to the minuend. These strategies are more effective and 'have a tendency to change according to the numbers involved, to make the calculation easier' (Van de Walle, 2007). For example, to calculate the sum '43+26' strategy 1010 may be used quickly and easily, while to calculate the difference '54-18' strategy N10 may be preferred (Blote et al., 2000).

Beliefs relating to the importance of mental calculations in everyday life are supported internationally by many educationalists who recommend the introduction of the systematic teaching of mental arithmetic into the elementary school curriculum. Such efforts were the suggestion of the National Council of Teachers of Mathematics (NCTM) in America (Van de Walle, 2007), and the National Numeracy Strategy which was adopted in England in 1995 (Maclellan, 2001; Macintyre and Forrester, 2003; Lemonidis, 2008).

In Greece, according to the new curriculum for mathematics in elementary school which was introduced in September 2005 with the writing of new books, mental arithmetic calculations occupy an important place in the teaching and learning of mathematics. This is in contrast to the old programme, which did not place enough emphasis on the children's ability to perform operations in their head. So, according to the new curriculum, it is requested that each teacher helps the pupils develop skills which contribute to the adoption of suitable strategies for the mental calculation of numbers, unlike the approach in the old maths books.

From this position, in the research of Karantzis & Tollou (2009), an attempt was made to investigate which strategies are used and to what extent, by third-year elementary school pupils, when they calculate mentally the sum and difference of two-digit numbers. The research took place during the second year of the application of the maths curriculum (November-December 2007, when the pupils of the 3^{rd} grade had followed the new maths curriculum since the 2^{nd} grade) and 90 pupils took part from four (4)

Elementary Schools in the city of Patras. The test criteria consisted of 16 operations, of which 8 were addition and the remaining 8 subtraction. Of the 8 additions, half (4) were operations without a number carried over, while the other half (4), were with a number carried over. The same was true for the subtractions. The operations were given in the following order for all pupils: (Addition: 57+12, 85+14, 68+28, 36+15, 58+31, 27+14, 32+42, 49+33, Subtraction: 87-44, 69-56, 95-19, 51-26, 96-33, 83-48, 78-25, 64-37). The results showed that the children answered correctly to a satisfactory degree (86.11% for addition and 60.3% for subtraction). In addition, a fairly significant percentage (39.6% for addition and 33.07% for subtraction) used low level strategies, mainly the algorithm and the vertical operation, while others (46.25 for addition and 27.23 for subtraction) used high level strategies, mainly strategies 1010 and N10.

1.2 The purpose of study

As we are now in the fourth year of the implementation of the new curriculum for pupils in Greek primary schools, we wanted to re-examine the hypothesis of the aforementioned research, and to extend its scope using fourth year pupils. From this study we may perhaps draw some conclusions about the effect contemporary methods of teaching mathematics will have in the area of mental calculation. In particular, the objective of the present study is to investigate pupils' ability to add and subtract two digit numbers mentally, to observe the strategies they use, to investigate the effect of teaching based on the new mathematics curriculum (when the pupils had followed the new maths curriculum from the first grade) and to ascertain whether there is any notable difference in the performance between pupils in different year groups. With all this in mind, we designed and carried out the present study.

2. Method

2.1 Participants

The choice of the sample for the research came from six (6) Elementary Schools in the city of Patras. The research was conducted during the current school year (November-December 2009) and pupils of grade 3 and 4 of the above mentioned schools participated. In particular, 60 pupils of grade 3 (31 boys and 29 girls) and 62 pupils of grade 4 (31 boys and 31 girls) took part. The schools and the pupils from those schools that participated in the research were selected using the method of simple random sampling (Cohen and Manion, 1994).

2.2 Materials

The test criteria consisted of 16 operations, of which 8 were addition and the remaining 8 subtraction. Of the 8 additions, half (4) were operations without a number carried over, while the other half (4), were with a number carried over. The same was true for the subtractions. Our aim was not to give the children too many tasks, so as not to tire them as this would prove an obstacle to the success of the research. The operations were given in the following order for all pupils: (Addition: 57+12, 85+14, 68+28, 36+15, 58+31, 27+14, 32+42, 49+33, Subtraction: 87-44, 69-56, 95-19, 51-26, 96-33, 83-48, 78-25, 64-37).

The children were given cards on which the additions and subtractions were presented horizontally, as in this way it is less likely that the pupils will be encouraged to use the traditional algorithms of the operations (Van de Walle, 2007).

