Barrackpore Rastraguru Surendranath College

9/21/2014

Seminar On

Solving Traveling Salesman Problem by Ant Colony Optimization

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OUTLINE

- Introduction (Ant Colony Optimization)
- Traveling Salesman Problem (TSP)
- An Example of solving TSP Using Ant Colony Optimization (ACO)
- Reference

SECTION I

Ant Colony optimization

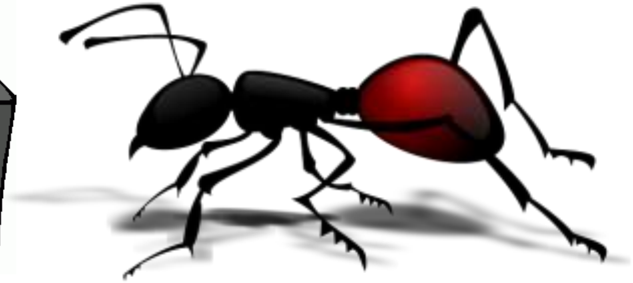
WHAT IS ANT COLONY OPTIMIZATION (ACO) ?

Ant Colony Optimization (ACO) studies artificial systems that take inspiration from the behavior of real ant colonies and which are used to solve discrete optimization problems.

In 1991, the **Ant Colony Optimization metaheuristic** was defined by Dorigo, Di Caro and Gambardella.

SECTION II

Travelling Salesman Problem (TSP)



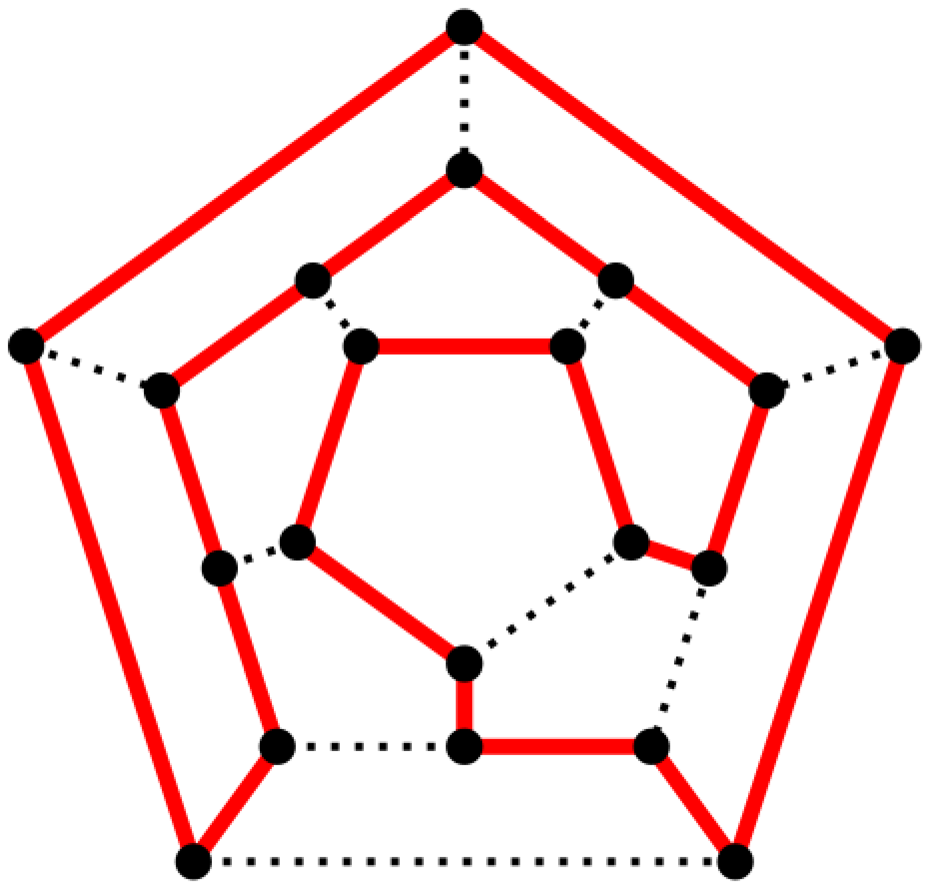
Traveling Salesman Problem (TSP)

Informally: The problem of finding the shortest tour through a set of cities starting at some city and going through all other cities once and only once, returning to the starting city

Formally: Finding the shortest Hamiltonian path in a fully-connected graph

TSP belongs to the class of NP-complete problems. Thus, it is assumed that there is no efficient algorithm for solving TSPs.

