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- Θεωρια Συνεχους Μεσου

ρευστο – ψευδορευστο

g   p

φαση $r : q_r , T_r , P_r , \rho_r$

- Μαζα μιγματος $\quad M = M_p + M_g$

- Ογκος μιγματος $\quad V = V_p + V_g$

- Ογκος σωματιδιων $\quad V_p = n_p V v_s$

- Μαζα σωματιδιων $\quad M_p = m_p n_p V$

- Πυκνοτητα σωματιδιων
  - φασικη $\quad \rho_{sp} = \dfrac{M_p}{V_p} = \dfrac{m_p}{v_s}$
  - μερικη $\quad \bar{\rho}_p = \dfrac{M_p}{V} = m_p n_p = \rho_{sp} v_s n_p = \rho_{sp} Z$

- Πυκνοτητα αερα
  - φασικη $\quad \rho_g = \dfrac{M_g}{V_g}$
  - μερικη $\quad \bar{\rho}_g = \dfrac{M_g}{V} = \dfrac{M_g}{V_g}\dfrac{V_g}{V} = (1-Z)\rho_g$

- Ογκομετρικη συγκεντρωση $\quad Z = \dfrac{V_p}{V}$

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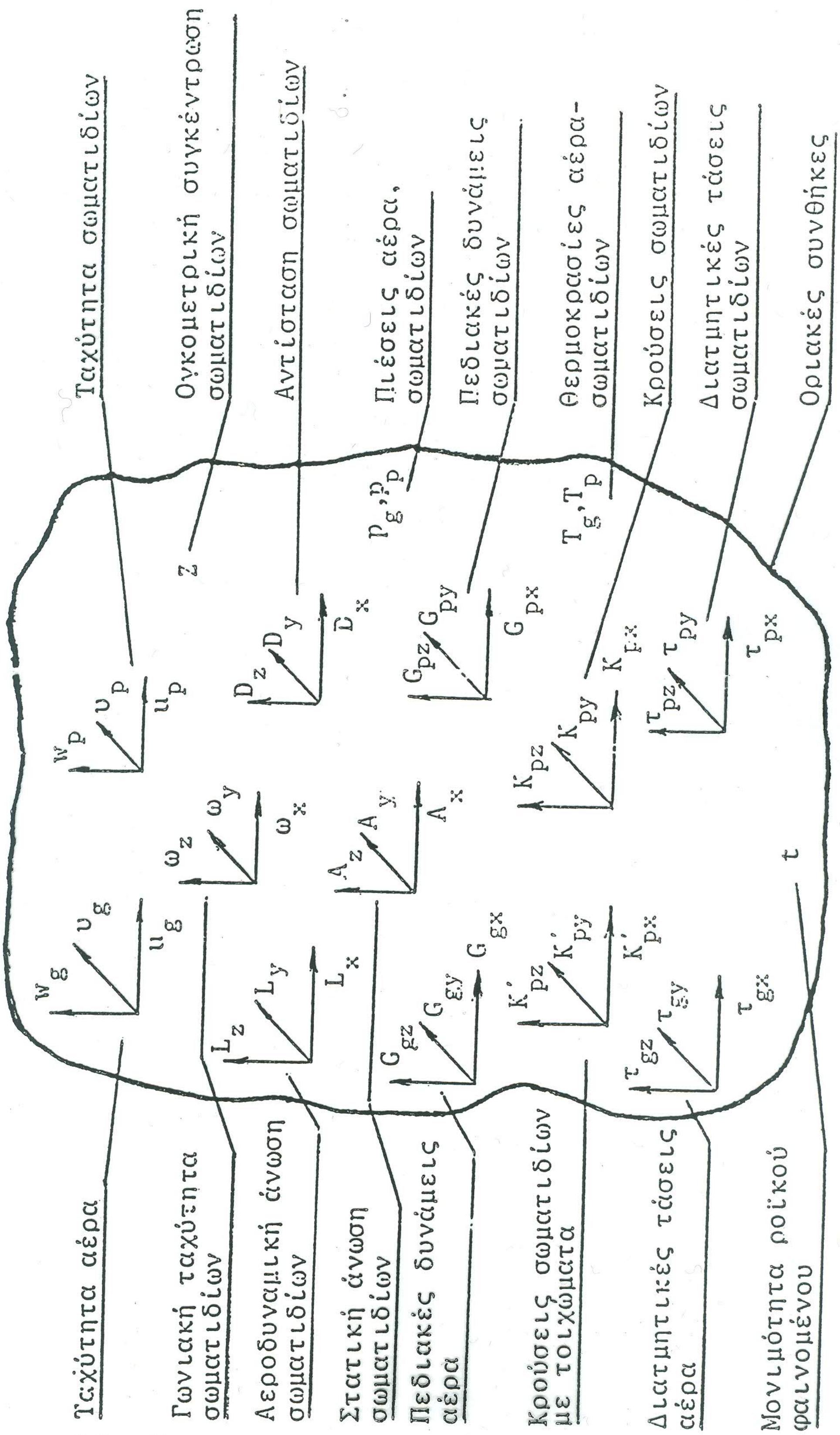

