Complex Numbers

Complex Numbers

The **imaginary unit** *i* is defined as

$$\sqrt{-1} = i$$

$$i^2 = -1$$

Example

$$\sqrt{-81} = \sqrt{-1 \cdot 81}$$

$$= i \cdot 9 = 9i$$

Complex Numbers

The set of all numbers in the form **a + b*i*** with

- real numbers *a* and *b*,
- *i* the imaginary unit,

is called the set of **complex numbers.**

Complex Numbers

The real number *a* is called the **real part**, and the real number *b* is called the **imaginary part**, of the complex number **a + b*i***.

Equality of Complex Numbers

$a+bi = c+di$

if and only if

a = c **and** b = d

Adding and Subtracting Complex Numbers

$$(a+bi) + (c+di) = (a+c) + (b+d)i$$

$$(a+bi) - (c+di) = (a-c) + (b-d)i$$

Multiplying Complex Numbers

$$(a+bi)(c+di) =$$

$$(ac) + (adi) + (cbi) + (bd)i^2 =$$

$$(ac-bd) + (ad+cb)i$$

Example

Simplify:

$$3+2i-6i-8$$

$$=(3-8)+(2-6)i$$

$$=-5-4i$$

Example

Multiply:

$$(2-i)(1+3i)$$

$$=2+6i-i-3i^2$$

$$=2+5i+3$$

$$=5+5i$$

The **complex conjugate** of the number a + b*i* is a - b*i*, and visa-versa. The product of a complex number and its conjugate is a real number.

$$(a+bi)(a-bi)=a^2+b^2$$

Example

Rationalize:

$$\frac{2}{1-i} = \frac{2}{1-i} \cdot \frac{1+i}{1+i}$$

$$= \frac{2+2i}{1-i^2} = \frac{2+2i}{1+1}$$

$$= \frac{2+2i}{2} = 1+i$$

For any positive real number b, the **principal square root** of the negative number -b is defined by

$$\sqrt{(-b)} = i\sqrt{b}$$

Example

Simplify:

$$\sqrt{-16} \cdot \sqrt{-9}$$

$$= 4i \cdot 3i$$

$$= 12i^2 = -12$$

Quadratic Formula

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples

Solve:

$$3x^2 - 2x + 4 = 0$$

$$x^2 - 2x + 2 = 0$$

Complex number

- Standard form

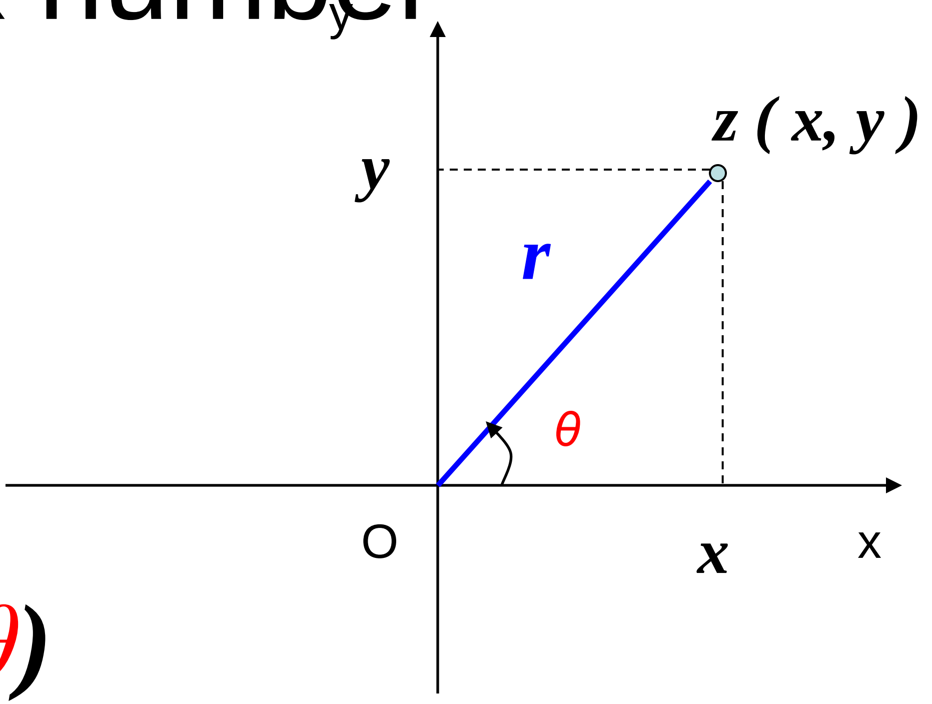
$$z = x + y\,i$$

- Polar form

$$z = r\,(\cos\theta + i\,\sin\theta)$$

- Exponential form

$$z = re^{i\theta}$$



$$r = \sqrt{x^2 + y^2}$$

$$Argz = \arctan\left(\frac{y}{x}\right) \in (-\pi, \pi]$$