A **Hamiltonian path (or traceable path)** is a path in an undirected graph which visits each vertex exactly once. A **Hamiltonian cycle** (or **Hamiltonian circuit**) is a cycle in an undirected graph which visits each vertex exactly once and also returns to the starting vertex. Determining whether such paths and cycles exist in graphs is the Hamiltonian path problem which is NP-complete.



A graph representing the vertices, edges, and faces of a dodecahedron, with a Hamiltonian cycle shown by red color edges.

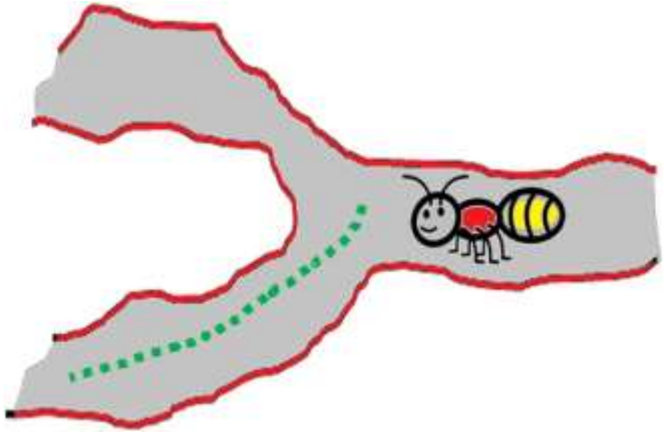
NP problem can not be solved in polynomial time.

TECHNIQUES FOR SOLVING TSP

TSP can be solved through various way. Some of those are,

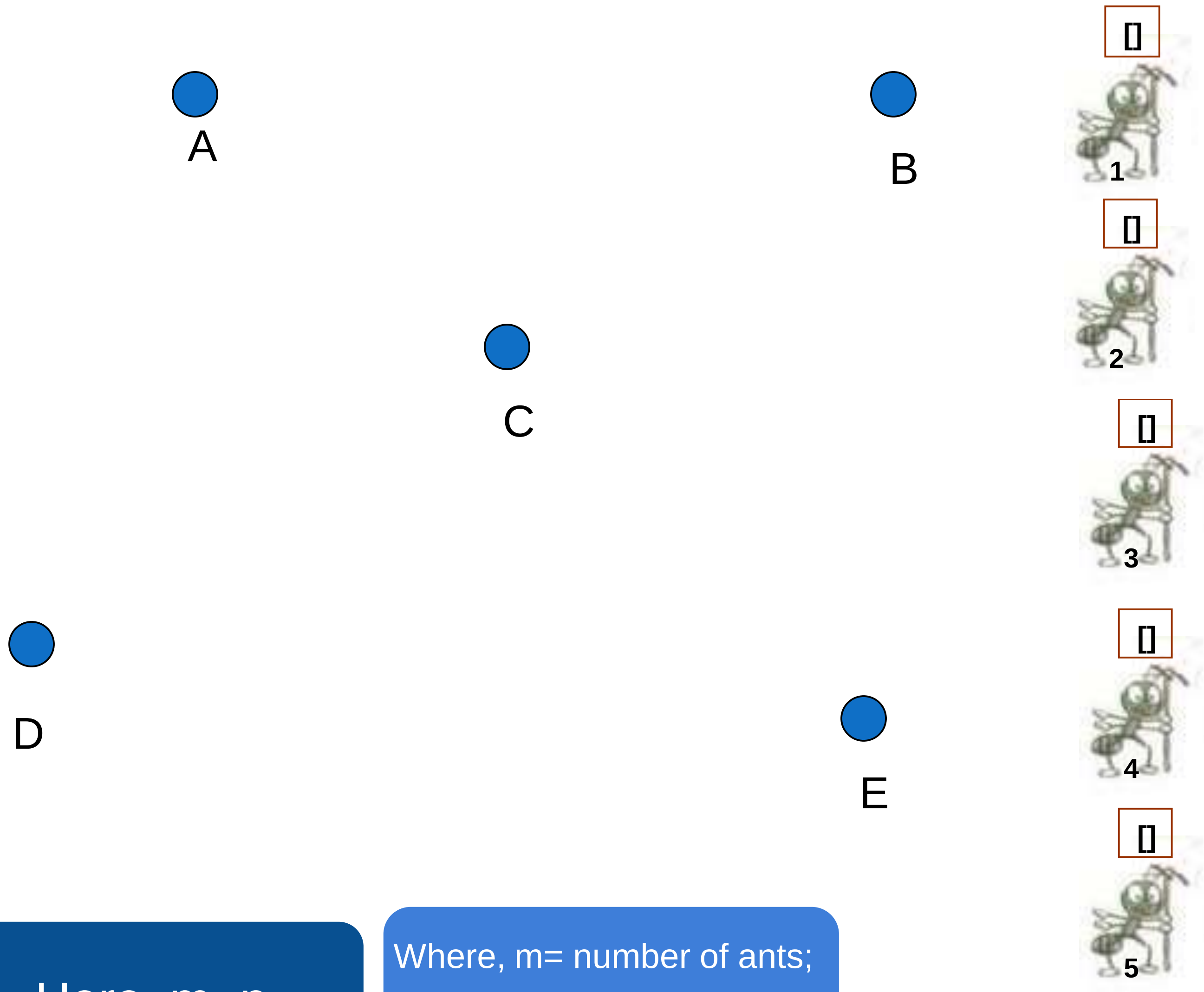
- **Exact algorithms:** The most direct solution would be to try all permutations (ordered combinations) and see which one is cheapest (using brute force search).
- **Computational complexity**
  - Complexity of approximation
- **Heuristic and approximation algorithms**
  - 4.3.1 Constructive heuristics
  - 4.3.2 Iterative improvement
  - 4.3.3 Randomized improvement
    - 4.3.3.1 Ant colony optimization
- **Special cases**
  - 4.4.1 Metric TSP
  - 4.4.2 Euclidean TSP
  - 4.4.3 Asymmetric TSP
  - 4.4.3.1 Solving by conversion to symmetric TSP, etc.

