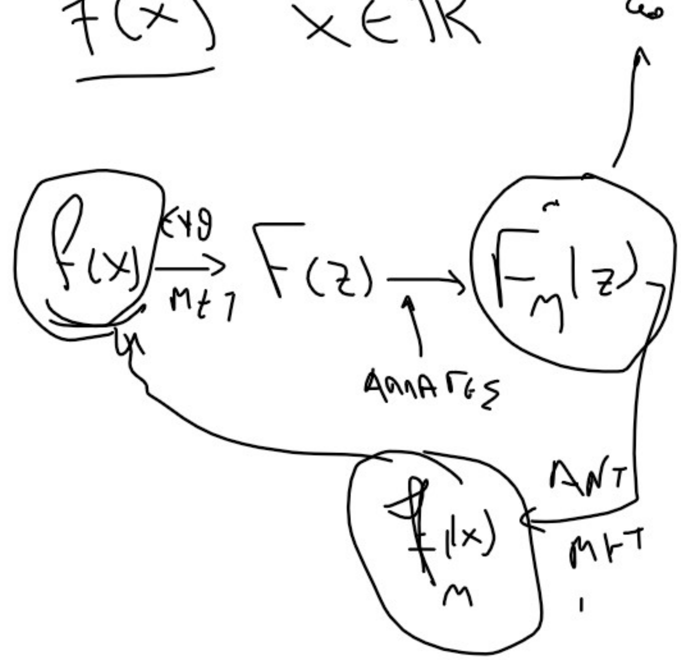
Fourier

ΑΡΧΙΖΟΥΜΕ 16:15

ΟΛΟΚΛΗΡΩΤΙΚΟΣ ΜΕΤΑΣΧΗΜΑΤΙΣΜΟΣ

$$\boxed{F(z) = \int f(x)\, t_r(x, z)\, dx}$$

$f(x) \quad x \in \mathbb{R}$



ΑΝΤΙΣΤΡΟΦΟΣ (ΑΝ ΥΠΑΡΧΕΙ)

$$f(x) = \int F(z) \cdot \underbrace{t_r^{ANT}(x, z)}\, \underbrace{dz}$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(x)\, e^{-j\omega x}\, dx$$

ΨΗΦΙΑΚΑ ΣΗΜΑΤΑ

$x[m]$, $\cancel{m \in \mathbb{Z}}$, $m \in \mathbb{N} \cup \{0\}$

~~ΣΥΝΕΧΗΣ~~

ΜΕΤΑΣΧ. FOURIER ΓΙΑ ΔΙΑΚΡΙΤΑ ΣΗΜΑΤΑ

$$F(\omega) = \sum^{+\infty} x[m]\, e^{-j\omega m}$$

$-\pi \le \omega < \pi$ $\qquad$ $0 \le \omega < 2\pi$

$-\pi \le \omega < \pi$ $\quad$ $0 \le \omega < \pi$

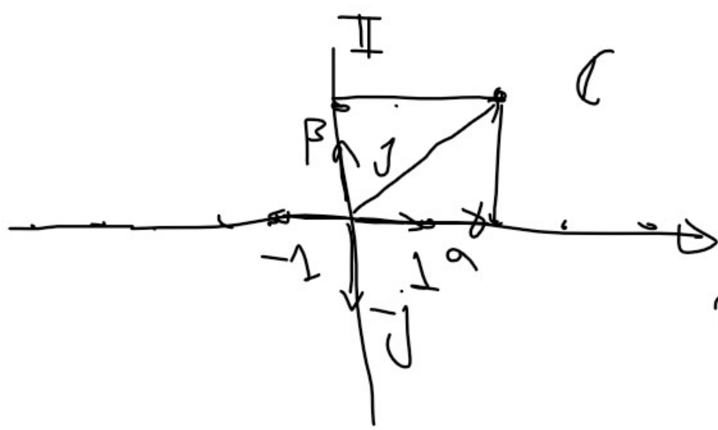
$$\boxed{F(\omega) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n}}$$

$n \in \mathbb{Z}$, $j \in \mathbb{I}$ $\quad |j| = 1$

$\omega \in \mathbb{R}$ $\qquad x = \{1, 2, 1\}$

$$\boxed{x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\omega)\, e^{j\omega n}}$$

$$F(\omega) = \sum_{n=0}^{2} x[n]\, e^{-j\omega n} = e^{0} + 2e^{-j\omega} + e^{-j2\omega}$$



