Homogeneous flow model

$$\frac{dP_F}{dz} = \frac{2 f_{TP} \cdot \dot{m}^2}{D \cdot \rho_H}$$

$$\rho_H = \frac{\rho_G \rho_L}{x \rho_L + (1-x)\rho_G}$$

$$f_{TP} = f(Re_{TP})$$

$$Re_{TP} = \frac{\dot{m} D}{\mu_{TP}}$$

$$\frac{1}{\mu_{TP}} = \frac{x}{\mu_G} + \frac{1-x}{\mu_L}$$

$$\Phi_L^2 = \left(1 + x \cdot \frac{\rho_L - \rho_G}{\rho_G}\right) \cdot \left(1 + x \cdot \frac{\mu_L - \mu_G}{\mu_G}\right)^{-1/4}$$

$$Re_L = \frac{\dot{m}(1-x) \cdot D}{\mu_L}$$

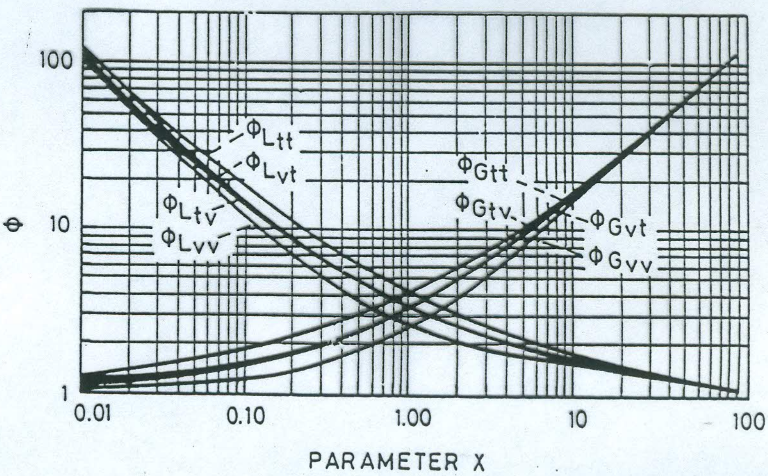
$$\left(\frac{dP_F}{dz}\right)_L = \frac{2 f_L \cdot \dot{m}^2 \cdot (1-x)^2}{D \cdot \rho_L}$$

$$f = 0.079 \left(\frac{\dot{m} \cdot D}{\mu}\right)^{-1/4}$$

$$(dP_F/dz) = \Phi_L^{\,2} \cdot (dP_F/dz)_L$$

Lockhart - Martinelli model

$$-\frac{dP_F}{dz} = \Phi^2{}_G \left(\frac{dP_F}{dz}\right)_G = \Phi^2{}_L \left(\frac{dP_F}{dz}\right)_L$$



$$\left(\frac{dP_F}{dz}\right)_L = \frac{2 f_L \dot{m}^2 (1-x)^2}{D \rho_L}$$

$$\left(\frac{dP_F}{dz}\right)_G = \frac{2 f_G \dot{m}^2 x^2}{D \rho_G}$$

$$Re_G = \frac{\dot{m} x D}{\mu_G} \qquad Re_L = \frac{\dot{m}(1-x) D}{\mu_L}$$

$$\Phi^2{}_L = 1 + \frac{c}{X} + \frac{1}{X^2}$$

$$f_L = \frac{16}{Re_L} \quad for \quad Re_L < 2000$$

$$\Phi^2{}_G = 1 + cX + X^2$$

$$\frac{1}{\sqrt{f_L}} = 1.74 - 2\log\left(\frac{2\epsilon}{D} + \frac{18.7}{Re_L \sqrt{f_L}}\right) \quad for \quad Re_L > 2000$$

$$X^2 = \frac{\left(\frac{dP_F}{dz}\right)_L}{\left(\frac{dP_F}{dz}\right)_G}$$

$$f_G = \frac{16}{Re_G} \quad for \quad Re_G < 2000$$

$$\frac{1}{\sqrt{f_G}} = 1.74 - 2\log\left(\frac{2\epsilon}{D} + \frac{1.87}{Re_G \sqrt{f_G}}\right) \quad for \quad Re_G > 2000$$