Section III



An Example of solving TSP Using Ant Colony Optimization (ACO)

A simple TSP example



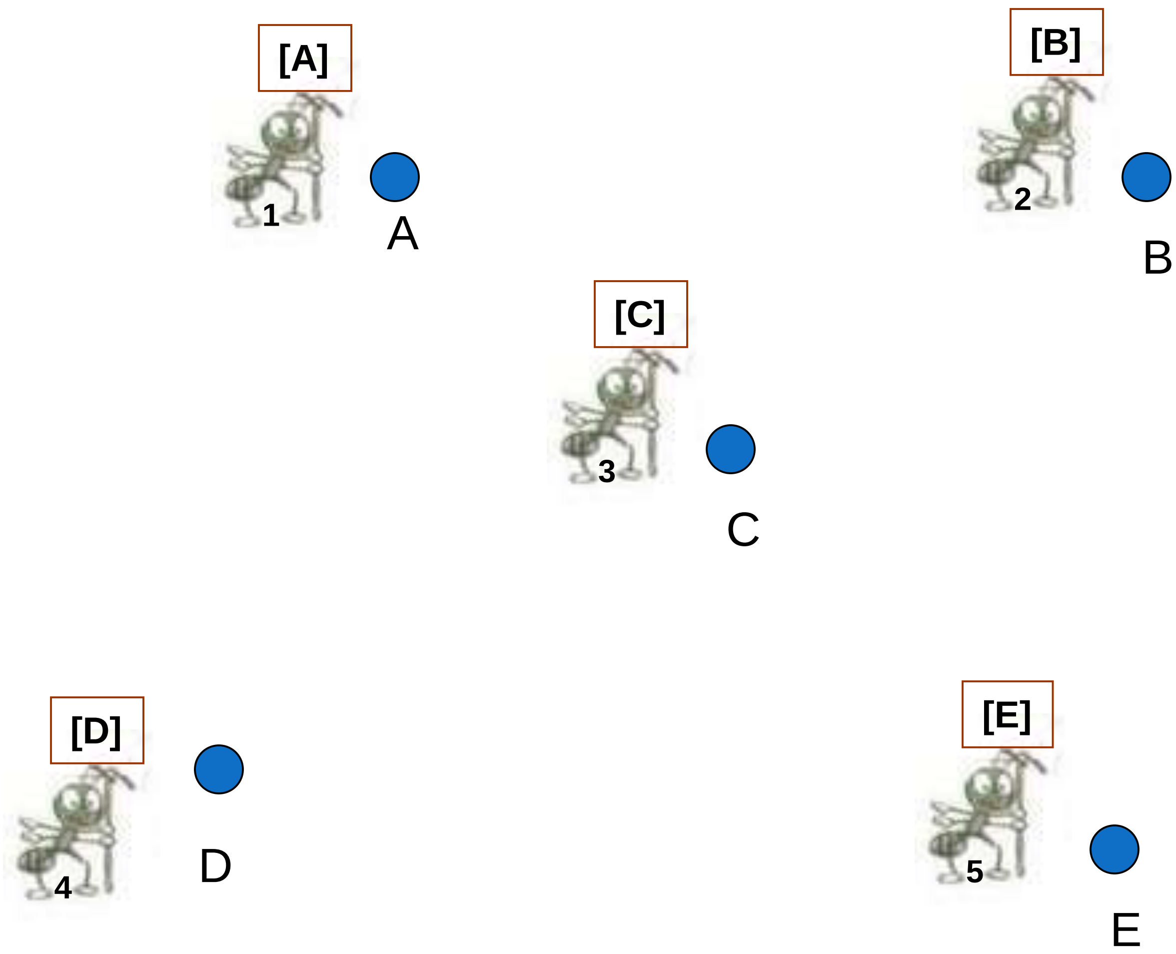
Distances Between Cities:

d_{AB} =100;
d_{AD} = 50;
d_{AE} = 70;
d_{AC} = 60;
d_{BE} = 60;
d_{BC} = 60;
d_{CD} = 90;
d_{CE} = 40;
d_{DE} = 150;

Here, m=n

Where, m= number of ants;
n= number of cities

ITERATION 1



And at first iteration , each of them are assigned to different city.

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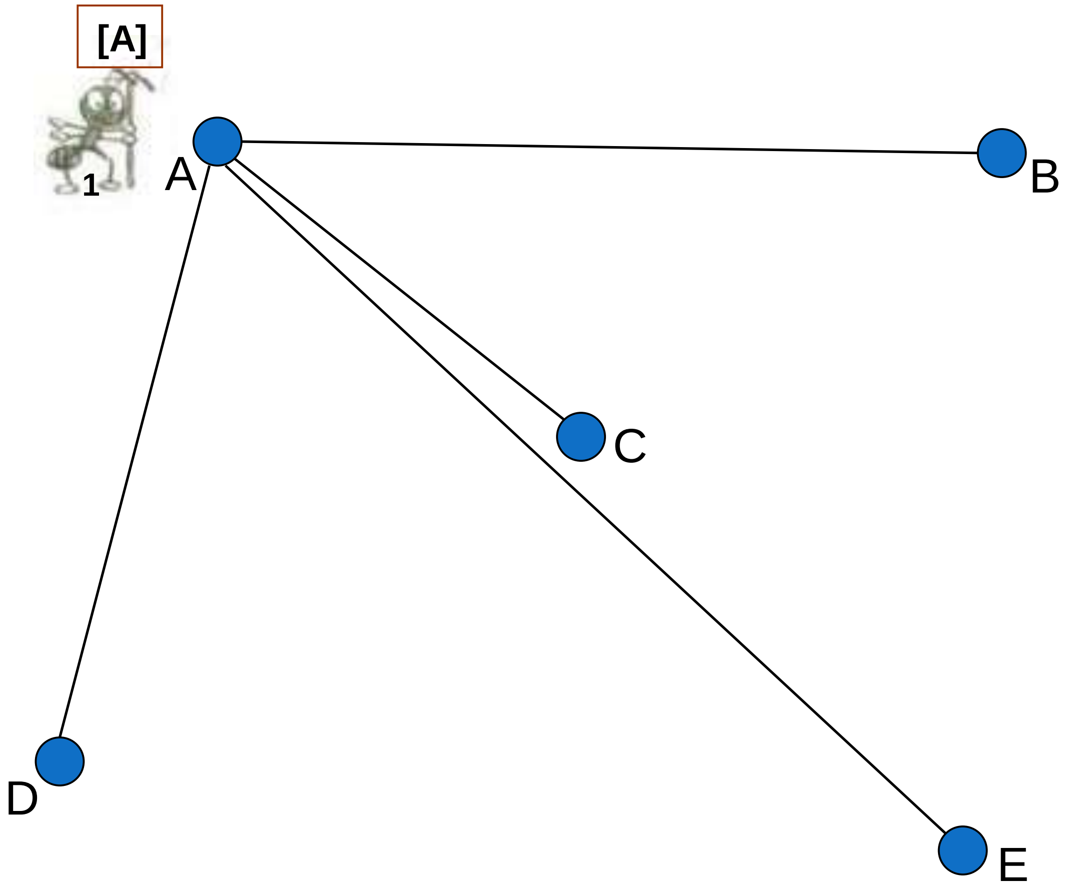
HOW TO BUILD NEXT SUB-SOLUTION?