$$\alpha x^2 + \beta x + \gamma = 0$$

$$\Downarrow$$

$$x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2a}$$

$$q = \alpha + \beta j$$

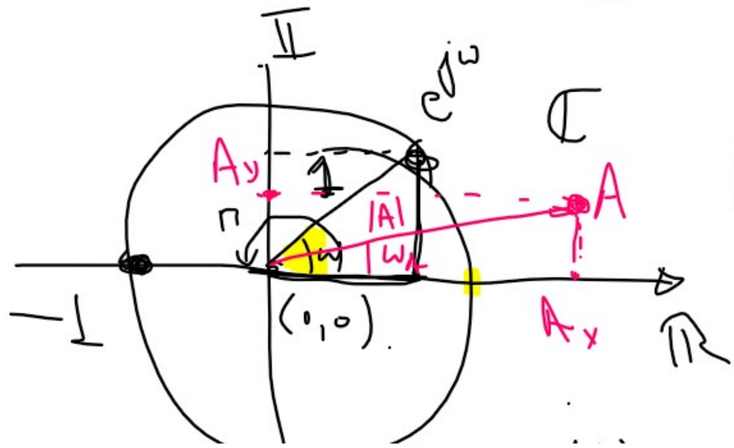
$$\beta^2 - 4\alpha\gamma < 0$$

$$\boxed{e^{jx} = \cos x + j \sin x}$$

$$j = \sqrt{-1}$$

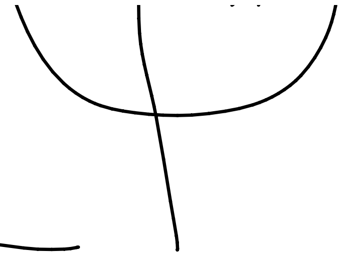
$$x = \frac{-\beta \pm \sqrt{(-1)\,|\beta^2 - 4\alpha\gamma|}}{2\alpha}$$

$$= \frac{-\beta \pm j\sqrt{|\beta^2 - 4\alpha\gamma|}}{2\alpha}$$



$A \in \mathbb{C}$

$$A = A_x + jA_y = |A| \cdot e^{j\omega_A}$$



$$A = A_x + jA_y = |A|\, e^{j\omega_A}$$

ΚΑΡΤ — ΠΟΛΙΚΕΣ

$$e^{j\omega} = \cos\omega + j\sin\omega$$

$$\boxed{e^{j\pi} = -1}$$

$$|e^{j\omega}| = 1$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = h \circledast x$$

ΠΟΛΥ ΣΗΜΑΝΤΙΚΕΣ ΙΔΙΟΤΗΤΕΣ

$$\boxed{x[n] \xrightarrow{ΣΜΦ} X(\omega)}$$

$n \in \mathbb{Z}$ — ΔΙΑΚΡΙΤΟ

$\omega \in \mathbb{R}$ — ΣΥΝΑΡΤΗΣΗ

(1) $x[n] \xrightarrow{ΣΜΦ} X(\omega) \overset{ΙΣΟΔ}{\longleftrightarrow} X(e^{j\omega})$

$$\boxed{x[n-T] \xrightarrow{ΣΜΦ} e^{-jT\omega} X(\omega) = \underbrace{e^{-jT\omega}}_{1} X(\omega)}$$

$$\boxed{|X(\omega)|}$$

$$f(x) \approx \sum_{k=\phi}^{N} \alpha_k \cos(\omega_k x + \varphi_k)$$

$$\underline{\alpha x[n] + \beta y[n]} = z[n] \rightarrow \underline{\alpha X(\omega) + \beta Y(\omega)}$$

$$x[n] \rightarrow X(\omega)$$

$$y[n] \rightarrow Y(\omega)$$

(2)

$$\boxed{\begin{aligned} y[n] &= h \circledast x \\ &\downarrow \quad \downarrow \\ Y(\omega) &= H(\omega) \cdot X(\omega) \end{aligned}}$$

$$\boxed{y[n] = \sum_{k=0}^{N} b_k x[n-k] + \sum_{k=1}^{M} \alpha_k y[n-k]}$$

$$\Downarrow$$

ΕΜΓ.ΔΕ. $\sum_{k=0}^{N} b_k x[n-k] + \sum_{k=\phi}^{M} \alpha_k y[n-k] = \phi$

$\alpha_0 = -1$

$$\sum_{k=0} b_k x[n-k] + \sum_{k=0} \alpha_k y[n-k] \ldots$$

$$\alpha_0 = -1$$

$$x[n] \to X(\omega)$$

$$y[n] \to Y(\omega)$$

$$\sum_{k=0}^{N} b_k e^{-j\omega k} \frac{X(\omega)}{X(\omega)} + \sum_{k=0}^{M} \alpha_k e^{-j\omega k} \frac{Y(\omega)}{X(\omega)} = 0$$

$$\boxed{\frac{Y(\omega)}{X(\omega)} = -\frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{k=0}^{N} \alpha_k e^{-j\omega k}}}$$

$$0 < \omega < 2\pi$$

$$\left|\frac{Y(\omega)}{X(\omega)}\right| = f(\omega) \geq 0$$

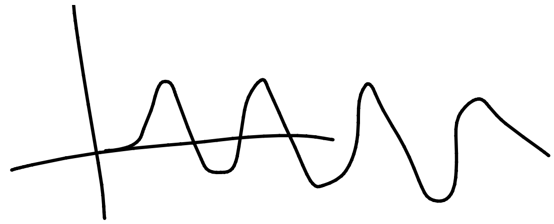
$$x[n] \to \boxed{h[n]} \to y[n]$$

$$\alpha x[n] \to \alpha y[n]$$

$$\alpha x[n-k] \to \alpha y[n-k]$$

$$x[n] = a\sin(\omega_0 n + k)$$

$x[n] = \alpha \sin(\omega_0 n + k)$



$y[n] = \beta \sin(\omega_0 n + \varphi)$

$$\left| \frac{Y(\omega)}{X(\omega)} \right| = \frac{\beta}{\alpha}$$

$y[n] = h[n] * x[n]$

$\Downarrow$

$$\frac{Y(\omega)}{X(\omega)} = \frac{h(\omega) \cdot \cancel{X(\omega)}}{\cancel{X(\omega)}} \Rightarrow$$

$$|h(\omega)| = \left| \frac{Y(\omega)}{X(\omega)} \right|$$