2.3 Procedure

The participants were tested individually and the tester, having asked each pupil orally, recorded their oral answers. The test lasted about 20 minutes for each pupil and took place in a quiet part of the school, away from the classroom, so that the children could express their thoughts freely and without anxiety. In the beginning, before the procedure started and to create a pleasant, stress-free environment, the pupil was asked about his lessons, his family and so on, and the whole procedure took the form of a game. The tester referred to the conditions, in accordance with which the pupil should calculate mentally the sum of, or the difference between the numbers, explaining his reasoning each time. Before the start of the test proper, there were a few trial runs, until each subject was judged capable of following the test procedure. In other words, the pupil was given two cards with simple additions. He found the answer and then reported which strategy he had used to find the solution. Later (when the additions were finished with), he was given two other cards with simple subtractions and again reported his reasoning.

When the actual test began, the pupil looked at the cards one by one in front of him and used mental calculation to find the answers to the operations. The tester recorded his performance and asked him to justify his answer. In addition, the tester listened carefully to the pupil and from his answers, but also from his behaviour generally while he was performing the mental calculations (observing for example if the pupil used his fingers in order to calculate), recorded the strategy used. Finally, the performance of each pupil in addition and subtraction was calculated for each strategy like this: (number of correct answers/8)* 100. It should be pointed out that in the present study we focus only on the correct answers. The analysis of the incorrect answers will be presented in another study.

Concluding the section on methodology, it should be pointed out that in the present study, as was mentioned above, we only evaluated the strategies that produced correct answers from the pupils. Perhaps, in a future investigation it would be interesting to consider the strategies used in cases where pupils gave a wrong answer, and search for explanations as to why the pupils, despite having chosen a suitable strategy, were still unable to produce the correct answer.

3. Results - discussion

The statistical analysis of the data was accomplished with the aid of the statistical programme SPSS v. 15. In particular, means and standard deviations were calculated for students' performance per class, operation, strategy and the carrying (or not) of a digit. A mixed analysis of variance model was estimated with one between – group factor (class) and two within – group factors (operation and carrying digit), ANOVA 2X2X2 (Snodgrass, 1977).

All factors had two levels. In addition, t-test was applied, for the comparison of the variables "low or high level strategies" within each class, because our sample was random and of a relatively satisfactory size and our variables are evenly spaced. (Snodgrass, 1977).

The mean results of the participants' performances from both classes (Grades 3 and 4) in the mental calculation of addition and subtraction, with or without a carrying digit, regardless of the strategy used, are presented in Table 1, while in Table 3 their performance in each strategy for mental calculation, is presented. In particular, in Table 1 mean (%) and standard deviation of pupils' performance in addition and subtraction with and without carrying digit are presented [e.g. in each case (number of correct answers/4) with and without carrying digit operations)* 100) was calculated]. In Table 3 presented mean (%) and standard deviation of pupils' performance in each strategy of mental calculation for addition and subtraction (e.g., if a student gave the correct sum for 6 additions, applying twice strategy 1010, three times the MA and once the N10 strategy, then this student's score is respectively 25, 37.5, 12.5). In the strategy "Counting on fingers and Mental algorithm (COF+MA)", the pupils use of the two strategies in the same operation should be pointed out.

From a first examination of the results of the present study, it emerges that the pupil, regardless of year group or strategy used, display quite a satisfactory performance in the mental calculation of addition and subtraction with two digit numbers. However, as expected, their performance in addition shows a statistically significant difference to their performance in subtraction, Anova $F(1,120)=94.27$, $p<0.000$. In other words, it seems that pupils encounter more difficulties when dealing with the mental calculation of subtraction, than when dealing with addition.

From these findings we can conclude that addition is easier for the pupils than subtraction. As we know, these operations are 'logical' and their formation is based on certain rules. One of these is the rule of 'inversion', according to which, at any moment a logical operation can be mentally processed in the opposite direction and be cancelled out (Cole and Cole, 2001). In this way, subtraction is considered the opposite-reverse operation of addition (Matthews, 1981), and during mental calculation presents a complexity that creates difficulties, especially for younger pupils (Cebulski & Bucher, 1986; Macintyre & Forrester, 2003; Lemonidis, 2003; Lemonidis & Lygouras, 2008).