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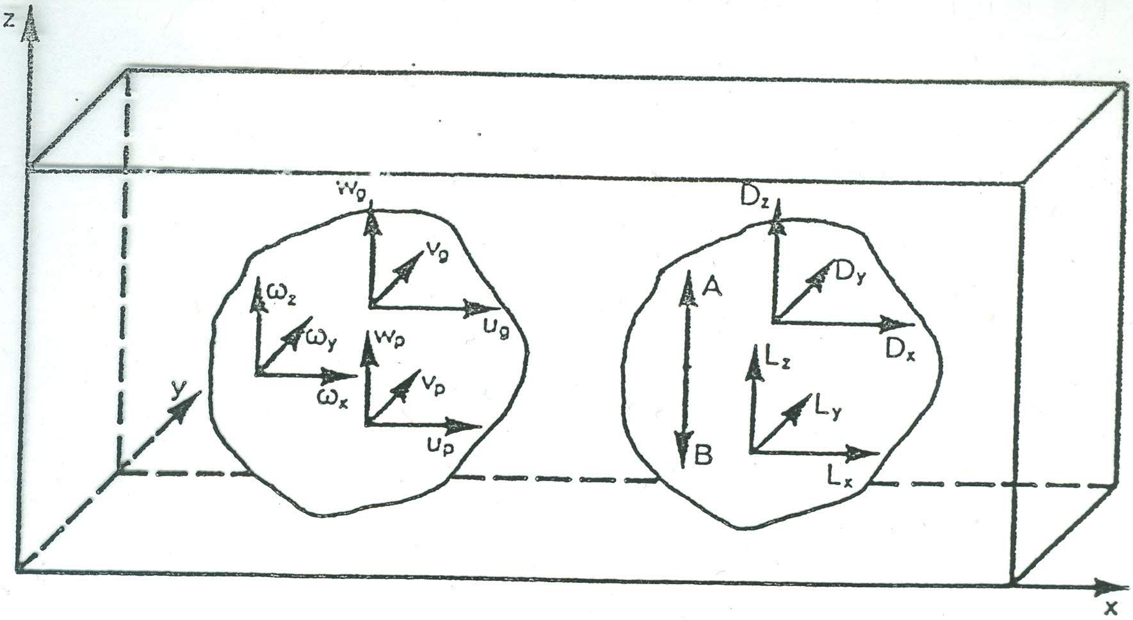

- Καταστατικη εξισωση

$$p_g = \bar{\rho}_g R T_g = (1-Z)p \qquad p_p = Zp \qquad p = p_g + p_p$$

$$\rho_{sp} = \sigma\tau\alpha\vartheta.$$

- Εξισωση συνεχειας

$$\frac{\partial \bar{\rho}_r}{\partial t} + \frac{\partial (\bar{\rho}_r u_r^i)}{\partial x^i} = 0 \qquad \vec{q}_r = \vec{i} u_r + \vec{j} \upsilon_r + \vec{k} w_r$$

