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ΕΞΙΣΩΣΕΙΣ ΟΡΙΑΚΟΥ ΣΤΡΩΜΑΤΟΣ ΔΙΦΑΣΙΚΗΣ ΡΟΗΣ

$$\frac{\partial \rho_g u_g}{\partial x} + \frac{\partial \rho_g \upsilon_g}{\partial y} = 0$$

$$\rho_g u_g \frac{\partial u_g}{\partial x} + \rho_g \upsilon_g \frac{\partial u_g}{\partial y} = \mu_g \frac{\partial^2 u_g}{\partial y^2} - D\rho (u_g - u_p)^2$$

$$\frac{\partial \rho_p u_p}{\partial x} + \frac{\partial \rho_p \upsilon_p}{\partial y} = 0$$

$$\rho_p u_p \frac{\partial u_p}{\partial x} + \rho_p \upsilon_p \frac{\partial u_p}{\partial y} = D\,\rho\,(u_g - u_p)^2$$

όπου

$$\rho_g = (1-Z)\,\rho$$

$$Z = \frac{V_p}{V} = \frac{\text{όγκος σωματιδίων}}{\text{όγκος μίγματος}}$$

$$\rho_p = Z\,\rho_{sp}$$

$$D = \frac{3}{4\,d_p}\, C_D\, Z$$

$$C_D = \frac{A}{Re_p} + \frac{B}{\sqrt{Re_p}} + C \qquad Re_p = \frac{\rho(u_g - u_p)d_p}{\mu}$$

<u>Πίνακας 1</u> Τιμές των συντελεστών A,B,C για τον υπολογισμό του συντελεστή αντίστασης σωματιδίων διαφόρων σχημάτων

Σχέση υπολογισμού C_D : $C_D = \frac{A}{Re_p} + \frac{B}{\sqrt{Re_p}} + C$					
Συντελεστές			Χαρακτηριστικό μέγεθος σωματιδίων	Σχήμα σωματιδίων	Αναφορά
A	B	C			
24	0	0	διάμετρος, d_p	σφαίρα	Stokes (3,4,6,7,11)
24	4	0.4	διάμετρος, d_p	σφαίρα	Molerus (8,9)
21.5	6.5	0.23	διάμετρος, d_p	σφαίρα	Muschelknautz (10)
24	6	0.35	1,1 * α α=μέγιστη διάσταση	πολύεδρο	(10)
23	6	0.5	1,08 * d_z d_z=διάμετρος βάσης	κύλινδρος l/d_z = 1	(10)
27	4.5	0.65	1,24 * α α=ακμή	κύβος	(10)

ΛΥΣΗ ΤΩΝ ΕΞΙΣΩΣΕΩΝ ΟΡΙΑΚΟΥ ΣΤΡΩΜΑΤΟΣ ΔΙΦΑΣΙΚΗΣ ΡΟΗΣ

$$U^2 \frac{d\delta^*_{2g}}{dx} = \frac{\tau_o}{\rho_{g\infty}} + \frac{\rho_{p\infty}}{\rho_{g\infty}} (UC_1\delta^*_{pg1} + U^{3/2}C_2\delta^*_{pg2} + U^2C_3\delta^*_{pg3})$$

$$U^2 \frac{d\delta_{2p}}{dx} = - (UC_1\delta^*_{pg1} + U^{3/2}C_2\delta^*_{pg2} + U^2C_3\delta^*_{pg3})$$

όπου

$$C_1 = \frac{3}{4}\left[\frac{\mu_g}{\rho_{sp}d_p^2}\right] A$$

$$C_2 = \frac{3}{4}\left[\frac{\rho^{1/2}\mu_g^{1/2}}{\rho_{sp}d_p^{3/2}}\right] B$$

$$C_3 = \frac{3}{4}\left[\frac{\rho}{\rho_{sp}d_p}\right] C$$

$$\delta^*_{2g} = \int_0^\delta \frac{\rho_g u_g}{\rho_{g\infty}U} (1- \frac{u_g}{U}) \, dy$$

$$\delta^*_{2p} = \int_0^\delta \frac{\rho_p u_p}{\rho_{p\infty}U} (1- \frac{u_p}{U}) \, dy$$