Friedel model

$$\Phi_L^{\ 2} = E + \frac{3.24FH}{Fr^{0.045} \cdot We^{0.035}}$$

$$E = (1-x^2) + x^2 \cdot \frac{\rho_L f_G}{\rho_G f_L}$$

$$F = x^{0.78} \cdot (1-x)^{0.24}$$

$$H = \left(\frac{\rho_L}{\rho_G}\right)^{0.91} \cdot \left(\frac{\mu_G}{\mu_L}\right)^{0.91} \cdot \left(1 - \frac{\mu_G}{\mu_L}\right)$$

$$Fr = \frac{\dot{m}^2}{gD\rho_{TP}^2} \qquad We = \frac{\dot{m}^2 D}{\rho_{TP} \cdot \sigma}$$

$$\rho_{TP} = \left(\frac{x}{\rho_G} + \frac{1-x}{\rho_L}\right)^{-1}$$

Baroczy - Chisholm model

$$\left(\frac{dP_F}{dz}\right) = \Phi_L^2 \cdot \left(\frac{dP_F}{dz}\right)_L$$

$$\left(\frac{dP_F}{dz}\right)_G = \frac{2f_G \dot{m}^2 \cdot x^2}{D\rho_G}$$

$$\Phi_L^2 = 1+(Y^2-1)\cdot\left[Bx^{(2-n)/2}\cdot(1-x)^{(2-n)/2}+x^{2-n}\right]$$

$$\left(\frac{dp_F}{dz}\right)_L = \frac{2f_L \dot{m}^2 \cdot (1-x)^2}{D\rho_L}$$

$$B = \frac{55}{\dot{m}^{1/2}} \quad for \quad 0<Y<9.5$$

$$Re_G = \frac{\dot{m}Dx}{\mu_G} \qquad Re_L = \frac{\dot{m}D(1-x)}{\mu_L}$$

$$B = \frac{520}{Y\cdot\dot{m}^{1/2}} \quad for \quad 9.5<Y<28$$

$$f_L = \frac{16}{Re_L} \quad for \quad Re_L<2000$$

$$B = \frac{15000}{Y^2\cdot\dot{m}^{1/2}} \quad for \quad 28<Y$$

$$\frac{1}{\sqrt{f_L}} = 1.74-2\log\left(\frac{2\epsilon}{D}+\frac{18.7}{Re_L\sqrt{f_L}}\right) \quad for \quad Re_L>2000$$

$$Y^2 = \frac{(dP_F/dz)_G}{(dP_F/dz)_L}$$

$$f_G = \frac{16}{Re_G} \quad for \quad Re_G<2000$$

$$\frac{1}{\sqrt{f_G}} = 1.74-2\log\left(\frac{2\epsilon}{D}+\frac{18.7}{Re_G\sqrt{f_G}}\right) \quad for \quad Re_G>2000$$

Beggs - Brill model

$$H_L(\alpha) = H_L(0)\cdot\Psi \qquad H_L(0) = \frac{\alpha_1\cdot\lambda_L{}^b}{N_{FR}^C}$$

$$N_{FR} = \frac{V^2}{g\cdot D}$$

$$V = V_L + V_G \qquad \lambda_L = \frac{V_L}{V}$$

$$\Psi = 1 + C[\sin(1.8\alpha) - 0.333\sin^3(1.8\alpha)]$$

$$C = (1-\lambda_L)\cdot\ln(\alpha_2\lambda_L{}^e\, N_{NL}{}^7\, N_{FR}^9)$$

$$N_{LV} = 1.938 V_L\left(\frac{\rho_L}{\sigma}\right)^{0.25}$$

$$\rho_s = \rho_L H_L + \rho_G H_G \qquad H_G = 1 - H_L$$

$$\left(\frac{dP}{dz}\right)_{el} = g\cdot\rho_s\cdot\sin\alpha$$

$$E_k = \frac{\rho_s V\cdot V_G}{P}$$

$$\frac{dP_F}{dz} = \frac{f_{TP}\cdot\rho_n\cdot V^2}{2D}$$

$$\rho_n = \rho_L\lambda_L + \rho_G\lambda_G$$

$$\lambda_G = \frac{V_G}{V}$$

$$f_{TP} = f_n\cdot\frac{f_{TP}}{f_n} \qquad \frac{f_{TP}}{f_n} = e^S$$

$$Re_n = \frac{\rho_n\cdot V\cdot D}{\mu_n} \qquad \mu_n = \mu_L\lambda_L + \mu_G\lambda_G$$

$$S = \ln y/[-0.0523 + 3.182\ln y - 0.8725\ln^2 y + 0.01853\ln^4 y]$$

$$y = \frac{\lambda_L}{H^{2_L}(\alpha)}$$

$$S = \ln(2.2y - 1.2) \qquad \text{for} \qquad 1 < y < 1.2$$

$$\left(\frac{dP}{dZ}\right)_{acc} = \frac{\rho_s V\cdot V_G}{P}\cdot\frac{dP}{dz}$$

$$\left(\frac{dP}{dZ}\right)_{total} = \frac{\left(\frac{dP}{dz}\right)_{el} + \frac{dP_F}{dz}}{1 - E_k}$$