- Εξισωση ορμης

$$\bar{\rho}_r \left(\frac{\partial}{\partial t} + u_r^i \frac{\partial}{\partial x^i} \right) \vec{q}_r = \bar{\rho}_r \cdot \frac{D_r \vec{q}_r}{Dt} = -\nabla p_r + \nabla \cdot \tau_r + \vec{F}_{br} + \vec{F}_r$$

- Εξισωση ενεργειας

$$\frac{D}{Dt}\left[\bar{\rho}_r \left(U_{mr} + \frac{1}{2} q_r^2 + \Phi_r \right) \right] + \frac{\partial}{\partial x^j}\left[\delta^{ij} u_r^i p_r - u_r^i \tau_r^{ij} - Q_{cr}^j \right] = k_T \Delta T$$

- **Γενικη σχεση αντιστασης**

$$\vec{F}_p = \vec{D} = c_D n_p \frac{1}{2}\rho_g (\vec{q}_g - \vec{q}_p)^2 A_p = c_D n_p p_d A_p$$

- **Σχεσεις υπολογισμου c_D**

$$c_D = \frac{24}{Re_p} \qquad \vec{F}_p = 6\pi r_p \mu_g n_p (\vec{q}_g - \vec{q}_p) = \frac{1}{2} c_D n_p A_p \rho_g \vec{c}^2$$

$$c_D = \frac{24}{Re_p}\left(1 + \frac{3}{16} Re_p\right)$$

$$\boxed{c_D = \frac{24}{Re_p} + \frac{4}{\sqrt{Re_p}} + 0.4} \qquad Re_{p_x} = \frac{(u_g - u_p)\rho_g d_p}{\mu_g}$$

$$D_x = c_{D_x} n_p \frac{1}{2}(u_g - u_p)^2 A_p = c_{D_x} n_p p_{d_x} A_p$$

- **Μη σφαιρικα σωματιδια**

σφαιρικοτητα

$$\Phi = \frac{\pi\left(\frac{6V_s}{\pi}\right)^{2/3}}{S_p} = (6\sqrt{\pi})^{2/3} \cdot \frac{V_s^{2/3}}{S_p} = \frac{\pi D_{\iota\sigma}^2}{S_p}$$