$$\delta^*_{pg1} = \int_0^\delta \frac{\rho_p}{\rho_{p\infty}} \left[(1- \frac{u_p}{U}) - (1 - \frac{u_g}{U})\right] dy$$

$$\delta^*_{pg2} = \int_0^\delta \frac{\rho_p}{\rho_{p\infty}} \left[(1 - \frac{u_p}{U}) - (1 - \frac{u_g}{U})\right]^{3/2} dy$$

$$\delta^*_{pg3} = \int_0^\delta \frac{\rho_p}{\rho_{p\infty}} \left[(1 - \frac{u_p}{U})-(1 - \frac{u_g}{u})\right]^2 dy$$

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$$\frac{u_g}{U} = \left(\frac{y}{\delta}\right)^{1/7} = n^{1/7} = \bar{u}_g \qquad \frac{\rho_p}{\rho_{p\infty}} = 1-(1-\rho_{pw})(1-\bar{u}_g)$$

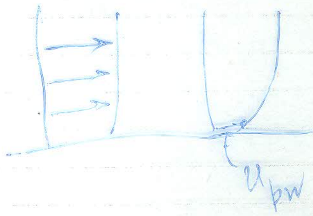
$$\frac{u_p}{U} = 1-(1-u_{pw})(1-\bar{u}_g) \qquad \frac{\rho_g}{\rho_{g\infty}} = 1-(1-\rho_{gw})(1-\bar{u}_g)$$

Για $n = 0$

$$u_g = 0 \;,\; u_p/U = u_{pw} \;,\; \rho_p/\rho_{p\infty} = \rho_{pw} \;,\; \rho_g/\rho_{g\infty} = \rho_{gw}$$

και για $n = 1$

$$\frac{u_g}{U} = 1 \;,\; \frac{u_p}{U} = 1 \;,\; \frac{\rho_p}{\rho_{p\infty}} = 1 \;,\; \frac{\rho_g}{\rho_{g\infty}} = 1$$



$$\rho_{g\infty} U \delta = \int_0^\delta \rho_g u_g d_y$$

$$\rho_{p\infty} U \delta = \int_0^\delta \rho_p u_p d_y$$

$$\frac{d\rho_{gw}}{d\bar{x}} = 0$$

$$\frac{d\rho_{pw}}{d\bar{x}} = -\frac{2\rho_{pw}+7}{2u_{pw}+7}\frac{du_{pw}}{d\bar{x}}$$

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$$\left(\frac{14}{720}\rho_{gw}+\frac{7}{90}\right)\frac{d\delta}{d\bar{x}}=\frac{\tau_o L}{\rho_{g\infty}U^2}+\frac{\rho_{p\infty}L\delta}{\rho_{g\infty}U^2}\left[UC_1\left(-\frac{2}{72}\rho_{pw}u_{pw}-\frac{7}{72}u_{pw}\right)+\right.$$

$$\left.+U^{3/2}C_2\left(0.025\,u_{pw}^{3/2}-1.99\,\rho_{pw}u_{pw}^{3/2}\right)+U^2C_3\left(\frac{6}{720}\rho_{pw}u_{pw}^2+\frac{14}{720}u_{pw}^2\right)\right]$$

$$\frac{d\delta}{d\bar{x}}\frac{1}{720}\left(14\,\rho_{pw}+56-8\,\rho_{pw}u_{pw}-42\,u_{pw}-6\,\rho_{pw}u_{pw}^2-14\,u_{pw}^2\right)=$$

$$=\frac{du_{pw}}{d\bar{x}}\frac{1}{720}\left(8\,\rho_{pw}\,\delta+42\,\delta+12\,\delta\,\rho_{pw}\,u_{pw}+\delta\,u_{pw}\right)-$$

$$-\frac{d\rho_{pw}}{d\bar{x}}\frac{1}{720}\left(14\,\delta-8\,\delta\,u_{pw}-6\,\delta\,u_{pw}^2\right)-\frac{L\delta}{U^2}\left[UC_1\left(-\frac{2}{72}\rho_{pw}u_{pw}-\frac{7}{72}u_{pw}\right)+\right.$$

$$\left.+U^{3/2}C_2\left(0.025\,u_{pw}^{3/2}-1.99\,\rho_{pw}\,u_{pw}^{3/2}\right)+U^2C_3\left(\frac{6}{720}\rho_{pw}\,u_{pw}^2+\frac{14}{720}u_{pw}^2\right)\right]$$

