No. 1 Date

Άσκηση 1 ((Σ) 3x3 με Gauss).

Έχω:
$$x_1 + x_2 + x_3 = -5$$
$$x_1 - 4x_2 + x_3 = 35$$
$$x_1 + 3x_2 + 4x_3 = -18.$$

Ο Επαυξημένος πίνακας του (Σ) είναι:

$$\underset{\sim}{E} = [\underset{\sim}{A} : \underset{\sim}{b}] = \begin{bmatrix} 1 & 1 & 1 & -5 \\ 1 & -4 & 1 & 35 \\ 1 & 3 & 4 & -18 \end{bmatrix} \xrightarrow{(\Gamma_1 - \Gamma_2) \to \Gamma_2} \begin{pmatrix} 1 & 1 & 1 & -5 \\ 0 & 5 & 0 & -40 \\ 1 & 3 & 4 & -18 \end{pmatrix}$$

$$\xrightarrow{(\Gamma_3 - \Gamma_1) \to \Gamma_3} \begin{pmatrix} 1 & 1 & 1 & -5 \\ 0 & 5 & 0 & -40 \\ 0 & 2 & 3 & -13 \end{pmatrix} \xrightarrow[-2\Gamma_2 + \Gamma_3 \to \Gamma_3]{\frac{\Gamma_2}{5} \to \Gamma_2} \begin{pmatrix} 1 & 1 & 1 & -5 \\ 0 & 1 & 0 & -8 \\ 0 & 2 & 3 & -13 \end{pmatrix} \xrightarrow{(-2\Gamma_2 + \Gamma_3) \to \Gamma_3}$$

$$\begin{pmatrix} 1 & 1 & 1 & -5 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 3 & 3 \end{pmatrix} \xrightarrow{\frac{\Gamma_3}{3} \to \Gamma_3} \begin{pmatrix} 1 & 1 & 1 & -5 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Οπότε: $x_1 + x_2 + x_3 = -5$

$$\boxed{x_2 = -8}$$
$$\boxed{x_3 = 1}$$

Άρα: $x_1 = -5 + 8 - 1 \Leftrightarrow$
$$\boxed{x_1 = 2}$$

Άσκηση 2 ((Σ) 3x3 με Gauss «Αόριστο»).

Έχω:
$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 2$$
$$5x_1 + x_2 + 4x_3 = 3.$$

Ο επαυξημένος πίνακας του (Σ) είναι:

$$\underset{\sim}{E} = [\underset{\sim}{A} : b] = \begin{pmatrix} 2 & -1 & 3 & 1 \\ 3 & 2 & 1 & 2 \\ 5 & 1 & 4 & 3 \end{pmatrix} \xrightarrow{(3\Gamma_1 - 2\Gamma_2)\to\Gamma_2} \begin{pmatrix} 2 & -1 & 3 & 1 \\ 0 & -7 & 7 & -1 \\ 5 & 1 & 4 & 3 \end{pmatrix}$$

$$\xrightarrow{(2\Gamma_3 - 5\Gamma_1)\to\Gamma_3} \begin{pmatrix} 2 & -1 & 3 & 1 \\ 0 & -7 & 7 & -1 \\ 0 & 7 & -7 & 1 \end{pmatrix} \xrightarrow{\left(\frac{\Gamma_1}{2}\right)\to\Gamma_1} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & -7 & 7 & -1 \\ 0 & 7 & -7 & 1 \end{pmatrix}$$

$$\xrightarrow{(\Gamma_3 + \Gamma_2)\to\Gamma_3} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & -7 & 7 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\left(-\frac{\Gamma_2}{7}\right)\to\Gamma_2} \begin{pmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & -1 & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Άρα το (Σ) Αόριστο ($\text{rank}\,\underset{\sim}{A} = \text{rank}\,\underset{\sim}{E}$)

$\boxed{\text{Άσκηση 3}}$ ((Σ) 3×3 με Gauss «Αδύνατο»).