σχεση Muschelknautz

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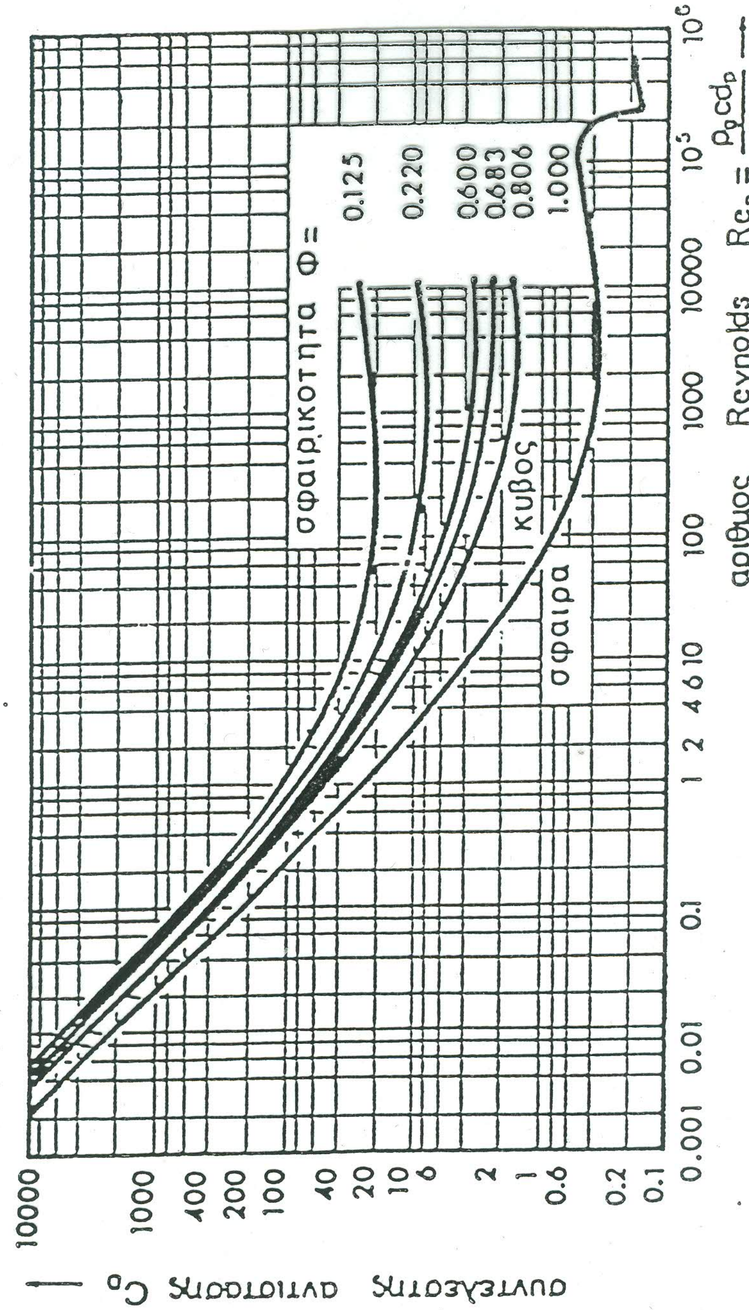

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σχημα σωματιδιου	σφαιρικοτητα
σφαιρα	1.
οκταεδρο	0.847
κυβος	0.806
πρισματα	
αxαx2α	0.767
αx2αx2α	0.761
αx2αx3α	0.725
κυλινδροι	
h=2r	0.874
h=3r	0.860
h=10r	0.691
h=20r	0.580
δισκοι	
h=1.33 r	0.858
h=r	0.827
h=r/3	0.594
h=r/10	0.323
h=r/15	0.254
ελλειψοειδες εκ περιστροφης	
ℓ=3d	0.683

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• σχεση <u>Muschelknautz</u>

$$c_D = \sum_{j=1}^{3} A_j Re_p^{\frac{j-3}{2}}$$

$$A_j \big|_{j=1,2,3}$$

$$Re_p = \frac{\rho_g (u_g - u_p) d_p}{\mu_g}$$

<u>ΠΙΝΑΚΑΣ 1</u>. Τιμες των συντελεστων A_j για τον υπολογισμο του συντελεστη αντιστασης σωματιδιων διαφορων σχηματων

Συντελεστες			Περιοχη Re_p	Χαρακτηριστικο μεγεθος σωματ.	Σχημα σωματιδ.	Αναφορα
A_1	A_2	A_3				
24	0	0	$Re_p<1$	διαμετρος, d_p	σφαιρα	Stokes
24	4	0.4	$Re_p<10^4$	διαμετρος, d_p	σφαιρα	Molerus
21.5	6.5	0.23	$0.5<Re_p<1000$	διαμετρος, d_p	σφαιρα	Muschel.
24	6	0.35	$0.5<Re_p<800$	1.1*μεγ.διαστ.	πολυεδρο	>>
23	6	0.5	$0.5<Re_p<600$	1.08*διαμ.βασης	κυλινδρος	>>
27	4.5	0.65	$0.5<Re_p<400$	1.24*ακμη	κυβος	>>

