• $\dfrac{d}{dx}(x^n) = \dfrac{dx^n}{dx} = n \cdot x^{n-1}$

$f(x) = x^n$

π.χ. $\dfrac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$

• $g(x) = \underbrace{f(x) \pm h(x)}$ [πρόσθεση ή αφαίρεση συναρτήσεων].

• $\dfrac{d}{dx} g(x) = \dfrac{d}{dx}\left[f(x) \pm h(x)\right] = \dfrac{d f(x)}{dx} \pm \dfrac{d h(x)}{dx}$

π.χ. Έστω $g(x) = \underbrace{x^4}_{f(x)} + \underbrace{2}_{h(x)}$

$\dfrac{d}{dx} g(x) = \dfrac{d}{dx}\left[x^4 + 2\right] = \dfrac{d}{dx} x^4 + \cancel{\dfrac{d}{dx} 2}^{\,0} = 4x^3$

ΠΑΡΑΤΗΡΗΣΗ (2η παράγωγος) $(\ldots)'' = \dfrac{d^2}{dx^2}$

$\dfrac{d^2}{dx^2} g(x) = \dfrac{d^2}{dx^2}\left[x^4 + 2\right] = \dfrac{d^2}{dx^2}(x^4) + \cancel{\dfrac{d^2}{dx^2}(2)}^{\,0} =$

$= \dfrac{d}{dx}(4x^3) = 4 \cdot 3x^2 = 12x^2$

- $$\frac{d}{dx}\left(a \cdot f(x)\right) = a \cdot \frac{d}{dx}\left(f(x)\right) = \ldots\ldots$$

π.χ $$\frac{d}{dx}\left(5 \cdot x^6\right) = 5 \cdot \frac{d}{dx}\left(x^6\right) = 5 \cdot 6 \cdot x^5 = 30x^5$$

- $$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \frac{df(x)}{dx} \cdot g(x) + f(x) \cdot \frac{dg(x)}{dx}$$

π.χ Έστω $f(x) = 5x+1$ και $g(x) = x^2$.

$$\boxed{h(x)} = f(x) \cdot g(x) = (5x+1) \cdot x^2.$$

Α΄ ΤΡΟΠΟΣ

$$\frac{d}{dx} h(x) = \frac{d}{dx}\left[(5x+1) \cdot x^2\right] = \frac{d}{dx}(5x+1) \cdot x^2 +$$

$$+ (5x+1) \cdot \frac{d}{dx}(x^2) = (5+0) \cdot x^2 + (5x+1) \cdot 2x$$

$$= 5x^2 + 10x^2 + 2x = \boxed{15x^2 + 2x}$$

Β΄ ΤΡΟΠΟ : $h(x) = 5x^3 + x^2$

$$\frac{d}{dx}\left(h(x)\right) = \frac{d}{dx}\left(5x^3 + x^2\right) = \frac{d}{dx}(5x^3) + \frac{d}{dx}(x^2) =$$

$$= 15x^2 + 2x$$

Έστω $f(x)$ και $g(x)$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2} =$$

$$= \frac{\frac{d f(x)}{dx} g(x) - f(x)\cdot \frac{d g(x)}{dx}}{(g(x))^2}$$

π.χ. Έστω $f(x) = x^3$ και $g(x) = x+1$.

$$\frac{d}{dx}\left(\frac{x^3}{x+1}\right) = \frac{\frac{d}{dx}(x^3)\cdot(x+1) - x^3\cdot\frac{d}{dx}(x+1)}{(x+1)^2} =$$

$$= \frac{3x^2\cdot(x+1) - x^3\cdot 1}{(x+1)^2} = \frac{3x^3+3x^2-x^3}{(x+1)^2} =$$

$$= \frac{2x^3+3x^2}{(x+1)^2}$$

π.χ. Έστω $f(x) = 10x^4$ και $g(x) = 5x+8$.

$$\frac{d}{dx}\left(\frac{10x^4}{5x+8}\right) = \frac{(10x^4)'\cdot(5x+8) - 10x^4\cdot(5x+8)'}{(5x+8)^2} =$$

$$= \frac{40x^3\cdot(5x+8) - 10x^4(5+0)}{(5x+8)^2} = \frac{200x^4+320x^3-50x^4}{(5x+8)^2}$$

$$= \frac{150x^4+320x^3}{(5x+8)^2}$$

$$\frac{d}{dx}\left(f^{n}(x)\right) = n \cdot f^{n-1}(x) \cdot \frac{d f(x)}{dx}$$