όπου $\bar{x}=x/L$ και

$$\tau_o=0.0225\,\rho\,U^2\left(\frac{\nu}{U\delta}\right)^{1/4}$$

$\bar{x}=0$: $\delta=0$, $u_{pw}=1$, $\rho_{pw}=1$

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$$\frac{\partial \varrho_i u_i}{\partial x} + \frac{\partial \varrho_i v_i}{\partial y} = 0$$

$$\varrho_i u_i \frac{\partial u_i}{\partial x} + \varrho_i v_i \frac{\partial u_i}{\partial y} = (2-i)\,\mu_i \frac{\partial^2 u_i}{\partial y^2} + (-1)^i D \varrho_1 (u_1 - u_2)^2$$

$$\varrho_i = [(2-i) + (-1)^i Z]^* \varrho_i$$

$$Z = V_2 / V_{12} = \text{volume of particles/volume of mixture}$$

$$D = \frac{3}{4 d_2} c_D Z$$

$$c_D = \sum_{j=1}^{3} A_j Re_2^{\frac{j-3}{2}}$$

$$Re_2 = {}^*\varrho_1 (u_1 - u_2) d_2 / \mu_1$$

Constants			Reynolds range	Characteristic dimension	Shape of particle	Reference
A_1	A_2	A_3				
24	0	0	$Re_2 < 0.1$	diameter, d_2	sphere	Stokes [3, 4, 6, 7]
24	4	0.4	$Re_2 < 10^4$	diameter, d_2	sphere	Molerus [8, 9]
21.5	6.5	0.23	$0.5 < Re_2 < 1000$	diameter, d_2	sphere	Muschelknautz [10]
24	6	0.35	$0.5 < Re_2 < 800$	$1.5 \times a$ a = largest dimen.	polyedron	Muschelknautz [10]
23	6	0.5	$0.5 < Re_2 < 600$	$1.08 \times d_z$ d_z = diam. of base	cylinder $l/d_z = 1$	Muschelknautz [10]
27	4.5	0.65	$0.5 < Re_2 < 400$	$1.24 \times a$ a = edge	cube	Muschelknautz [10]

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$$\frac{\mathrm{d}\delta^*_{2i}}{\mathrm{d}x} = (2-i)\,\frac{\tau_0}{\varrho_{1\infty}U^2} + (-1)^i\,\frac{\varrho_{2\infty}}{\varrho_{i\infty}U^2}\sum_{j=1}^{3} C_j \delta^*_{12j}\, U^{\frac{j+1}{2}}$$

where

$$\delta^*_{2i} = \int_0^\delta \frac{\varrho_i u_i}{\varrho_{i\infty} U}\left(1 - \frac{u_i}{U}\right)\mathrm{d}y\,\Bigg|_{i=1,2}$$

$$\delta^*_{12j} = \int_0^\delta \frac{\varrho_2}{\varrho_{2\infty}}\left[\left(1-\frac{u_1}{U}\right) - \left(1 - \frac{u_2}{U}\right)\right]^{\frac{j+1}{2}}\mathrm{d}y\,\Bigg|_{j=1,2,3}$$

$$C_j = \frac{3}{4}A_j\left[\frac{{}^*\varrho_1^{j-1}\mu_1^{3-j}}{\mathrm{d}_2^{5-j}\,{}^*\varrho_2^{2}}\right]^{1/2}\Bigg|_{j=1,2,3}$$

$$\bar{u}_2 = \frac{u_2}{U} = 1 - (1 - \bar{u}_{2w})\,(1 - \bar{u}_1)$$

$$\bar{\varrho}_i = \frac{\varrho_i}{\varrho_{i\infty}} = 1 - (1 - \bar{\varrho}_{iw})\,(1 - \bar{u}_1)$$