Έχω:

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ 4x_1 + 4x_2 + x_3 &= 2 \\ 6x_1 + 6x_2 + 2x_3 &= -11 \end{aligned}$$

$$\underset{\sim}{E} = [\underset{\sim}{A} : \underset{\sim}{b}] = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 4 & 4 & 1 & 2 \\ 6 & 6 & 2 & -11 \end{pmatrix} \xrightarrow{(4\Gamma_1 - \Gamma_2)\to\Gamma_2} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 3 & 22 \\ 6 & 6 & 2 & -11 \end{pmatrix}$$

$$\xrightarrow{(\Gamma_3 - 6\Gamma_1)\to\Gamma_3} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 3 & 22 \\ 0 & 0 & -4 & -47 \end{pmatrix} \xrightarrow[\frac{\Gamma_3}{-4}\to\Gamma_3]{\frac{\Gamma_2}{3}\to\Gamma_2} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 22/3 \\ 0 & 0 & 1 & \frac{47}{4} \end{pmatrix} \xrightarrow{\Gamma_2 - \Gamma_3 \to \Gamma_3}$$

$\text{rank}(\underset{\sim}{E}) = 3$

$\text{rank}(\underset{\sim}{A}) = 2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 22/3 \\ 0 & 0 & 0 & \frac{22}{3} - \frac{47}{4} \end{array}\right) \to 0x_1 + 0x_2 + 0x_3 = -\frac{53}{12}$$ ✓

$$x_1 + x_2 + x_3 = 6$$

$$\left.\begin{aligned} x_3 &= \tfrac{22}{3} \\ x_3 &= \tfrac{47}{4} \end{aligned}\right\} \text{Αδύνατο.}$$

No. 1 Date 10/11/

Άσκηση 1 (...).

Σύστημα Cramer 3x3. (Με μοναδική λύση).

$$\begin{cases} \overset{x_1}{x} + \overset{x_2}{y} + \overset{x_3}{z} = -5 \\ \overset{x_1}{x} - 4\overset{x_2}{y} + \overset{x_3}{z} = 35 \\ \overset{x_1}{x} + 3\overset{x_2}{y} + 4\overset{x_3}{z} = -18 \end{cases}$$

$x = \frac{D_x}{D}$

$y = \frac{D_y}{D}$

$z = \frac{D_z}{D}$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -4 & 1 \\ 1 & 3 & 4 \end{vmatrix} = 1\cdot(-4\cdot4 - 1\cdot3) - 1(1\cdot4 - 1\cdot1) + 1\cdot(1\cdot3 + 4\cdot1)$$

$$= 1(-16-3) - (4-1) + (3+4) = -19 - 3 + 7 = -15.$$

$$D_x = \begin{vmatrix} -5 & 1 & 1 \\ 35 & -4 & 1 \\ -18 & 3 & 4 \end{vmatrix} = \ldots = -30.$$

$$D_y = \begin{vmatrix} 1 & -5 & 1 \\ 1 & 35 & 1 \\ 1 & -18 & 4 \end{vmatrix} = \ldots = 120 \;, \quad D_z = \begin{vmatrix} 1 & 1 & -5 \\ 1 & -4 & 35 \\ 1 & 3 & -18 \end{vmatrix} = -15.$$

Άρα: $x = \frac{D_x}{D} = \frac{-30}{-15} = 2.$ $\quad y = \frac{D_y}{D} = \frac{120}{-15} = -8.$

$z = \frac{D_z}{D} = \frac{-15}{-15} = 1.$

Άσκηση 2 (Παράδειγμα βιβλίου σελ163)

Cramer 3x3.

$$\begin{cases} x_1 - 2x_2 - 3x_3 = 7 \\ 2x_1 - x_2 + 2x_3 = -2 \\ -3x_1 + 4x_2 - x_3 = -1 \end{cases}$$

Έγινε 1/11/20.

No 2 Date 10/11/...

$$|\underset{\sim}{A}| = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ -3 & 4 & -1 \end{bmatrix} = 1\begin{vmatrix} -1 & 2 \\ 4 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ -3 & -1 \end{vmatrix} + (-3)\cdot\begin{vmatrix} 2 & -1 \\ -3 & 4 \end{vmatrix} = -30 \neq 0$$

$$|\underset{\sim}{A_1}| = \begin{vmatrix} 7 & 2 & -3 \\ -2 & -1 & 2 \\ -1 & 4 & -1 \end{vmatrix} = 7\begin{vmatrix} -1 & 2 \\ 4 & -1 \end{vmatrix} - 2\begin{vmatrix} -2 & 2 \\ -1 & -1 \end{vmatrix} - 3\begin{vmatrix} -2 & -1 \\ -1 & 4 \end{vmatrix} = -30$$

$$|\underset{\sim}{A_2}| = \begin{vmatrix} 1 & 7 & -3 \\ 2 & -2 & 2 \\ -3 & 4 & -1 \end{vmatrix} = 1\begin{vmatrix} -2 & 2 \\ 4 & -1 \end{vmatrix} - 7\begin{vmatrix} 2 & 2 \\ -3 & -1 \end{vmatrix} - 3\begin{vmatrix} 2 & -2 \\ -3 & 4 \end{vmatrix} = 0$$

$$|\underset{\sim}{A_3}| = \begin{vmatrix} 1 & 2 & 7 \\ 2 & -1 & -2 \\ -3 & 4 & -1 \end{vmatrix} = 1\begin{vmatrix} -1 & -2 \\ 4 & -1 \end{vmatrix} - 2\cdot\begin{vmatrix} 2 & -2 \\ -3 & -1 \end{vmatrix} + 7\begin{vmatrix} 2 & -1 \\ -3 & 4 \end{vmatrix} = 60$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{-30}{-30} = 1 \qquad x_2 = \frac{|A_2|}{|A|} = \frac{0}{-30} = 0.$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{60}{-30} = -2.$$