Attempting a comparison within each operation and regardless of the year group of the pupils, we notice, as expected, that their performance in mental calculation without a carrying digit display a statistically significant difference to their performance in operations with a carrying digit, Anova ($F(1,120)=103.02$, $p<0.000$). In other words, operations with a carrying digit create significantly greater difficulties for pupils in both year groups. In parallel, however, a statistically significant interaction between the factors 'operation' and 'carrying digit', Anova ($F(1,120)=63.74$, $p<0.000$, Table 1), also emerges. The results are clearly apparent in Table 2. We observe that almost all the pupils, from the two grades, give correct answers for the two sums operations without a carrying digit and in addition with a carrying digit, for a large proportion of the operations they were given. However, for subtraction with carrying digit only about 50% of the pupils answer a large proportion of the operations they were given correctly.

These results are confirmed by the research of Wolters, Beishuizen, Broers & Knoppert (1990), Macintyre & Forrester (2203), Lemonidis (2003). These findings show that the degree of difficulty of the problem, such as the carrying digit, burdens the central executive (unit) of the working memory (Baddeley, 1995) which is responsible for the control of mental operations, the articulatory loop and other probable subsystems of the working memory that retain information temporarily until the solution to the problem is complete. Consequently, the time required to solve the problem increases, and the mistakes multiply (Hitch, 1978; Karantzis, 2004). Naturally, the pupils' flexibility in manipulating the carrying digit during the mental calculation of addition and subtraction may well improve with teaching and maturity. We are led to this conclusion by the statistically significant interaction which was observed between the factors ‘operation’ and ‘carrying digit’ (regardless of the type of operation), Anova $F(1,120)=5.43$, $p<0.02$. In other words, it seems that the performance of grade 4 pupils in the mental calculation of operations with addition and subtraction with a carrying digit is significantly better than that of grade 3 pupils (Table 1).

Another finding of the present study is that, in order to process maths problems mentally, pupils used strategies that they believed would help them to solve the problems successfully. This finding is also supported by the findings of previous researchers (Beishuitzen, 1993; Lemonidis, 2003; Lemonidis & Lygouras, 2008), who observed that the pupils developed skills to interchange their strategies according to the numerical data of each mental exercise. The participants in our research relied for the most part on traditional mental algorithm (MA) for both operations, when they chose low level strategies, and on strategy 1010 and strategy N10 when they used high level strategies.

The results are clearly apparent in Table 4. We observe that approximately 25% of grade 3 pupils and 40% of grade 4 pupils use the strategy traditional mental algorithm (MA), answering more than about half of the operations they given correctly. Most pupils from two grades prefer strategy 1010, with success in more than half of the operations they were given. Finally, fewer pupils from both grades use strategy N10, but more successfully (about 70-75%), in the operations they were given. The greater success, though of fewer pupils, in strategy N10, as compared with strategy 1010 can be explained by the fact that strategy 1010 is a strategy with a number of steps (compared with strategy N10) and as a result, as the number of steps required to solve a problem increases, so the number of mistakes made in the process of solving the problem in creases (Cebulski & Bucher, 1986). Finally, commenting on the fact that in strategy 1010 the performance of the pupils in both grades drops noticeably in operations involving subtraction, this may be due to the fact that this particular strategy causes difficulties for the pupils in subtraction with a carrying digit and leads them to make mistakes such as, for example: 8-4=4 instead of 14-8=6 during the mental calculation of the subtraction 54-18 (Macintyre & Forrester, 2003; Blote et al., 2000; Beishuizen & Anghileri, 1998).

These three strategies, as also emerged from the research of Heirdsfield & Cooper (2004), Askew & Brown (2006) and Varol and Farran (2007), are the strategies most commonly used by children when they perform the mental calculation of addition and subtraction of two digit numbers. It is worth mentioning that pupils in grade 3, as much as pupils in grade 4, choose high level strategies more than low level strategies, and this difference is statistically significant [for grade 3, $t(59)=6.94$, $p<0.000$ for addition and $t(59)=5.11$, $p<0.000$ for subtraction, and for grade 4, $t(61)=4.12$, $p<0.000$ for addition and $t(61)=3.21$, $p<0.002$ for subtraction], in other words, they choose strategies which make an important contribution to the consolidation of the relationships which exist between numbers and which promote mathematical thought.

This finding of the present study (concerning the performance of grade 3 pupils) stands in contrast to the results of the research of Karantzis & Tollou (2009). In this study, it was found that the strategy of traditional mental algorithm (MA) was used by pupils to a greater degree than strategies 1010 and N10. As we mentioned before, it followed the same methodology and used the same materials with the pupils, and took place in the same area (the city of Patras). The only difference was that this research took place in the second year of the application of the new mathematics curriculum (the pupils of the 3rd grade had been following the new maths curriculum since 2nd grade).