$$\bar{u}_1 = \sum_{k=l}^{m} \alpha_k \eta^k, \qquad \eta = y/\delta$$

$$\eta = 0:\ \bar{u}_2 = \bar{u}_{2w}, \qquad \bar{\varrho}_i = \bar{\varrho}_{iw}$$

$$\eta = 1:\ \bar{u}_2 = 1, \qquad \bar{\varrho}_i = 1$$

$$\varrho_{i\infty} U \delta = \int_0^\delta \varrho_i u_i \,\mathrm{d}y$$

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$$\frac{d\bar{\varrho}_{1w}}{d\bar{x}} = 0 \rightarrow \bar{\varrho}_{1w} \neq \bar{\varrho}_{1w}(x)$$

$$\frac{d\bar{\varrho}_{2w}}{d\bar{x}} = -\frac{\bar{\varrho}_{2w}\,[I^{(2)} - 2\,I^{(1)} + 1] + [I^{(1)} - I^{(2)}]}{\bar{u}_{2w}\,[I^{(2)} - 2\,I^{(1)} + 1] + [I^{(1)} - I^{(2)}]}\,\frac{d\bar{u}_{2w}}{d\bar{x}}$$

$$\delta^*_{21} = \delta\{\bar{\varrho}_{1w}\,[I^{(1)} - 2\,I^{(2)} + I^{(3)}] + [I^{(2)} - I^{(3)}]\}$$

$$\begin{aligned}\delta^*_{22} = \delta\{&\bar{u}_{2w}\,[I^{(1)} - 3\,I^{(2)} + 2\,I^{(3)}] - \bar{u}^2_{2w}\,[I^{(1)} - 2\,I^{(2)} + I^{(3)}]\\ &+ \bar{\varrho}_{2w}\,[I^{(1)} - 2\,I^{(2)} + I^{(3)}]\\ &+ \bar{\varrho}_{2w}\bar{u}_{2w}\,[1 - 4\,I^{(1)} + 5\,I^{(2)} - 2\,I^{(3)}]\\ &+ \bar{\varrho}_{2w}\bar{u}^2_{2w}\,[-1 + 3\,I^{(1)} - 3\,I^{(2)} + I^{(3)}] + [I^{(2)} - I^{(3)}]\}\end{aligned}$$

$$\begin{aligned}\delta^*_{12j} = \bar{u}_{2w}^{\frac{j-1}{2}}\,\delta\Bigg\{&\bar{\varrho}_{2w} + \left[1 - \frac{j+3}{2}\,\bar{\varrho}_{2w}\right] I^{(1)}\\ &+ \left[-\frac{j+1}{2} + \frac{(j+1)(j+3)}{8}\,\bar{\varrho}_{2w}\right] I^{(2)}\\ &+ \left[\frac{j^2-1}{8} - \frac{(j^2-1)(j+3)}{48}\,\bar{\varrho}_{2w}\right] I^{(3)}\\ &+ \left[-\frac{(j^2-1)(j-3)}{48} + \frac{(j^2-1)(j^2-9)}{384}\,\bar{\varrho}_{2w}\right] I^{(4)}\\ &+ \left[\frac{(j^2-1)(j-3)(j-5)}{384}\right.\\ &\left.- \frac{(j^2-1)(j-3)(j-5)}{384}\,\bar{\varrho}_{2w}\right] I^{(5)}\Bigg\}\end{aligned}$$

where

$$I^{(k)} = \int_0^1 (\bar{u}_1)^k\,d\eta \quad \text{with} \quad k = 1, 2, 3, 4, 5$$

$$\bar{x} = x/L = 0 : \delta = 0, \quad \bar{u}_{2w} = 1, \quad \bar{\varrho}_{2w} = 1$$

