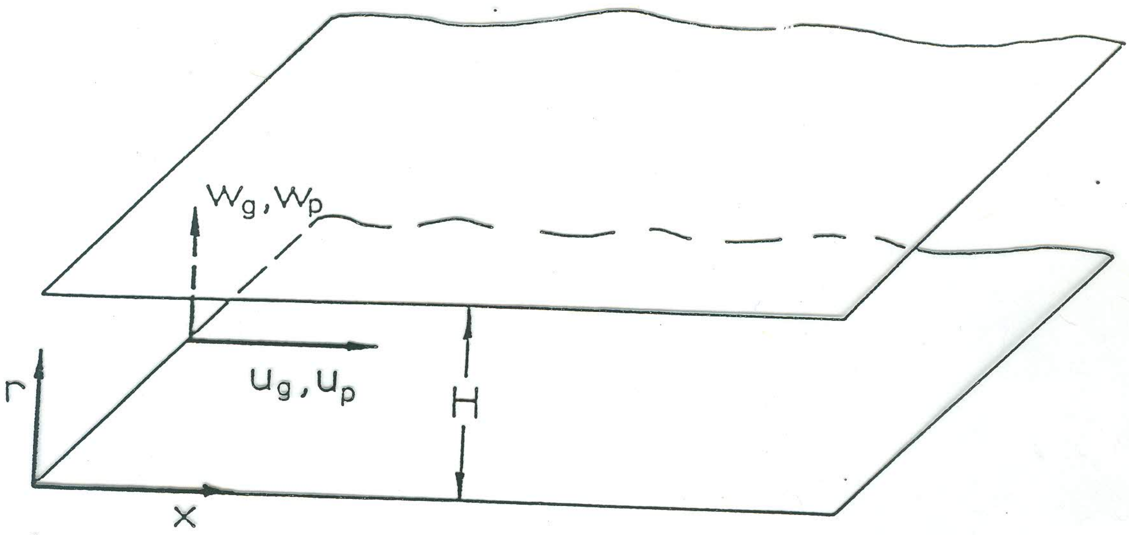
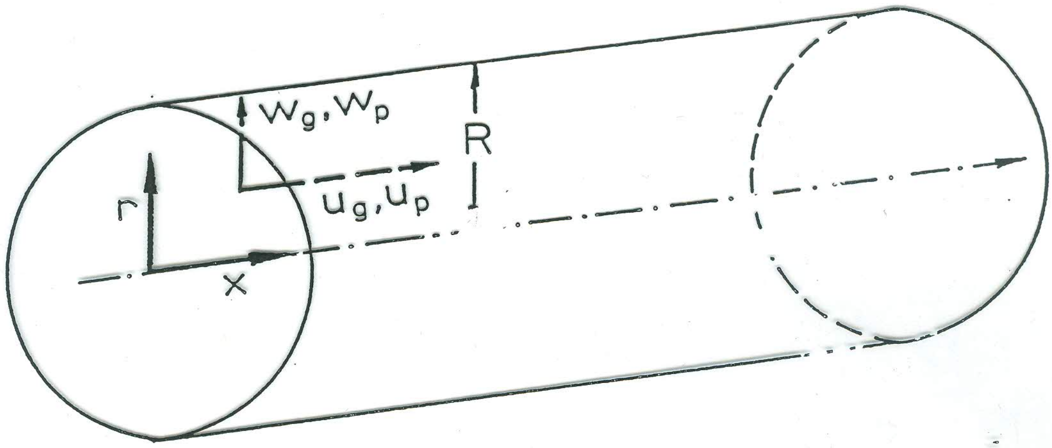
Subject ΔΙΦΑΣΙΚΗ ΡΟΗ	Date	Project
	Author	Report

- <u>Ορθογωνικο συστημα</u>



- <u>Κυλινδρικο συστημα</u>



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ΔΙΦΑΣΙΚΗ ΡΟΗ	Author	Report

- **Εξισωση συνεχειας**

$$\frac{\partial}{\partial x}(\bar{\rho}_r u_r) + \frac{1}{r^k}\frac{\partial}{\partial r}(\bar{\rho}_r w_r r^k) = 0$$