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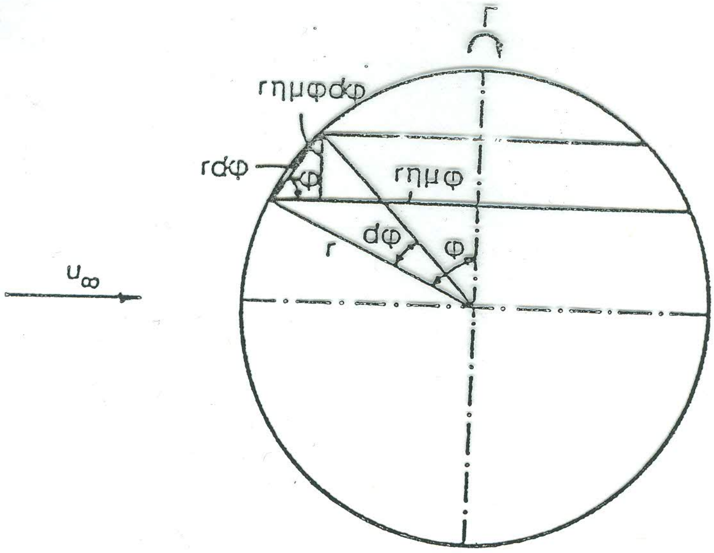

- **Γενικη σχεση ανωσης**

$$L = c_{Lid}\frac{1}{2}\rho_g u^2 A_p$$

- **Συντελεστης ανωσης**

$$c_{Lid} = \frac{16}{3}\frac{\omega r_p}{u_\infty} = \frac{16}{3} M_{Re}$$

$$M_{Re} = \frac{Re_{Rot}}{Re_{Tran}} = \frac{\dfrac{\rho_g d_p \omega r_p}{\mu_g}}{\dfrac{\rho_g d_p u_\infty}{\mu_g}} = \frac{\omega r_p}{u_\infty}$$

$$L_x = \frac{16}{3} n_p A_p \left[\frac{J_z r_p \rho_{dy}}{\upsilon_g - \upsilon_p}\frac{1}{2}\left(\frac{\partial \upsilon_g}{\partial x} - \frac{\partial u_g}{\partial y}\right) + \frac{J_y r_p \rho_{dz}}{w_g - w_p}\frac{1}{2}\left(\frac{\partial u_g}{\partial z} - \frac{\partial w_g}{\partial x}\right)\right]$$

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$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

γωνιακη ταχυτητα αερα

$$\omega_x = \frac{1}{2}\left(\frac{\partial w_g}{\partial y} - \frac{\partial \upsilon_g}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u_g}{\partial z} - \frac{\partial w_g}{\partial x}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial \upsilon_g}{\partial x} - \frac{\partial u_g}{\partial y}\right)$$

$$\vec{\omega}' = \omega'_x \vec{i} + \omega'_y \vec{j} + \omega'_z \vec{k}$$

γωνιακη ταχυτητα σωματιδιων.

$$\omega'_x = J_x \omega_x$$

$$\omega'_y = J_y \omega_y$$

$$\omega'_z = J_z \omega_z$$

κυκλοφορια

$$dL = \rho_g u_\infty \Gamma r_p \eta\mu\varphi d\varphi$$

$$\Gamma = 2\pi\omega r_p^2 \eta\mu^2\varphi$$

$$L = \int_0^\pi 2\pi\omega\rho_g u_\infty r_p^3 \eta\mu^3\varphi d\varphi$$

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- <u>συντελεστης αδρανειας σωματιδιων</u>

$$\vec{T} = -8\,\mu\pi\,\omega_o\, r_p^3 \qquad \vec{T}_p = \vec{T}_g$$

$$\vec{T}_p = I_p\,\vec{\alpha}_p \qquad \vec{T}_g = I_g\,\vec{\alpha}_g$$

$$I_p\,\vec{\alpha}_p = I_g\,\vec{\alpha}_g \quad \text{ή} \quad \frac{I_g}{I_p} = \frac{\vec{\alpha}_p}{\vec{\alpha}_g}$$