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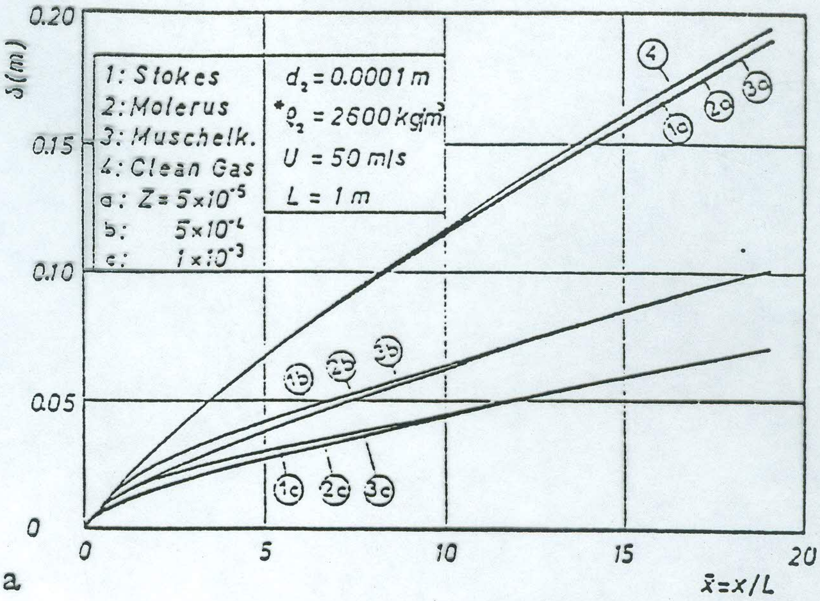

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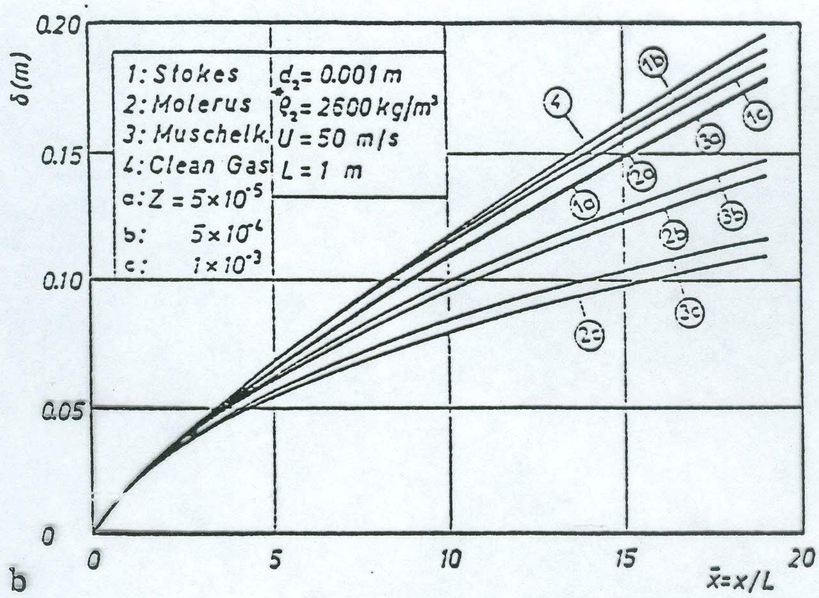

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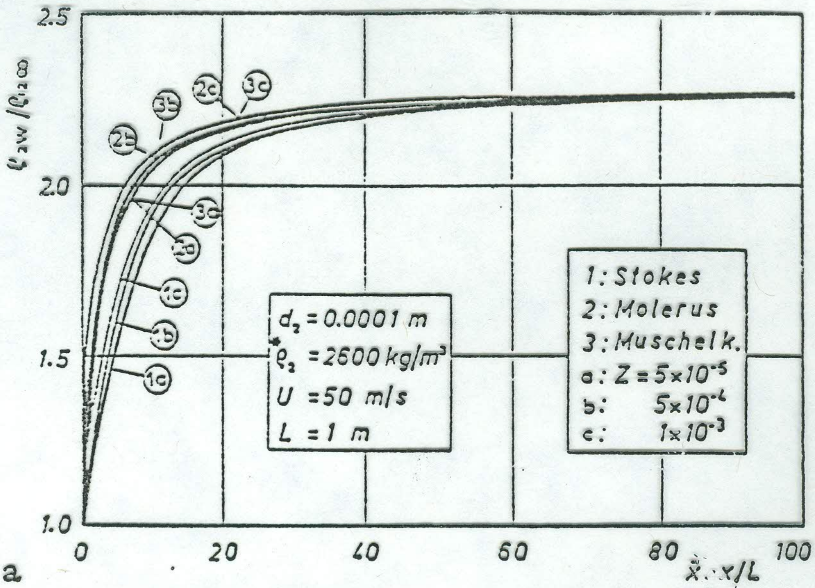