- **Εξισωση ορμης x -διευθυνση**

$$\begin{aligned}
\bar{\rho}_r u_r \frac{\partial u_r}{\partial x} + \bar{\rho}_r w_r \frac{\partial u_r}{\partial r} = {} & - \frac{\partial p_r}{\partial x} + \frac{\partial}{\partial x}\Big[2\mu_r \frac{\partial u_r}{\partial x} - \\
& - \frac{2}{3}\mu_r\Big(\frac{1}{r^k}\frac{\partial}{\partial r}(r^k w_r) + \frac{\partial u_r}{\partial x}\Big)\Big] + \frac{1}{r^k}\frac{\partial}{\partial r}\Big[r^k \mu_r\Big(\frac{\partial u_r}{\partial r} + \\
& + \frac{\partial w_r}{\partial x}\Big)\Big] + D_{x_r} + L_{x_r}
\end{aligned}$$

- **Εξισωση ορμης r -διευθυνση**

$$\begin{aligned}
\bar{\rho}_r u_r \frac{\partial w_r}{\partial x} + \bar{\rho}_r w_r \frac{\partial w_r}{\partial r} = {} & - \frac{\partial p_r}{\partial r} + \frac{1}{r^k}\frac{\partial}{\partial r}\Big[2\mu_r r^k \frac{\partial w_r}{\partial r} - \\
& - \frac{2}{3}\mu_r r^k\Big(\frac{1}{r^k}\frac{\partial}{\partial r}(r^k w_r) + \frac{\partial u_r}{\partial x}\Big)\Big] + \frac{\partial}{\partial x}\Big[\mu_r\Big(\frac{\partial u_r}{\partial r} + \\
& + \frac{\partial w_r}{\partial x}\Big)\Big] + D_{r_r} + L_{r_r} + G_{r_r}
\end{aligned}$$

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- **Αδιαστατα μεγεθη**

$$\bar{u}_g = \frac{u_g}{u_{gR}} \qquad \bar{w}_g = \frac{w_g}{u_{gR}} \qquad \bar{u}_p = \frac{u_p}{u_{gR}} \qquad \bar{w}_p = \frac{w_p}{u_{gR}}$$

$$\bar{\bar{\rho}}_g = \frac{\bar{\rho}_g}{\rho_{gR}} \qquad \bar{\bar{\rho}}_p = \frac{\bar{\rho}_p}{\rho_{gR}} \qquad \bar{\mu}_g = \frac{\mu_g}{\mu_{gR}} \qquad \bar{\mu}_p = \frac{\mu_p}{\mu_{gR}}$$

$$\bar{x} = \frac{x}{L_R} \qquad \bar{r} = \frac{r}{L_R} \qquad \bar{r}_p = \frac{r_p}{L_R} \qquad \bar{p} = \frac{p}{\rho_{gR} u_{gR}^2}$$

- **Ταξη μεγεθους των ορων**

$\delta \ll L$

- **χαρακτηριστικη ιδιοτητα**

$$O(1) : \bar{u}_r , \frac{\partial \bar{u}_r}{\partial \bar{x}} , \frac{\partial \bar{w}_r}{\partial \bar{r}} , \frac{\partial^2 \bar{u}_r}{\partial \bar{x}^2}$$

$$O\left(\frac{1}{\delta}\right) : \frac{\partial \bar{u}_r}{\partial \bar{r}} , \frac{\partial^2 \bar{w}_r}{\partial \bar{r}^2}$$

$$O\left(\frac{1}{\delta^2}\right) : \frac{\partial^2 \bar{u}_r}{\partial \bar{r}^2} , \mathrm{Re}$$

$$O(\delta) : \bar{w}_r , \frac{\partial \bar{w}_r}{\partial \bar{x}} , \frac{\partial^2 \bar{w}_r}{\partial \bar{x}^2}$$

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- εξισωση συνεχειας.