$$I_g = \frac{2}{5}\, m_g\, r_g^2 \quad \text{με} \quad m_g = \rho_g\, V_g$$

$$I_p = \frac{2}{5}\, m_p\, r_p^2 \quad \text{με} \quad m_p = \rho_p\, V_p$$

$$\frac{I_g}{I_p} = \frac{\rho_g}{\rho_p} = \frac{\vec{\alpha}_p}{\vec{\alpha}_g} \qquad \vec{\alpha}_p = \frac{\rho_g}{\rho_p}\,\vec{\alpha}_g$$

$$\frac{d\vec{\omega}_p}{dt} = \frac{\rho_g}{\rho_p}\,\frac{d\vec{\omega}_g}{dt} \qquad \vec{\omega}_p = \frac{\rho_g}{\rho_p}\,\vec{\omega}_g$$

$$\vec{\omega}_p = J\cdot\vec{\omega}_g \qquad J = \rho_g/\rho_p \qquad J_x = J_y = J_z = J$$

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- Στατικη ανωση

$$\vec{F}_{p(b)} = -Z\rho_g \vec{g}$$

- Πεδιακες δυναμεις

$$\vec{F}_{bp(grav)} = \bar{\rho}_p \vec{g} = Z\rho_{sp} \vec{g}$$

$$\vec{F}_{bg(grav)} = \bar{\rho}_g \vec{g} = (1-Z)\rho_g \vec{g}$$

- Συνδυασμος

$$\vec{F}_{bp(grav)} + \vec{F}_{p(b)} = Z\rho_{sp} \vec{g} - Z\rho_g \vec{g} = Z(\rho_{sp} - \rho_g)\vec{g}$$

$$\vec{F}_{bg(grav)} + \vec{F}_{g(b)} = (1-Z)\rho_g \vec{g} + Z\rho_g \vec{g} = \rho_g \vec{g}$$

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- **σωματιδια**

$$Z\rho_{sp}\left(\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} + \upsilon_p\frac{\partial u_p}{\partial y} + w_p\frac{\partial u_p}{\partial z}\right) = -\frac{\partial p_p}{\partial x} +$$

$$+\frac{\partial}{\partial x}\left[2\mu_p\frac{\partial u_p}{\partial x} - \frac{2}{3}\mu_p\left(\frac{\partial u_p}{\partial x}+\frac{\partial u_p}{\partial y}+\frac{\partial w_p}{\partial z}\right)\right] +$$

$$+\frac{\partial}{\partial y}\left[\mu_p\left(\frac{\partial u_p}{\partial y}+\frac{\partial \upsilon_p}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu_p\left(\frac{\partial w_p}{\partial x}+\frac{\partial u_p}{\partial z}\right)\right] + c_{Dx}n_pA_pP_{dx}$$

$$+\left[\left(\frac{\partial\upsilon_g}{\partial x}-\frac{\partial u_g}{\partial y}\right)\frac{J_zP_{dx}}{\upsilon_g-\upsilon_p} + \left(\frac{\partial u_g}{\partial z}-\frac{\partial w_g}{\partial x}\right)\frac{J_yP_{dz}}{w_g-w_p}\right]\frac{8}{3}n_pr_pA_p$$

- **αερας**

$$(1-Z)\rho_g\left(\frac{\partial u_g}{\partial t} + u_g\frac{\partial u_g}{\partial x} + \upsilon_g\frac{\partial u_g}{\partial y} + w_g\frac{\partial u_g}{\partial z}\right) = -\frac{\partial p_g}{\partial x} +$$

$$+\frac{\partial}{\partial x}\left[2\mu_g\frac{\partial u_g}{\partial x} - \frac{2}{3}\mu_g\left(\frac{\partial u_g}{\partial x}+\frac{\partial \upsilon_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)\right] +$$

$$+\frac{\partial}{\partial y}\left[\mu_g\left(\frac{\partial u_g}{\partial y}+\frac{\partial \upsilon_g}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu_g\left(\frac{\partial w_g}{\partial x}+\frac{\partial u_g}{\partial z}\right)\right] - c_{Dx}n_pA_pP_{dx}$$