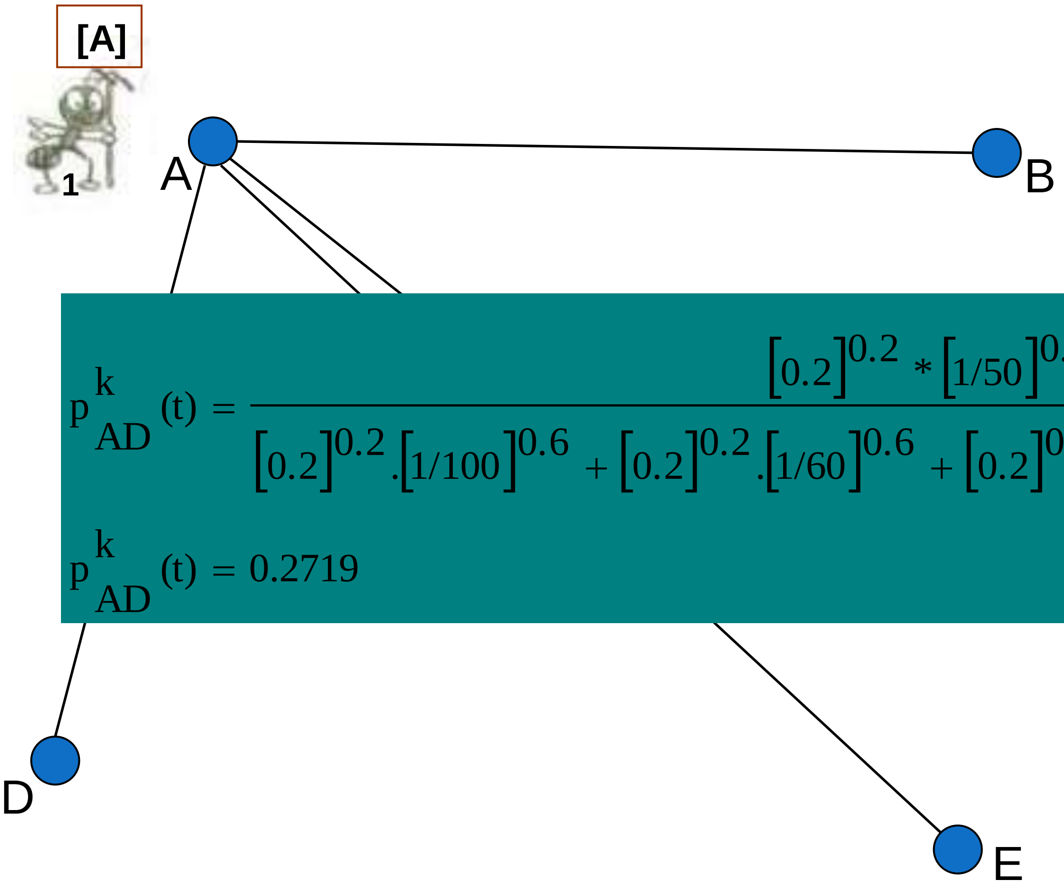
A_1 has four different option i.e. **AB=100**,**AC=60**, **AD=50** and **AE=70**

A_1 will choose path by a probabilistic value

$$p_{ij}^{k}(t) = \frac{\left[\tau_{ij}(t)\right]^{\alpha} . \left[\eta_{ij}\right]^{\beta}}{\sum_{\ell \in J_i^k} \left[\tau_{il}(t)\right]^{\alpha} . \left[\eta_{il}\right]^{\beta}}$$

Depending on a random number.