$$\frac{\partial}{\partial\bar{x}}(\bar{\bar{\rho}}_r\bar{u}_r) + \frac{1}{\bar{r}^k}\frac{\partial}{\partial\bar{r}}(\bar{\bar{\rho}}_r\bar{w}_r\bar{r}^k) = 0$$

- εξισωση ορμης x -διευθυνση

$$\bar{\bar{\rho}}_r\bar{u}_r\frac{\partial\bar{u}_r}{\partial\bar{x}} + \bar{\bar{\rho}}_r\bar{w}_r\frac{\partial\bar{u}_r}{\partial\bar{r}} = -\frac{\partial\bar{p}_r}{\partial\bar{x}} + \frac{1}{Re_R}\bar{\mu}_r\frac{\partial^2\bar{u}_r}{\partial\bar{r}^2} +$$

$$+\frac{1}{Re_R}\frac{\partial\bar{\mu}_r}{\partial\bar{r}}\frac{\partial\bar{u}_r}{\partial\bar{r}} + \frac{1}{Re_R}\frac{k}{\bar{r}}\bar{\mu}_r\frac{\partial\bar{u}_r}{\partial\bar{r}} \pm \frac{3}{8\bar{r}_p}c_{Dx}Z\bar{\rho}_g(\bar{u}_g-\bar{u}_p)^2 \pm$$

$$\pm J_yZ\bar{\rho}_g(\bar{w}_g-\bar{w}_p)\frac{\partial\bar{u}_g}{\partial\bar{r}}$$

- εξισωση ορμης r -διευθυνση

$$0 = -\frac{\partial\bar{p}_r}{\partial\bar{r}} \pm \frac{3}{8\bar{r}_p}c_{Dr}Z\bar{\rho}_g(\bar{w}_g-\bar{w}_p)^2 \pm$$

$$\pm J_yZ\bar{\rho}_g(\bar{u}_g-\bar{u}_p)\frac{\partial\bar{u}_g}{\partial\bar{r}} - G_r$$

$$G_p = \frac{Z}{F_{r_R}}(\bar{\rho}_p - \bar{\rho}_g)$$

$$G_g = \frac{Z}{F_{r_R}}\bar{\rho}_g$$

$$Re_R = \frac{\rho_{gR}u_{gR}L_R}{\mu_{gR}} \qquad F_{r_R} = \frac{u^2_{gR}}{gL_R}$$

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• Αρχικες και Οριακες Συνθηκες

$$\bar{p} = \bar{p}(\bar{r},0) = \bar{p}_{\sigma\tau\alpha\theta}$$

$$\bar{M} = \bar{M}(\bar{r},0)$$

$$\frac{\partial \bar{u}_r}{\partial \bar{r}} = 0 \qquad \text{και} \qquad \bar{w}_r = 0$$

$$\frac{\partial \bar{\bar{\rho}}_r \bar{u}_r}{\partial \bar{x}} + 2^k \frac{\partial \bar{\bar{\rho}}_r \bar{w}_r}{\partial \bar{r}} = 0$$

$$\bar{\bar{\rho}}_r \bar{u}_r \frac{\partial \bar{u}_r}{\partial \bar{x}} = - \frac{\partial \bar{p}_r}{\partial \bar{x}} + \frac{2\bar{\mu}_r}{Re_R} \frac{\partial^2 \bar{u}_r}{\partial \bar{r}^2} \pm \frac{3}{8\bar{r}_p} c_{Dx} Z \bar{\rho}_g (\bar{u}_g - \bar{u}_p)^2$$

$$\bar{u}_r(x,R) = 0 \qquad \text{και} \qquad \bar{w}_r(x,R) = 0$$

ΣΥΣΤΗΜΑ ΕΞΙΣΩΣΕΩΝ ΓΙΑ ΤΥΡΒΩΔΗ ΡΟΗ

$u_r = \bar{u}_r + u'_r \qquad w_r = \bar{w}_r + w'_r \qquad p = \bar{p} + p'$

$$\bar{\rho}_r \bar{u}_r \frac{\partial \bar{u}_r}{\partial x} + \bar{\rho}_r \bar{w}_r \frac{\partial \bar{u}_r}{\partial r} = - \frac{\partial \bar{p}_r}{\partial x} + \frac{1}{Re_R} \frac{1}{r^k} \frac{\partial}{\partial r}\left(\mu_r r^k \frac{\partial \bar{u}_r}{\partial r}\right) \pm$$

