ΕΦΑΡΜΟΓΗ

· Ολοκλήρωμα δύναμης όταν το διαφορικό είναι ή μπορεί να γίνει όμοιο με τη βάση της δύναμης.

$$\int (x-2)^{-3}\,dx =$$

Θέτω $\boxed{x-2=w} \Rightarrow$

$$\Rightarrow \frac{dw}{dx} = 1 \Rightarrow$$

$$\Rightarrow \boxed{dw = dx}$$

$$= \int w^{-3}\,dw = \frac{w^{-2}}{-2} + C_1 = -\frac{1}{2w^2} + C_1 =$$

$$= \boxed{-\frac{1}{2(x-2)^2} + C_1}$$

$$\boxed{(x^n)' = n \cdot x^{n-1}}$$

$$\left(\ ?\ \right)' = w^{-3}$$

$$\boxed{n-1=-3} \Rightarrow n = -3+1 = -2$$

ΔΟΚΙΜΗ:

$$(w^{-2})' = -2 \cdot w^{-2-1} = -2 \cdot w^{-3}$$

$$\Rightarrow \boxed{\left(\frac{w^{-2}}{-2}\right)' = w^{-3}}$$

$$\left(\frac{w^{-2}}{-2}\right)' = \cancel{-}\cancel{\tfrac{1}{2}} \cdot (\cancel{-2})\, w^{-3} = w^{-3}$$

$$x-2 = w(x)$$

$$\frac{dw}{dx} = 1 \Rightarrow$$

$$\boxed{dw = dx}$$

ΕΦΑΡΜΟΓΗ

$$\int \frac{1}{\cancel{x}+1}\,dx = \left[\int (\cancel{x}+1)^{-1}\,dx\right] = \boxed{\ln|x+1| + C}$$

$$\cancel{\ln|x^2+1|+C}$$

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C} \leftarrow \text{"ΚΑΝΟΝΑΣ" ΟΛΟΚΛΗΡΩΣΗΣ}$$

$$\int \frac{1}{x+1}\,dx =$$

Θέτω

$$\boxed{x+1} = u \Rightarrow \frac{du}{dx} = 1 \Rightarrow \boxed{du = dx}$$

$$= \int \frac{1}{u}\,du = \ln|u| + C =$$

$$(\ln x)' = \frac{1}{x}$$

$$(\ln u)' = 1/u.$$

$$= \boxed{\ln|x+1| + C}$$

ΕΦΑΡΜΟΓΗ

$$\int \frac{x\,dx}{x^2+1} = \int \frac{2\,x\,dx}{2\,(x^2+1)} = \frac{1}{2}\int \frac{\boxed{2x}\,dx}{x^2+1} =$$

$$= \frac{1}{2}\int \frac{(x^2+1)'}{x^2+1}\,dx = \frac{1}{2}\ln|x^2+1| + C$$

$$\int \frac{x\,dx}{x^2+1} = \int \frac{\frac{du}{2}}{\frac{u+1}{1}} = \int \frac{du}{2(u+1)} = \frac{1}{2}\int \frac{du}{u+1} = \frac{1}{2}\int \frac{1}{u+1}du = \frac{1}{2}\ln|u+1| + c_1 = \frac{1}{2}\ln|x^2+1| + c_1$$

θέτω ή

$$\boxed{x^2+1 = u} \Rightarrow \frac{du}{dx} = 2x \Rightarrow du = 2x\,dx \Rightarrow \boxed{\frac{du}{2} = x\,dx}$$

θέτω $u = x^2$

$du = 2x\,dx$

$$\boxed{\frac{du}{2} = x\,dx}$$

ΕΦΑΡΜΟΓΗ

$$\int \frac{x+\overset{3}{1}}{x^2+2x+\underset{5}{10}}\,dx = \int \frac{1}{2u}du = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + c_1 = \frac{1}{2}\ln|x^2+2x+10| + c_1$$

$$\Rightarrow \int \frac{x}{x^2+2x+10}dx + \int \frac{1}{x^2+2x+10}dx$$

θέτω

$$x^2+2x+10 = u$$

$$\frac{du}{dx} = 2x+2 \Rightarrow du = (2x+2)\cdot dx \Rightarrow du = 2\cdot(x+1)\,dx \Rightarrow \frac{du}{2} = (x+1)\,dx$$

$$\cancel{\int \frac{f(x)}{g(x)}dx = \frac{\int f(x)\,dx}{\int g(x)\,dx}}$$

ΜΕΓΑ ΛΑΘΟΣ !!!!