BJA model

* Frictional pressure loss

$$S_L = H_L - \lambda_L \qquad F_{TP} = 1 - S_L$$

$$D_E = D \cdot F_{TP}^{0.5}$$

$$\rho_{TP} = \rho_L \cdot \lambda'_L + \rho_G \cdot \lambda'_G$$

$$\mu_{TP} = \mu_L \cdot \lambda'_L + \mu_G \cdot \lambda'_G$$

where:

$$\lambda'_L = \frac{\lambda_L}{F_{TP}} \qquad \lambda'_G = 1 - \lambda'_L$$

$$V_{TP} = \frac{Q_L + Q_G}{A \cdot F_{TP}}$$

$$Re_{TP} = \frac{D_E \cdot V_{TP} \cdot \rho_{TP}}{\mu_{TP}}$$

$$We = \frac{\rho_G \cdot V_G^2 \cdot E_i}{\sigma}$$

$$N_\mu = \frac{\mu_L}{\rho_L \cdot \sigma \cdot E_i}$$

$$E_i = \frac{34 \cdot C_1 \cdot \sigma}{\rho_G \cdot V \cdot C_2} \qquad \text{if} \quad We \cdot N_\mu \le 0.005$$

$$E_i = \frac{170 \cdot C_1 \cdot \sigma \cdot (We \cdot N_\mu)^{0.3}}{\rho_G \cdot V \cdot C_2} \qquad \text{if} \quad We \cdot N_\mu > 0.005$$

$$C_1 = 0.04$$
$$C_2 = 0.5$$

$$E_i = E_p \qquad \text{if} \quad E_i < E_p$$

$$E_i = \frac{D}{2} \qquad \text{if} \quad E_i > \frac{D}{2}$$

$$E_i = \frac{h_L}{4} \qquad \text{if} \quad E_i > \frac{h_L}{4}$$

$$E_\alpha = \frac{L_i E_i + L_G E_p}{L_i + L_G}$$

$$L_i = D \sin(\theta/2)$$

$$L_G = \pi \cdot D - L_L$$

$$L_L = \theta \cdot \frac{D}{2}$$

$$\frac{1}{f_{TP}^{\;0.5}} = 2\log_{10} \cdot \left[\frac{2.51}{Re_{TP} f_{TP}^{\;0.5}} + \frac{(E/D)e}{3.7}\right]$$

$$(E/D)_e = \frac{E_\alpha}{D_e}$$

$$\frac{dP_F}{dz} = \frac{f_{TP} \cdot \rho_{TP} \cdot V_{TP}^2}{2D_e}$$

* Elevation pressure drop

$$\left(\frac{dP}{dz}\right)_{el} = g \cdot \rho_G \sin\alpha \qquad \textit{for downwards flow}$$

$$\left(\frac{dP}{dz}\right)_{el} = g \cdot \rho_A \sin\alpha \qquad \textit{for horizontal or upwards flow}$$

where:

$$\rho_A = H_L \cdot \rho_L + (1 - H_L)\rho_G$$

* Total pressure loss

$$\left(\frac{dP}{dz}\right)_{total} = \frac{dP_F}{dz} + \left(\frac{dP}{dz}\right)_{el} + \left(\frac{dP}{dz}\right)_{acc}$$

Dukler - Flanigan model

* Frictional component

$$\frac{dP_F}{dz} = \frac{f_n f_{TP} \cdot \rho_k \cdot V^2}{0.14623D}$$

$$\rho_k = \frac{\rho_L \lambda_L^2}{H_L} + \frac{\rho_G \cdot (1-\lambda_L)^2}{1-H_L}$$

$$\lambda_L = \frac{Q_L}{Q_L + Q_G}$$

$$H_L = H_L(\lambda_L, Rey)$$

$$f_n = 0.0056 + 0.5(Rey)^{-0.32}$$

$$Rey = \frac{124 \cdot \rho_k \cdot V \cdot D}{\mu_n} \qquad where:\ V = V_L + V_G$$

$$\mu_n = \mu_L \cdot \lambda_L + \mu_G (1-\lambda_L)$$

* Elevation component

$$\Delta P_{el} = \frac{\rho_L \cdot H_L}{144} \cdot \Sigma Ze$$

* Total pressure drop

$$\left(\frac{dP}{dz}\right)_{total} = \frac{dP_F}{dz} + \left(\frac{dP}{dz}\right)_{el}$$