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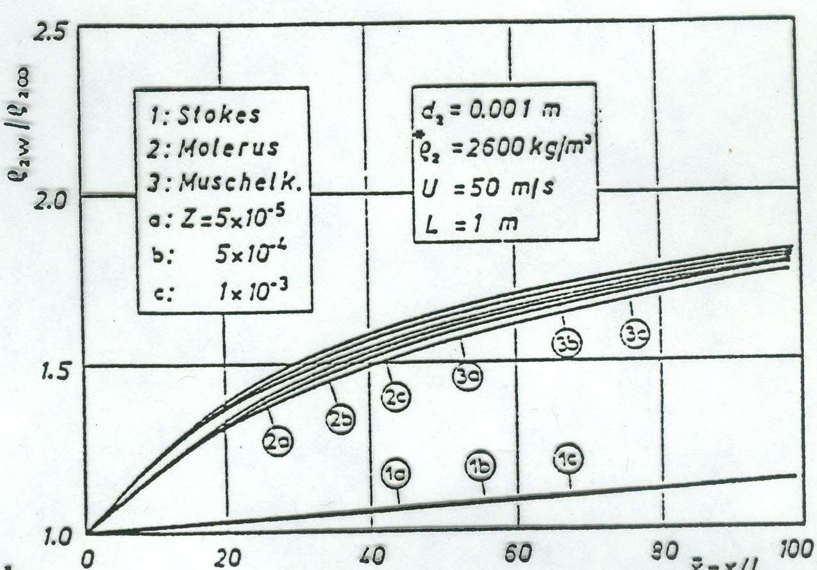

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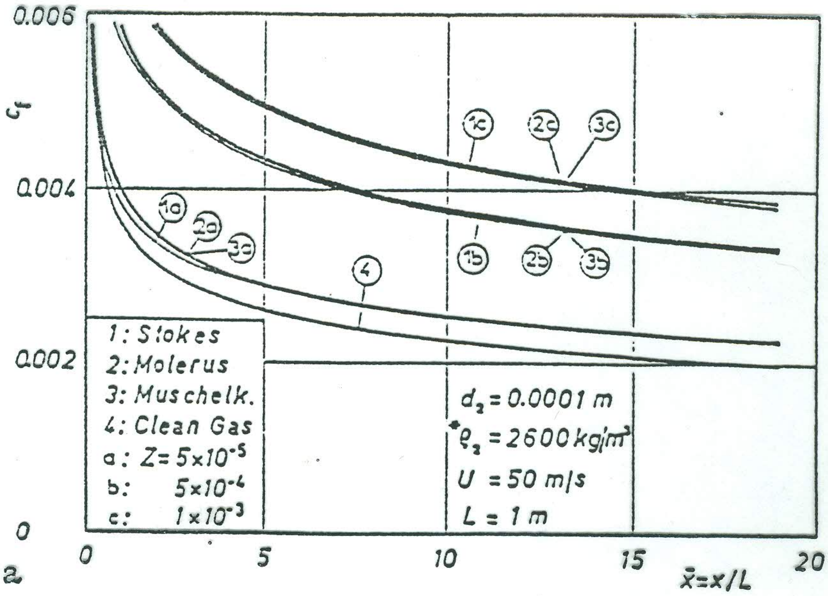