$$\pm \frac{3}{8 r_p} c_{Dx} Z \rho_g \left[(\bar{u}_g - \bar{u}_p)^2 + (\overline{u'_g} - \overline{u'_p})^2 \right] \pm$$

$$\pm J_y Z \rho_g \left[(\bar{w}_g - \bar{w}_p) \frac{\partial \bar{u}_g}{\partial r} + (\overline{w'_g} - \overline{w'_p}) \frac{\partial \overline{u'_g}}{\partial r} \right] - \rho_r \overline{\frac{\partial u'_r}{\partial x}} - \rho_r \overline{\frac{\partial u'_r w'_r}{\partial r}}$$

$$\bar{\rho}_r \bar{u}_r \frac{\partial \bar{u}_r}{\partial x} + \bar{\rho}_r \bar{w}_r \frac{\partial \bar{u}_r}{\partial r} = - \frac{\partial \bar{p}_r}{\partial x} + \frac{1}{Re_R} \frac{1}{r^k} \frac{\partial}{\partial r}\left(\mu_r r^k \frac{\partial \bar{u}_r}{\partial r}\right) \pm$$

$$\pm \frac{3}{8 r_p} c_{Dx} Z \rho_g (\bar{u}_g - \bar{u}_p)^2 \pm J_y Z \rho_g (\bar{w}_g - \bar{w}_p) \frac{\partial \bar{u}_g}{\partial r} - \frac{\partial}{\partial r}(\overline{\rho_r u'_r w'_r})$$

$$\tau_t = - \overline{\rho_r u'_r w'_r} = \mu_t \frac{\partial \bar{u}_r}{\partial r}$$

$$\tau_{o\lambda} = \tau_l + \tau_t = \mu_l \frac{\partial \bar{u}_r}{\partial r} + \mu_t \frac{\partial \bar{u}_r}{\partial r} = (\mu_l + \mu_t) \frac{\partial \bar{u}_r}{\partial r}$$

ΣΤΡΩΤΗ ΡΟΗ

- **Αερας**

$$\frac{\mu_g}{\mu_{go}} = \frac{S+T_o}{S+T}\left(\frac{T}{T_o}\right)^{1.5}$$

σχεση Sutherland

$S = 123.6$ K

$$\boxed{\mu_g = \frac{\beta T^{1.5}}{S+T}}$$

$\beta = 1.458 \times 10^{-6}$
$S = 110.4$ K

- **Σωματιδια**

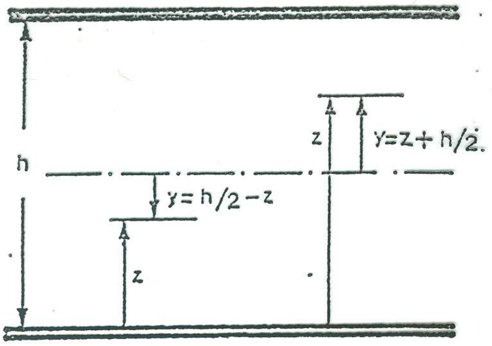
$$\nu_m = \frac{\nu_g}{1+n}$$

σχεση Pai

$$n = \frac{k_p}{1-k_p} \quad \text{και} \quad k_p = \frac{Z\rho_{sp}}{Z\rho_{sp}+(1-Z)\rho_g} = \frac{\bar{\rho}_p}{\rho_m}$$

$$\mu_m = \mu_g - Z\mu_g$$

$$\boxed{\mu_p = Z\mu_g}$$



Σχ. 1. Επεξήγηση των διαφόρων συντεταγμένων.

$$\tau_t = \tau_o \frac{y}{h/2}$$ όπου $y = |z-h/2|$ με $y = 0$ για $z = h/2$.

$$\tau_t = 2\,\tau_o \bar{y}$$ όπου $\bar{y} = |\bar{z} - 1/2|$ με $\bar{y} = y/h$. και $\bar{z} = z/h$.