$$\cancel{\int f(x)\cdot g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx}$$

όμως $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$ ✓

ΕΦΑΡΜΟΓΗ

sin = ημ.
cos = συν.

$$\frac{d}{dx}(\sin x) = \cos x.$$

$$\frac{d}{dx}(\cos x) = -\sin x.$$

$$\int \sin(2x+1)\, dx = $$

$$= \cancel{\cos(-x^2-x) + C_1}$$

Θέτω

$$2x+1 = k \Rightarrow$$

$$\Rightarrow \frac{dk}{dx} = 2 \Rightarrow dk = 2dx.$$

$$\Rightarrow \boxed{\frac{dk}{2} = dx}$$

$$\int \sin(k) \cdot \frac{dk}{2} = \frac{1}{2}\int \sin(k) \cdot dk =$$

$$= -\frac{1}{2}\cos(k) + C_1 =$$

$$= \boxed{-\tfrac{1}{2}\cos(2x+1) + C_1}$$ ✓

ΕΦΑΡΜΟΓΗ

$$\int e^{3x/2}\, dx =$$

Θέτω ✓

$$u = \frac{3x}{2} \Rightarrow$$

$$\Rightarrow \frac{du}{dx} = \frac{3}{2} \Rightarrow du = \tfrac{3}{2}\, dx$$

$$\Rightarrow \boxed{dx = \frac{2}{3}\, du}$$ ✓

$$= \int e^u \cdot \frac{2}{3}\, du = \frac{2}{3}\int \boxed{e^u} du =$$

$$(e^x)' = e^x$$

$$= \frac{2}{3}e^u + C_1 =$$

$$= \boxed{\frac{2}{3}e^{3x/2} + C_1}$$

ΕΦΑΡΜΟΓΗ

$x^{1/2} = \sqrt{x}$

$\sin^{1/2}x = \sqrt{\sin x}$

$$\int \sin^{1/2}x \cdot \cos x \, dx =$$

$$= \int \sqrt{\sin x} \cdot \underbrace{\boxed{\cos x \, dx}}_{= du} =$$

$$= \int \sqrt{u} \cdot du = \int u^{1/2} du =$$

$$= \frac{2}{3} u^{3/2} + c_1 =$$

$$= \frac{2}{3} (\sin x)^{3/2} + c_1$$

$$= \boxed{\frac{2}{3} \cdot \sin^{3/2}x + c_1}$$

$\left(f^{n}(x)\right)' = n \cdot f^{n-1}(x) \cdot f'(x)$

Θέτω

$\sin^{1/2}x = u. \Rightarrow$

$\Rightarrow \frac{du}{dx} = \boxed{\frac{1}{2}} \sin^{-1/2}x \cdot \cos x$

ΘΕΤΩ

$\boxed{\sin x = u} \Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow \boxed{du = \cos x \cdot dx}$

$\left(\ldots \right)' = u^{1/2}$

$(x^n)' = n \cdot x^{n-1}$

$n - 1 = \frac{1}{2} \Rightarrow n = \frac{1}{2} + 1$

$\Rightarrow \boxed{n = \frac{3}{2}}$

$\left(u^{3/2}\right)' = \frac{3}{2} u^{1/2}$

$$\boxed{\left(\frac{2}{3} u^{3/2}\right)' = u^{1/2}}$$

- $\int \frac{1}{\sqrt{x}} + \frac{2}{\sqrt[3]{x}} + \frac{4}{\sqrt[5]{x}} \, dx$

- $\int x^2 \cdot \sqrt{2 - 3x^3} \, dx$