$$-\left[\left(\frac{\partial\upsilon_g}{\partial x}-\frac{\partial u_g}{\partial y}\right)\frac{J_zP_{dy}}{\upsilon_g-\upsilon_p} + \left(\frac{\partial u_g}{\partial z}-\frac{\partial w_g}{\partial x}\right)\frac{J_yP_{dz}}{w_g-w_p}\right]\frac{8}{3}n_pr_pA_p$$

$$\tau_r^{ij} = \mu_r\left[\left(\frac{\partial u_r^i}{\partial x^j}+\frac{\partial u_r^j}{\partial x^i}\right) - \frac{2}{3}\left(\frac{\partial u_r^k}{\partial x^k}\right)\delta^{ij}\right]$$

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$$
\begin{aligned}
& Z\rho_{sp} c_{pp}\left(\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + \upsilon_p \frac{\partial T_p}{\partial y} + w_p \frac{\partial T_p}{\partial z}\right) - \frac{\partial p_p}{\partial t} - u_p \frac{\partial p_p}{\partial x} - \\
& - \upsilon_p \frac{\partial p_p}{\partial y} - w_p \frac{\partial p_p}{\partial z} - \frac{\partial}{\partial x}\left(k_p \frac{\partial T_p}{\partial x}\right) - \frac{\partial}{\partial y}\left(k_p \frac{\partial T_p}{\partial y}\right) - \frac{\partial}{\partial z}\left(k_p \frac{\partial T_p}{\partial z}\right) - \\
& - \mu\Phi_p + c_{Dx} u_p n_p A_p p_{dx} + c_{Dy} \upsilon_p n_p A_p p_{dy} + c_{Dz} w_p n_p A_p p_{dz} + \\
& + \left[\left(\frac{\partial \upsilon_g}{\partial x} - \frac{\partial u_g}{\partial y}\right)\frac{J_z p_{dy}}{\upsilon_g - \upsilon_p} + \left(\frac{\partial u_g}{\partial z} - \frac{\partial w_g}{\partial x}\right)\frac{J_y p_{dz}}{w_g - w_p}\right] \frac{8}{3} n_p r_p A_p u_p + \\
& + \left[\left(\frac{\partial \upsilon_g}{\partial x} - \frac{\partial u_g}{\partial y}\right)\frac{J_z p_{dx}}{u_g - u_p} + \left(\frac{\partial w_g}{\partial y} - \frac{\partial \upsilon_g}{\partial z}\right)\frac{J_x p_{dz}}{w_g - w_p}\right] \frac{8}{3} n_p r_p A_p \upsilon_p + \\
& + \left[\left(\frac{\partial u_g}{\partial z} - \frac{\partial w_g}{\partial x}\right)\frac{J_y p_{dx}}{u_g - u_p} + \left(\frac{\partial w_g}{\partial y} - \frac{\partial \upsilon_g}{\partial z}\right)\frac{J_x p_{dy}}{\upsilon_g - \upsilon_p}\right] \frac{8}{3} n_p r_p A_p w_p + \\
& + Z\rho_g g w_p + Z\rho_{sp} g w_p = k_T (T_g - T_p)
\end{aligned}
$$

$$
\begin{aligned}
\Phi_p = {} & 2\left[\left(\frac{\partial u_p}{\partial x}\right)^2 + \left(\frac{\partial \upsilon_p}{\partial y}\right)^2 + \left(\frac{\partial w_p}{\partial z}\right)^2\right] + \left[\frac{\partial \upsilon_p}{\partial x} + \frac{\partial u_p}{\partial y}\right]^2 + \\
& + \left[\frac{\partial w_p}{\partial y} + \frac{\partial \upsilon_p}{\partial z}\right]^2 + \left[\frac{\partial u_p}{\partial z} + \frac{\partial w_p}{\partial x}\right]^2 - \frac{2}{3}\left[\frac{\partial u_p}{\partial x} + \frac{\partial \upsilon_p}{\partial y} + \frac{\partial w_p}{\partial z}\right]^2
\end{aligned}
$$