σχέση του Boussinesq $$\tau_{tg} = \mu_{tg} \frac{d\bar{u}_g}{dz}$$

$$\mu_{tg} = \frac{\tau_{tg}}{d\bar{u}_g/dz} = \frac{2\tau_{og}\bar{y}}{d\bar{u}_g/dz} \qquad \tau_{og} = 0{,}332\ \rho_{gR} u_{gR}^2 Re_x^{-1/2} \qquad Re_x = \frac{\rho_{gR} u_{gR} x}{\mu_{gR}}$$

$$\tau_{tp} = \mu_{tp} \frac{d\bar{u}_p}{dz} \qquad \mu_{tp} = \frac{\tau_{tp}}{d\bar{u}_p/dz} = \frac{2\tau_{op}\bar{y}}{d\bar{u}_p/dz}$$

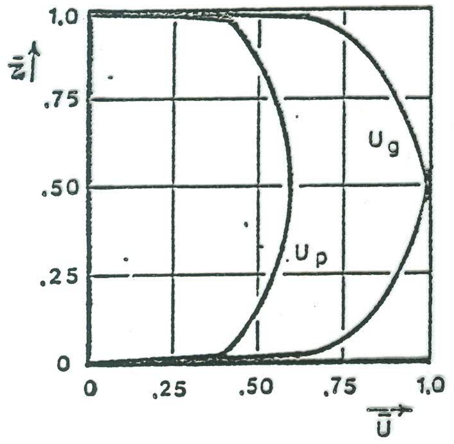
$$\tau_{ogp} = \frac{0.332\rho_{gR} u_{gR}^2}{Re_x^{1/2}} \left[1+n\right]^{1/2} \left[1+0.49\,\frac{Lv}{x}\,\frac{n}{1+n}\right] \qquad n = \frac{Z\rho_p}{(1-Z)\rho_g}$$

$$Lv = \frac{4\rho_p d_p^2 u_{gR}}{18\mu_{gR}}$$

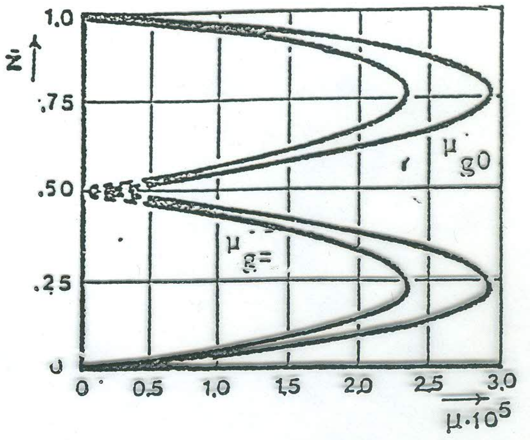
$$\tau_{ogp} = \tau_{og} + \tau_{op} \qquad \tau_{op} = \tau_{ogp} - \tau_{og}$$

$$\tau_{op} = \frac{0.332\rho_{gR} u_{gR}^2}{Re_x^{1/2}} \left[\left(1+n\right)^{1/2} \left(1+0.49\,\frac{Lv}{x}\,\frac{n}{1+n}\right) - 1 \right]$$

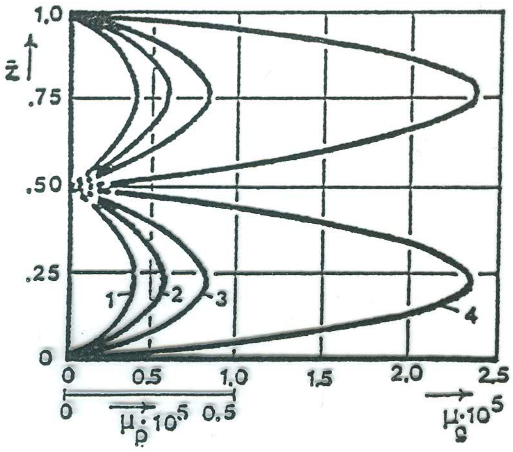
$$\left(\frac{du}{dz}\right)_i = \frac{u_{i+1} - u_{i-1}}{2\Delta z}$$