Bearing this in mind, a comparison of the results of the two studies would be appropriate. So, the results of the present study permit us, perhaps, to claim that pupils of grade 3 who followed from Year 1 the new maths curriculum, improved their performance quantitatively, but also qualitatively as far as their strategies are concerned, which suggests that the improvement in quality may be a result of a better understanding of the value of the position of the digits in a number. This proposition is supported by results from diachronic research which took place in Greece (Lemonidis, 2003) from which it appeared that the teaching of mathematics that is based on new approaches to the teaching and learning of mathematics, approaches which have been adopted by the new mathematics curriculum in primary schools, made a significant contribution to the quantitative and qualitative improvement in pupils' performance in mental arithmetic calculation. It is worth mentioning however that these conclusions need to the verified in the future with further diachronic research.

The statistical analysis of the data showed that between the year groups and regardless of the type of operation, there doesn't seem to be a statistically significant difference [Anova $F(1,120)=2.65$, $p=0.106$] but especially with low lever strategies [$t(120)=1,70$, $p<0.09$)] and high lever strategies [$t(120)=0,74$, $p<0.46$]. This result poses an interesting question – why wasn't there a statistically significant improvement in the performance of grade 4 pupils as compared with that of grade 3 pupils, in the mental calculation of addition and subtraction of two digit numbers. One explanation could be that in grade 2, and at the beginning of grade 3, according to the new curriculum, pupils spend more time on mental calculation and consequently in grade 3 develop a greater degree of flexibility in the area of operations. For the carrying digit of grade3 and in grade 4 the pupils spend more time on the execution of those operations with larger numbers and with the application of strategies based on the algorithm of vertical operations. As a result grade 4 pupils don't spend as much time on mental calculations and consequently that performance doesn't show a significant improvement from that of grade 3 pupils.

Another reason could be that grade 4 pupils make use of what is for them the 'easy' algorithm of vertical addition and subtraction. This could actually be the case since in the present study we observe an increase in grade 4 pupils' performance in mental calculation using the strategy of vertical addition and subtraction and at the same time a reduction in their performance of subtraction with strategy 1010 (Table 4). This result, according to the research of Cooper et al., (1996), Heirdsfield & Cooper (1996), Lemonidis (2003), could be due to the obvious effect of the teaching of written operations on the children's spontaneous mental strategies. More specifically, prior to teaching, the children presented a variety of effective mental strategies, while after teaching they have a tendency to use a mental strategy, which seems to reflect the written algorithm taught by the teacher. However, further research with a larger number of pupils may provide more certain results and perhaps reveal other factors witch could explain the results of the present study.

4. Conclusions – Implications for Teaching

The main research conclusions from the present study with reference to the performance of grade 3 and 4 primary school pupils in the mental calculation of the addition and subtraction of two digit numbers are the following:

As expected, it was found that both grade 3 and grade 4 primary school pupils performed significantly better on mental addition than on subtraction generally. Also, with reference to the pupils' performance in addition and subtraction with and without a carrying digit, it emerged that operations with a carrying digit present significantly more problems to the pupils than those without, which results in a significant increase in mistakes, which is perhaps due to factors concerning the processing and retention of information in the working memory. Naturally, the pupils' flexibility in manipulating the carrying digit during the mental calculation of addition and subtraction may well improve with teaching and maturity, a fact which is apparent in the significantly superior performances grade 4 pupils as against those in grade 3.

Finally, the participants in the research used, for both kinds of operation, strategies 1010 and N10 much more than other strategies when they used high level strategies, as well as the mental strategy which is based on the traditional algorithm (the vertical operation), when they used low level strategies. It seems that most pupils of both grades prefer strategy 1010 and successfully solved more than half of the operations they were given, using it. Finally, fewer pupils for both grades use strategy N10, but with greater success (approximately 70-75%) in the operations they were given.

Studying the results of the present research, which took place four years after the implementation of the new mathematics curriculum in primary schools, and comparing them with the results of similar research by Karantzis & Tollou, (2009), which took place just two years after the implementation of the new curriculum (the pupils of the 3rd grade had followed the new maths curriculum since the 2nd grade), we can perhaps claim, as we mentioned before, that the quantitative and qualitative improvement of the pupils in the present study is the result of teaching which is based on the new mathematics curriculum and on the teacher himself who better understood the spirit of the new curriculum, adapting the objectives of his teaching accordingly.