How to build next sub-solution?



The amount of pheromone on **edge(A,B)**,**edge(A,C)**, **edge(A,D)** and **edge(A,E)** are **$\tau_{A,B}(t)$**, **$\tau_{A,C}(t)$**, **$\tau_{A,D}(t)$** and **$\tau_{A,E}(t)$** respectively with same initial value 0.2.

The value of **η(A,B)**, **η(A,C)**, **η(A,D)** and **η(A,E)** are **1/100**, **1/60**, **1/50** and **1/70**

$$p^{k}_{AD}(t) = \frac{[0.2]^{0.2} * [1/50]^{0.6}}{[0.2]^{0.2}.[1/100]^{0.6} + [0.2]^{0.2}.[1/60]^{0.6} + [0.2]^{0.2}.[1/50]^{0.6} + [0.2]^{0.2}.[1/70]^{0.6}}$$

$$p^{k}_{AD}(t) = 0.2719$$

as shown here respectively.

Depending on a random number A_1 choose AD as path.
Assuming random no.=0.25

τ_{ij}^{k} = **Amount of Pheromone**

η_{ij}^{k}= **Reciprocal of the distance**

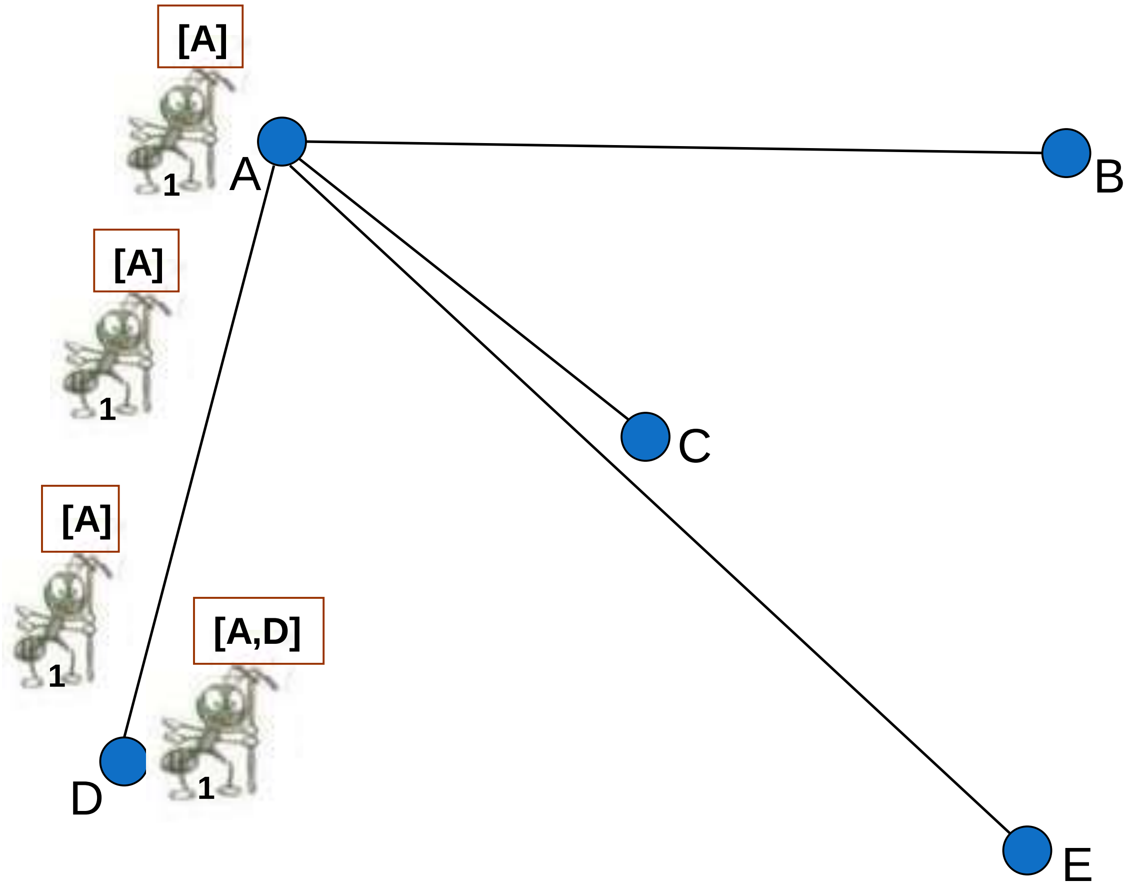
EVAPORATION RATE IN ITERATION 1

Pheromone decay: $\tau_{ij}(t) = (1-\rho).\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}(t)$