$\boxed{\text{Άσκηση 3}}$ √

<u>Κράμερ 3x3 (Αόριστο)</u>

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 1 \\ 3x_1 + 2x_2 + x_3 = 2 \\ 5x_1 + x_2 + 4x_3 = 3. \end{cases}$$

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \quad x_3 = \frac{|\underset{\sim}{A_3}|}{|\underset{\sim}{A}|}$$

$$|\underset{\sim}{A}| = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 5 & 1 & 4 \end{vmatrix} = 2(8-1)+1(12-5)+3(3-10)=0.$$

$$|\underset{\sim}{A_1}| = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 1(8-1)+1(8-3)+3(2-6)=0.$$

$$|\underset{\sim}{A_2}| = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 5 & 3 & 4 \end{vmatrix} = 2(8-3)-1(12-5)+3(9-10)=0.$$

$$|\underset{\sim}{A_3}| = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 2 \\ 5 & 1 & 3 \end{vmatrix} = 2(6-2)+1(9-10)+1(3-10)=0.$$

Αφού $|\underset{\sim}{A}|=0$ κι όλες οι ορίζουσες μηδέν το (Σ) έχει ΑΠΕΙΡΕΣ ΛΥΣΕΙΣ (ΑΟΡΙΣΤΟ).
Για να βρω τις λύσεις μπορώ να θέσω $x_3=c$ και να βρω λύσεις της μορφής $(x_1=f(c), x_2=f(c), x_3=c)$.

Άσκηση 4 Κράμερ 3x3 (Αδύνατο)

Έγινε 1/11/20

$$\begin{cases} x_1 + x_2 + x_3 = 6. \\ 4x_1 + 4x_2 + x_3 = 2. \\ 6x_1 + 6x_2 + 2x_3 = -11 \end{cases}$$

$$|\underset{\sim}{A}| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 1 \\ 6 & 6 & 2 \end{vmatrix} = 1\begin{vmatrix} 4 & 1 \\ 6 & 2 \end{vmatrix} - 1\begin{vmatrix} 4 & 1 \\ 6 & 2 \end{vmatrix} + 1\begin{vmatrix} 4 & 4 \\ 6 & 6 \end{vmatrix} =$$

$$= 1(8-6)-1(8-6)+24-24=$$

$$=2-2=0.$$

No. 4 Date . / . / .

$$|\underset{\sim}{A}_1| = \begin{vmatrix} 6 & +1 & 1 \\ 2 & 4 & 1 \\ -11 & 6 & 9 \end{vmatrix} = 6(8-6) - 1(4+11) + 1(12+44)$$

$$= 6 \cdot 2 - 1 \cdot 15 + 1 \cdot 56 = 53 \neq 0.$$

Άρα το (Σ) είναι <u>ΑΔΥΝΑΤΟ</u>.

No. 1 Date

$\boxed{\text{Άσκηση 1}}$ (Άσκηση με δύο παραμέτρους) (με Cramer)

$$
\begin{aligned}
-2x + y + z &= a \\
x - 2y + z &= b \\
x + y - az &= -1
\end{aligned}
$$

Λύση

$$|\underset{\sim}{A}| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -a \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} - a \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 6 - 3a.$$

$$|\underset{\sim}{A_1}| = \begin{vmatrix} a & 1 & 1 \\ b & -2 & 1 \\ -1 & 1 & -a \end{vmatrix} = a \begin{vmatrix} -2 & 1 \\ 1 & -a \end{vmatrix} - 1 \cdot \begin{vmatrix} b & 1 \\ -1 & -a \end{vmatrix} + \begin{vmatrix} b & -2 \\ -1 & 1 \end{vmatrix}$$

$$
\begin{aligned}
&= a(2a-1) + ab - 1 + b - 2 = \\
&= 2a^2 - a + ab - 1 + b - 2 = \\
&= 2a^2 + ab - a + b - 3
\end{aligned}
$$