Naturally, we should point out that, due to the fact that our research sample is very small, our results require further investigation with studies of diachronic nature, at least as far as the positive effect of teaching based on the principles of the new mathematics curriculum is concerned. However the results of long term research in Greece (Lemonidis, 2003) are encouraging as they show that the teaching of mathematics based on the new mathematics curriculum, which emphasizes mental calculation and actively involves pupils in the learning process with activities that promote mathematical thought, has a positive effect on their performance in mental arithmetic calculations.

Finally, the results of the present research showed that there doesn't seem to be a statistically significant difference between year groups, regardless of the type of operation (addition and subtraction) but particularly in low and high level strategies. The fact that for a large period of time in grade 3, and for the whole of grade 4, the pupils are occupied with the execution of these operations, firstly with larger numbers, and then with the application of strategies based on the algorithm of the vertical operation, mean that grade 4 pupils don't spend as much time on mental calculations and consequently that performance doesn't improve significantly against that of grade 3 pupils. Another reason could be that the effect of the teaching of written operations on the spontaneous mental calculations of the children is clear, this result is confirmed by the research of Cooper et al., (1996), Heirdsfield & Cooper (1996), Lemonidis (2003). In conclusion, we believe that further research may help us better understand results and perhaps also reveal other factors which could explain our results.

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Table 1
Mean (%) and standard deviation (in parenthesis) of pupils' performance (N=122) in addition and subtraction with or without carrying digit

	Grade 3		*Grade 4*			
	Addition	Subtraction	Addition	Subtraction	*Mean Addition*	*Mean Subtraction*
Without carrying digit	97.50 (7.6)	92.50 (16.8)	97.98 (6.9)	92.34 (20.5)	97.74	92.42
Mean Addit. & Subtrac.	95		95.16		////////////////	
With Carrying digit	88.75 (23.7)	43.3 (47.6)	91.53 (22.6)	62.5 (46.6)	90.14	52.90
Mean Addit. & Subtract.	66.03		77.02		//////////////////	
General Mean	93.13	67.92	94.76	77.42	/////////////////	

Table 2
Number of pupils (in parenthesis %) and number of operations (%) that they answer correctly in addition and subtraction

		Grade 3		Grade 4	
		Addition	Subtraction	Addition	Subtraction
Without carrying digit	N	60 (100)	60 (100)	62 (100)	61 (98.4)
	Nub. operat.	97.5	92.5	97.9	93.8
With Carrying digit	N	58 (96.7)	29 (48.3)	60 (96.8)	41 (66.1)
	Nub. operat.	91.8	89,7	94.6	94.5

Table 3
Mean (%) and standard deviation (in parentheses) of pupils' performance (N=122) in each strategy of mental calculation for addition and subtraction

Category of Strategies	Strategies of mental calculation	Grade 3		Grade 4	
		Addition	Subtraction	Addition	Subtraction
Low level Strategies	Counting on fingers and Mental algorithm (COF+MA)	1.25 (5.5)	0.63 (2.7)	0.40 (3.2)	0.20 (1.6)
	Mental algorithm (MA)	14.58 (30.7)	13.13 (26.5)	26.01 (39.4)	22.98 (35.2)
	Mean low level Strategies	15.83 (31.6)	13.75 (26.6)	26.41 (39.5)	23.18 (35.2)
High level Strategies	Wholistic (N10C)	0.83 (3.14)	0.00	0.81 (3.1)	0.40 (2.2)
	Strategy 1010	61.25 (44.3)	32.92 (29.6)	46.77 (42.2)	24.80 (33.5)
	Strategy (N10	15.21 (33.6)	20.63 (36.5)	20.77 (37.1	27.22 (40.6)
	Complementing the subtrahend	----	0.00	---	0.20 (1.6)
	Other Means	0.00	0.63 (3.6)	0.00	0.81 (6.4)
	Mean high level Strategies	77.29 (38.1)	54.17 (36.7)	68.35 (41.8)	53.43 (43.1)

Table 4
Number of pupils (in parenthesis %) and number of operations (%) that they answer correctly in MA, 1010, N10 strategies of mental calculation for addition and subtraction

Strategies of mental calculation		Grade 3		Grade 4	
		Addition	Subtraction	Addition	Subtraction
(MA)	N	16 (26.7)	16 (26.7)	24 (38.7)	24 (38.7)
	Numb. operat.	54.7	46.1	67.2	59.9
Strategy (1010)	N	45 (75)	41 (68.3)	44 (71)	26 (41.9)
	Numb. operat.	81.7	48.2	65.9	59.1
Strategy (N10)	N	12 (20)	17 (28.3)	18 (29)	23 (37.1)
	Numb. operat.	76	72.1	71.5	75