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

π.χ. Εστω $g(x) = (5x+1)^2$

$f(x) = 5x+1$

Α' ΤΡΟΠΟΣ

$$\frac{d}{dx}\left[(5x+1)^2\right] = 2 \cdot (5x+1)^1 \cdot \frac{d}{dx}(5x+1) =$$

$$= 2 \cdot (5x+1) \cdot 5 = 10 \cdot (5x+1) = \boxed{50x+10}$$

Β' ΤΡΟΠΟΣ

$$g(x) = (5x+1)^2 = 25x^2 + 10x + 1$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\frac{d}{dx}\left(25x^2+10x+1\right) = \frac{d}{dx}\left(25x^2\right) + \frac{d}{dx}(10x) + \cancel{\frac{d}{dx}(1)}^{\,0} =$$

$$= 25 \cdot 2x + 10 \cdot 1 = \boxed{50x+10}$$

π.χ. Εστω $g(x) = (8x^3+5)^{10}$

$$\frac{d}{dx} g(x) = \frac{d}{dx}\left((8x^3+5)^{10}\right) = 10 \cdot (8x^3+5)^9 \cdot \frac{d}{dx}\left(8x^3+5\right) =$$

$$= 10 \cdot (8x^3+5)^9 \cdot 24x^2 = 240x^2 \left(8x^3+5\right)^9$$

• Έστω $f(x) = e^x$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

• Έστω $f(x) = e^{g(x)}$

$$\frac{d}{dx}\left(f(x)\right) = \frac{d}{dx}\left(e^{g(x)}\right) = e^{g(x)} \cdot \frac{d}{dx}\left(g(x)\right)$$

π.χ Έστω $f(x) = e^{\underbrace{(5x+1)}_{g(x)}}$

$$\frac{d}{dx}\left(e^{(5x+1)}\right) = e^{5x+1} \cdot \frac{d}{dx}(5x+1) = 5 \cdot e^{5x+1}$$

π.χ Έστω $f(x) = e^{x^3}$, $x^3 \to g(x)$

$$(x^n)' = n \cdot x^{n-1}$$

$$\frac{d}{dx}\left(e^{x^3}\right) = e^{x^3} \cdot \frac{d}{dx}\left(x^3\right) =$$

$$= e^{x^3} \cdot 3x^2 = \boxed{3x^2 \cdot e^{x^3}}$$

- $f(x) = \ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

- $f(x) = \ln(g(x)) \Leftarrow$

$$\frac{d}{dx}\left(\ln(g(x))\right) = \frac{1}{g(x)} \cdot \frac{dg(x)}{dx}$$

π.χ. Εαν $f(x) = \ln(x^2+1)$

$$\frac{d}{dx}\left(\ln(x^2+1)\right) = \frac{1}{x^2+1} \cdot \frac{d}{dx}(x^2+1) =$$

$$= \frac{2x}{x^2+1}$$

π.χ Εαν $f(x) = \ln(10x^4)$

$$\frac{d}{dx}\left(\ln(10x^4)\right) = \frac{1}{10x^4} \cdot \frac{d}{dx}(10x^4) =$$

$$= \frac{1}{10x^4} \cdot 10 \cdot 4x^3 = \frac{4\cancel{0}\cancel{x^3}}{1\cancel{0}\,\cancel{x^4}} = \frac{4}{x} = \boxed{4 \cdot x^{-1}}$$

ΠΑΡΕΝΘΕΣΗ ΣΤΟΥΣ ΚΑΝΟΝΕΣ ΠΑΡΑΓΩΓΙΣΗΣ

$$\frac{1}{x} = x^{-1} \quad , \quad \frac{1}{x^3} = x^{-3}$$

$$\frac{1}{5x+1} = (5x+1)^{-1} \quad \text{ή} \quad \frac{1}{(5x)^2} = (5x)^{-2}$$

π.χ. Έστω $g(x) = \dfrac{x^4}{x+1}$

Α' τρόπος

$$\frac{d}{dx}\left(\frac{x^4}{x+1}\right) = \frac{(x^4)' \cdot (x+1) - x^4 \cdot (x+1)'}{(x+1)^2} = \ldots$$