$$|\underset{\sim}{A_2}| = \begin{vmatrix} -2 & a & 1 \\ 1 & b & 1 \\ 1 & -1 & -a \end{vmatrix} = -2 \begin{vmatrix} b & 1 \\ -1 & -a \end{vmatrix} - a \begin{vmatrix} 1 & 1 \\ 1 & -a \end{vmatrix} + \begin{vmatrix} 1 & b \\ 1 & -1 \end{vmatrix}$$

$$
\begin{aligned}
&= -2(-ab+1) - a(-a-1) + (-1-b) \\
&= 2ab - 2 + a^2 + a - b - 1 \\
&= a^2 + 2ab + a - b - 3.
\end{aligned}
$$

$$|\underset{\sim}{A_3}| = \begin{vmatrix} -2 & 1 & a \\ 1 & -2 & b \\ 1 & 1 & -1 \end{vmatrix} = -2 \begin{vmatrix} -2 & b \\ 1 & -1 \end{vmatrix} - \begin{vmatrix} 1 & b \\ 1 & -1 \end{vmatrix} + a \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$$= -2(2-b) - (-1-b) + a(1+2) = -4 + 2b + 1 + b + 3a$$

$$= 3a + 3b - 3.$$

No. 2 Date 24/1

- Εφόσον $|\underset{\sim}{A}| \neq 0 \Leftrightarrow 6-3a \neq 0$

$$a \neq 2.$$

$$x = \frac{|\underset{\sim}{A}_1|}{|\underset{\sim}{A}|} = \frac{2a^2+ab-a+b-3}{6-3a}$$

$$y = \frac{|\underset{\sim}{A}_2|}{|\underset{\sim}{A}|} = \frac{2a^2+2ab+a-b-3}{6-3a}$$

$$z = \frac{|\underset{\sim}{A}_3|}{|\underset{\sim}{A}|} = \frac{3a+3b-3}{6-3a} = \frac{\cancel{3}(a+b-1)}{\cancel{3}(2-a)} =$$

$$= \frac{a+b-1}{2-a}$$

- Εφόσον $|\underset{\sim}{A}| = 0 \Leftrightarrow a=2$ τότε:

$$|\underset{\sim}{A}_1| = |\underset{\sim}{A}_2| = |\underset{\sim}{A}_3| = 3b+3 \text{ οπότε:}$$

Η τιμή του b θα ορίσει αν το (Σ) μας είναι ΑΔΥΝΑΤΟ ή ΑΟΡΙΣΤΟ.

→ Αν $b=-1 \Rightarrow |\underset{\sim}{A}_1| = |\underset{\sim}{A}_2| = |\underset{\sim}{A}_3| = 0$ Το (Σ) είναι ΑΟΡΙΣΤΟ

→ Αν $b \neq -1 \Rightarrow |\underset{\sim}{A}_1| \neq 0$ κ' $|\underset{\sim}{A}_2| \neq 0$ κ' $|\underset{\sim}{A}_3| \neq 0$.
Το (Σ) είναι ΑΔΥΝΑΤΟ

Με μία Μεταβλητή

No. 1

Άσκηση 2.14 σελ 80 — 1.

Να προσδιοριστεί ο $\lambda \in \mathbb{R}$ ώστε το (Σ) να έχει μοναδική λύση (x, y, z) (διαδοχικοί όροι γεωμ. προόδου).

$$\begin{aligned} x + y - z &= 1 \\ 2x + 3y + \lambda z &= 3 \\ x + \lambda y + 3z &= 2 \end{aligned}$$

<u>Λύση</u>

<u>Με Cramer:</u>

$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & \lambda & 3 \\ 1 & \lambda & 3 & 2 \end{pmatrix}$$

$$|\underset{\sim}{A}| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & \lambda \\ 1 & \lambda & 3 \end{vmatrix} = \begin{vmatrix} 3 & \lambda \\ \lambda & 3 \end{vmatrix} - \begin{vmatrix} 2 & \lambda \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 2 & 3 \\ 1 & \lambda \end{vmatrix} = \ldots = -(\lambda+3)\cdot(\lambda-2).$$

$$|\underset{\sim}{A_1}| = \begin{vmatrix} 1 & 1 & -1 \\ 3 & 3 & \lambda \\ 2 & \lambda & 3 \end{vmatrix} = \begin{vmatrix} 3 & \lambda \\ \lambda & 3 \end{vmatrix} - \begin{vmatrix} 3 & \lambda \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 3 & 3 \\ 2 & \lambda \end{vmatrix} = -(\lambda+3)\cdot(\lambda-2).$$

$$|\underset{\sim}{A_2}| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & \lambda \\ 1 & 2 & 3 \end{vmatrix} = \ldots = -(\lambda-2).$$

$$|\underset{\sim}{A_3}| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & \lambda & 2 \end{vmatrix} = \ldots = -(\lambda-2).$$