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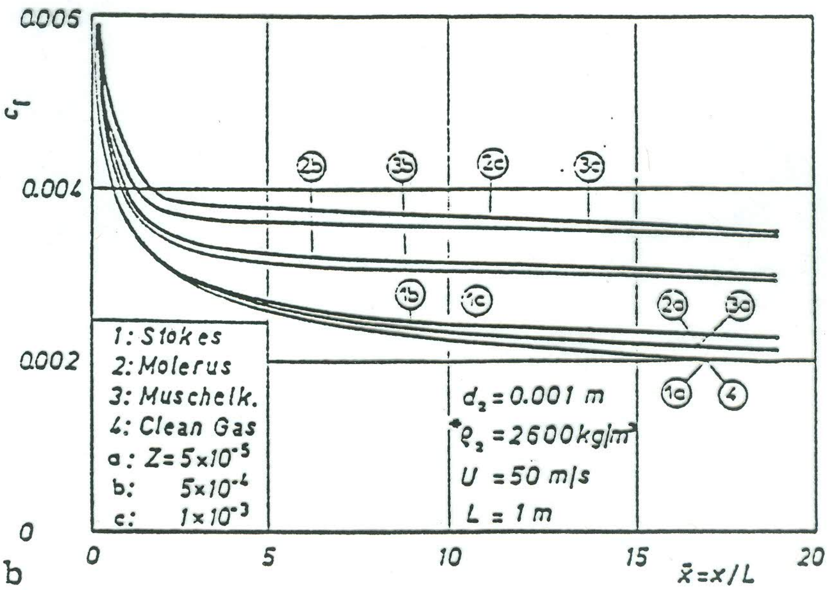

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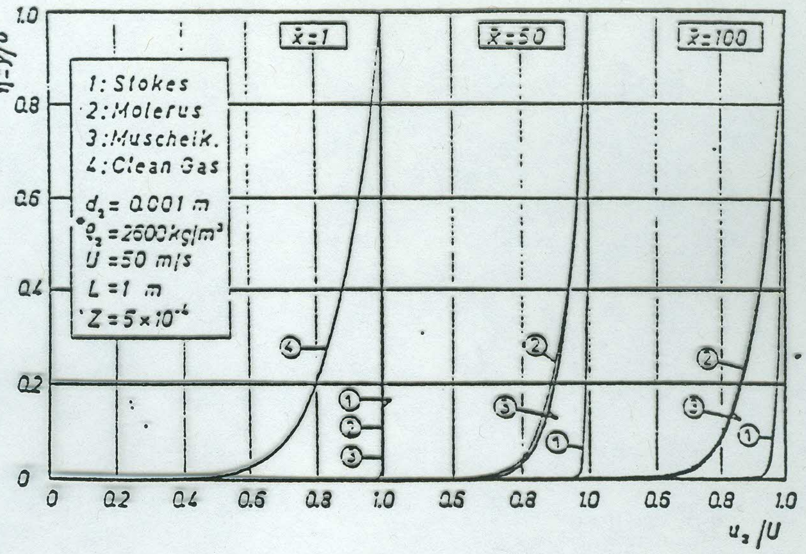

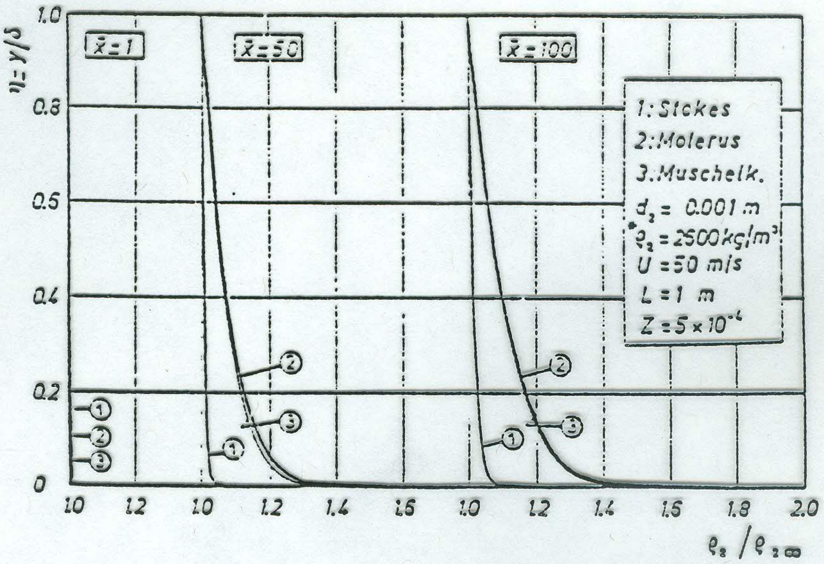

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