$$\int_1^3 x^2 \cdot \sqrt{x+1}\, dx =$$

$$\int_{\sqrt{2}}^{2} (u^2-1)^2\, 2u^2\, du = \ldots\ldots\ldots =$$

$$= \left[\frac{2}{7}u^7 - \frac{4}{5}u^5 + \frac{2}{3}u^3\right]_{\sqrt{2}}^{2} =$$

$$= \left(\frac{2}{7}2^7 - \frac{4}{5}2^5 + \frac{2}{3}2^3\right) - \left(\frac{2}{7}(\sqrt{2})^7 - \frac{4}{5}(\sqrt{2})^5 + \frac{2}{3}(\sqrt{2})^3\right) = \ldots$$

....

θέτω

$\boxed{\sqrt{x+1} = u}$ ....

$dx = \ldots\ldots$

$x=1 \longrightarrow u=\sqrt{2}$

$x=3 \longrightarrow u=\sqrt{4}=2$

ή ΠΑΡΑΓΟΝΤΙΚΗ ΟΛΟΚΛΗΡΩΣΗ.

ΜΕΘΟΔΟΣ ΟΛΟΚΛΗΡΩΣΗΣ – ΚΑΤΑ ΠΑΡΑΓΟΝΤΕΣ

$$\int f(x)\cdot g'(x)\, dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\, dx.$$

$$\int \frac{e^x}{x}\, dx = \frac{e^x}{x} + \int \frac{1}{x^2}\cdot e^x\, dx$$

$g'(x) = e^x \Rightarrow g(x) = e^x$

$f(x) = \frac{1}{x} \Rightarrow f'(x) = -\frac{1}{x^2}$

$$= e^x \ln x - \int e^x \ln x\, dx$$

$\int \frac{1}{x}dx = \ln x + c$

$g'(x) = \frac{1}{x} \Rightarrow g(x) = \ln x$

$f(x) = e^x \Rightarrow f'(x) = e^x$

$(\ln x)' = \frac{1}{x}$

$\left(\frac{1}{x}\right)' = (x^{-1})' = -1\cdot x^{-2} = -\frac{1}{x^2}$

$$\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx$$

Να αποδειχθεί ο τύπος ????

$(e^x)' = e^x$

$$\bullet \int x\cdot e^x\,dx = \int \underbrace{x}_{f(x)}\cdot\underbrace{(e^x)'}_{g'(x)}\,dx =$$

$$= x\cdot e^x - \int (x)'\cdot e^x\,dx = x\cdot e^x - \int 1\cdot e^x\,dx =$$

$$= x\cdot e^x - \int e^x\,dx = \boxed{x\cdot e^x - e^x + c_1}$$

$$\bullet \int x\cdot\cos x\,dx = \int x\,(\sin x)'\,dx = x\cdot\sin x - \int (x)'\cdot\sin x\,dx =$$

$$= x\cdot\sin x - \int 1\cdot\sin x\,dx = x\cdot\sin x - \int \sin x\,dx =$$

$$= x\cdot\sin x - (-\cos x) + c_1 =$$

$$= \boxed{x\cdot\sin x + \cos x + c_1}$$

$\int \sin x\,dx =$

ΟΡΙΣΜΕΝΟ ΟΛΟΚΛΗΡΩΜΑ (+) ΠΑΡΑΓΟΝΤΙΚΗ ΟΛΟΚΛΗΡΩΣΗ

$$\int_a^b f(x)\cdot g'(x)\,dx = f(x)\cdot g(x)\Big|_a^b - \int_a^b f'(x)\cdot g(x)\,dx =$$

$$= [f(b)\cdot g(b) - f(a)\cdot g(a)] - \int_a^b f'(x)\cdot g(x)\,dx \;\ldots\ldots$$

$$\int_2^{10} x e^x \, dx = \int_2^{10} x \cdot (e^x)' \, dx =$$

$$= x \cdot e^x \Big|_2^{10} - \int_2^{10} \cancel{(x)'}^{\,1} \cdot e^x \, dx =$$

$$= x \cdot e^x \Big|_2^{10} - e^x \Big|_2^{10} = \left(10e^{10} - 2 \cdot e^2\right) - \left(e^{10} - e^2\right) =$$

$$= 10e^{10} - 2e^2 - e^{10} + e^2 =$$

$$= \boxed{9e^{10} - e^2} = \boxed{198{,}230.8}$$

!!!

$$\int x^2 \ln x \, dx = \int \left(\frac{x^3}{3}\right)' \cdot \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} (\ln x)' \, dx$$

$$= \frac{x^3}{3} \cdot \ln x - \int \frac{\cancel{x^3}^{\,2}}{3} \cdot \frac{1}{\cancel{x}} \, dx = \frac{x^3}{3} \cdot \ln x - \int \frac{x^2}{3} \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 \, dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \frac{x^3}{3} + c = \boxed{\frac{x^3}{3} \ln x - \frac{x^3}{9} + c}$$

$$\boxed{\int x \cdot \sqrt{1 + x} \, dx}$$

ΟΛΟΚΛΗΡ. ΚΑΤΑ

$$\sin^2 x \, dx$$

$$(\sin x)^2$$

$$\int x^3 \cdot e^{2x} \, dx$$

ΟΛΟΚΛΗΡΩΣΗ ΚΑΤΑ ΠΑΡΑΓΟΝΤΕΣ