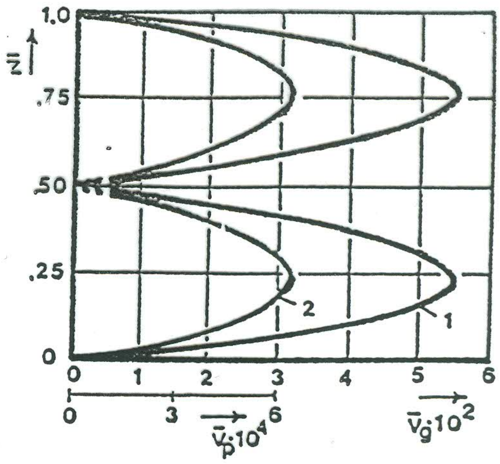
Σχ. 8. Κατανομές ταχύτητας αέρα και σωματιδίων για τον υπολογισμό του ιξώδους.



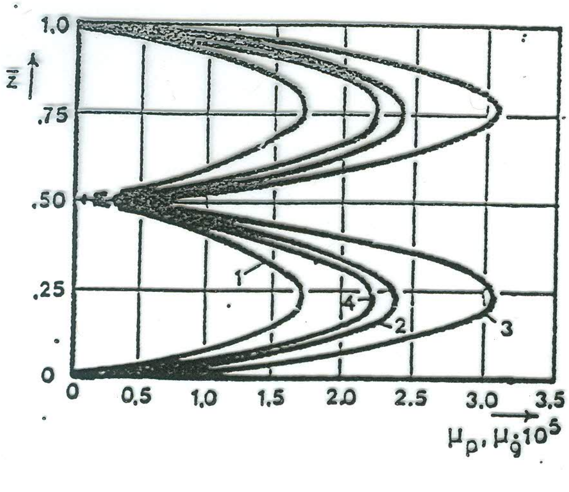
Σχ. 9. Κατανομή ιξώδους αέρα για ροή σε κυκλικό αγωγό μ_{g0}, και για ροή σε παράλληλες πλάκες $\mu_{g=}$.



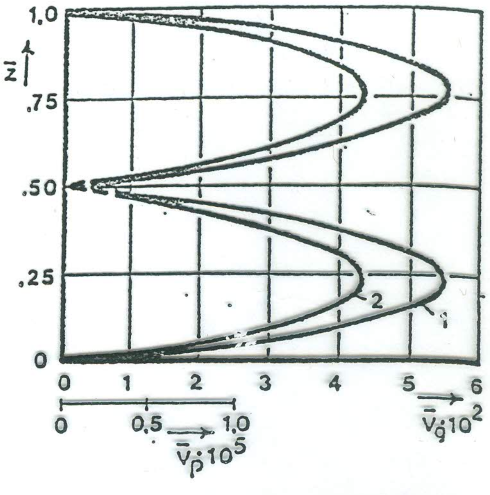
χ.10. Κατανομές ιξώδους αέρα και σωματιδίων για αραιή πνευματική μεταφορά (1: ρ_p = 1250, 2: ρ_p = 1800 3: ρ_p = 2600, 4: ρ_g = 1,2kg/m^3



Σχ.11. Κατανομή κινηματικού ιξώδους αέρα σωματιδίων σε αδιάστατη μορφή. (Στή καμπύλη 2 συμπίπτουν οι κατανομές και των τριών σωματιδίων του σχ.10)



Σχ.12. Κατανομές ιξώδους αέρα και σωματιδίων για πυκνή πνευματική μεταφορά (1:ρ_p= 1250, 2: ρ_p=1800, 3: ρ_p = = 2600, 4: ρ_g= 1,2 kg/m^3)



Σχ.13. Κατανομή κινηματικού ιξώδους αέρα σωματιδίων σε αδιάστατη μορφή. (Στή καμπύλη 2 συμπίπτουν οι κατανομές και των τριών σωματιδίων του σχ12).