Για να έχει το (Σ) μοναδική λύση πρέπει: $D \neq 0$. $-(\lambda+3)(\lambda-2) \neq 0 \Leftrightarrow$

$\lambda \neq -3$ κ' $\lambda \neq 2$

No. 2 Date 23/11

$$x_1 = \frac{|A_1|}{|A|} = \frac{-(\lambda+3)(\lambda-2)}{-(\lambda+3)(\lambda-2)} = 1.$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{-(\lambda-2)}{-(\lambda+3)(\lambda-2)} = \frac{1}{\lambda+3}$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{-(\lambda-2)}{-(\lambda+3)(\lambda-2)} = \frac{1}{\lambda+3}$$

Οι αριθμοί x_1, x_2, x_3 αποτελούν διαδοχικούς όρους γεωμετρικής προόδου, οπότε:

$$x_2^2 = x_1 \cdot x_3 \Leftrightarrow \left(\frac{1}{\lambda+3}\right)^2 = \cancel{\frac{1}{\lambda+3}} \Leftrightarrow$$

$$\frac{1}{\lambda+3} = 1 \Leftrightarrow \lambda+3 = 1 \Leftrightarrow$$

$$\boxed{\lambda = -2}$$

Με δύο μεταβλητές

No. 3 Date

$\boxed{\text{Άσκηση 2}}$ (2.1 (σελ 66))

$$\begin{aligned} -2x + y + z &= a \\ x - 2y + z &= b \\ x + y - az &= -1 \end{aligned}$$

<u>Λύση</u>

$$\underset{\sim}{E} = (\underset{\sim}{A} : \underset{\sim}{b}) = \begin{pmatrix} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -a & -1 \end{pmatrix} \longrightarrow$$

$$\xrightarrow{(\Gamma_2 - \Gamma_3) \to \Gamma_3} \begin{pmatrix} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 0 & -3 & 1+a & b+1 \end{pmatrix} \xrightarrow{(\Gamma_1 + 2\Gamma_2) \to \Gamma_2} \begin{pmatrix} -2 & 1 & 1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & -3 & 1+a & b+1 \end{pmatrix}$$

$$\xrightarrow{(\Gamma_2 - \Gamma_3) \to \Gamma_3} \begin{pmatrix} -2 & 1 & 1 & a \\ 0 & -3 & 3 & a+2b \\ 0 & 0 & 2-a & a+b-1 \end{pmatrix}.$$

• Αν $a = 2 \Rightarrow \begin{pmatrix} -2 & 1 & 1 & 2 \\ 0 & -3 & 3 & 2+2b \\ 0 & 0 & 0 & b+1 \end{pmatrix}$

<u>οπότε:</u> $b = -1 \Rightarrow \begin{pmatrix} -2 & 1 & 1 & 2 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow$

Το (Σ) είναι Αόριστο με έναν ελεύθερο άγνωστο. <u>Δηλ</u> $\left.\begin{aligned} -2x + z &= 2 - y \\ -3y + 3z &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} -2x - 2z &= 2 \\ z &= y \end{aligned}$

$\Rightarrow \begin{aligned} x &= y - 1 \\ z &= y. \end{aligned}$ Δηλ. $(x, y, z) = (y-1, y, y)$

$= (-1, 0, 0) + y(1, 1, 1)$

No. 4 Date 23/11

→ Αν $b \neq -1$ $\begin{pmatrix} -2 & 1 & 1 & 2 \\ 0 & -3 & 3 & 2+2b \\ 0 & 0 & 0 & b+1 \end{pmatrix} \Longrightarrow$ Το (Σ) είναι αδύνατο, διότι $rank(\underline{A}) = 2 \neq 3 = rank(\underline{E})$.

→ Αν $a \neq 2 \Rightarrow \underset{\sim}{E} = \begin{pmatrix} -2 & 1 & 1 & a \\ 0 & -3 & 3 & \alpha+2b \\ 0 & 0 & 1 & \frac{\alpha+\cancel{b}-1}{2-\alpha} \end{pmatrix} \Longrightarrow$

$z = \dfrac{a+b-1}{2-a}$, οπότε: $y = \dfrac{a+2b-3\frac{a+b-1}{2-a}}{-3}$

και $x = \dfrac{2\alpha^2+\alpha b-\alpha+b-3}{6-3\alpha}$