Β' τρόπος

$$g(x) = x^4 (x+1)^{-1}$$

$$\left\{ \left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x) \right\}$$

$$\frac{d}{dx}\left(x^4 \cdot (x+1)^{-1}\right) = \frac{d}{dx}(x^4) \cdot (x+1)^{-1} + x^4 \boxed{\frac{d}{dx}(x+1)^{-1}} =$$

$$\frac{d}{dx}(f(x))^n = n \cdot f(x)^{n-1} \cdot f'(x)$$

$$= 4x^3 \cdot (x+1)^{-1} + x^4 \cdot \left(-1 \cdot (x+1)^{-2} \cancel{(x+1)'}^{\,1}\right)$$

$$= \left| \frac{4x^3}{x+1} - \frac{x^4}{(x+1)^2} \right| = \ldots$$

ΓΕΝΙΚΕΣ ΠΑΡΑΤΗΡΗΣΕΙΣ

- $ημ(x) \equiv \sin(x)$.
- $συν(x) \equiv \cos(x)$

$εφ(x) = \dfrac{ημx}{συνx} =$

$\tan(x) = \dfrac{\sin x}{\cos x}$.

- $\dfrac{d}{dx}(\sin x) = \cos x$
- $\dfrac{d}{dx}(\cos x) = -\sin x$

- $\dfrac{d}{dx}\left(\sin(f(x))\right) = \cos f(x) \cdot \dfrac{d f(x)}{dx}$
- $\dfrac{d}{dx}\left(\cos(f(x))\right) = -\sin(f(x)) \cdot \dfrac{d f(x)}{dx}$

π.χ. έστω $f(x) = \sin(5x+1)$

$$\frac{d}{dx}\left(\sin(5x+1)\right) = \cos(5x+1) \cdot \frac{d}{dx}(5x+1) = 5 \cdot \cos(5x+1)$$

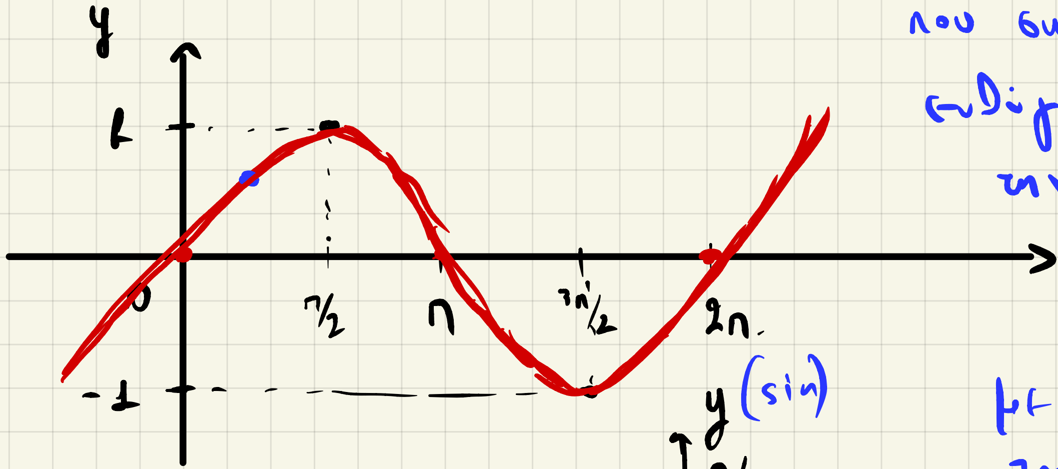
έστω ότι $f(x) = \cos(x^2+3)$

$$\frac{d}{dx}\left(\cos(x^2+3)\right) = -\sin(x^2+3) \cdot \frac{d}{dx}(x^2+3) = -2x \cdot \sin(x^2+3)$$

ΓΡΑΦΙΚΕΣ ΠΑΡΑΣΤΑΣΗΣ

EXTRA

$$\boxed{y(x) = \sin x}$$



Ποια είναι η σχέση που συνδέει το μπλε ευθύγραμμο τμήμα της δεύτερης γραφικής παράστασης με το μπλε σημείο της πρώτης γραφ. παράστασης?