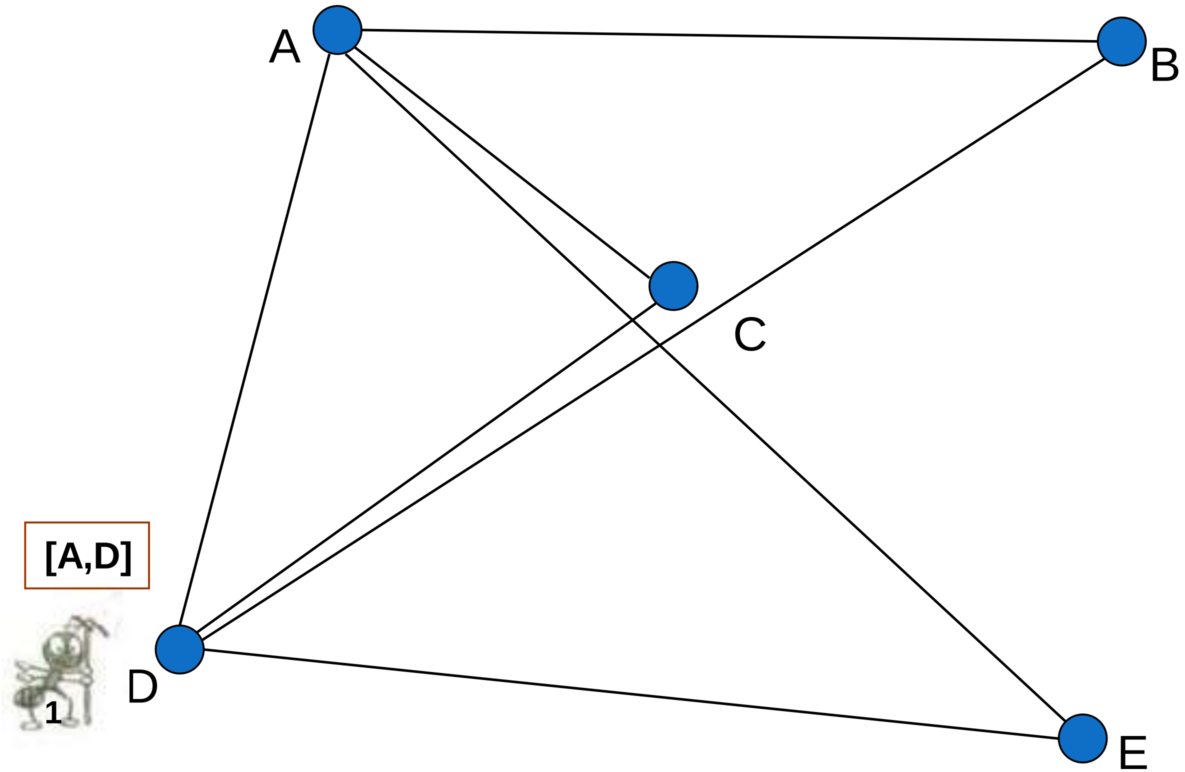
Assume that ρ =0.6

Iteration	Path	Decay Rate
1	AB	0.08
	AC	0.08
	AD	0.08
	AE	0.08
	BC	0.08
	BD	0.08
	BE	0.08
	CD	0.08
	CE	0.08
	DE	0.08

ITERATION 2



How to build next sub-solution?



Now A_1 has three different option i.e.
DB=70,**DC=90**,**DE=150**

Again, A_1 will choose path by the probabilistic value

$$p_{ij}^{k}(t) = \frac{\left[\tau_{ij}(t)\right]^{\alpha} . \left[\eta_{ij}\right]^{\beta}}{\sum_{\ell \in J_i^k} \left[\tau_{il}(t)\right]^{\alpha} . \left[\eta_{il}\right]^{\beta}}$$

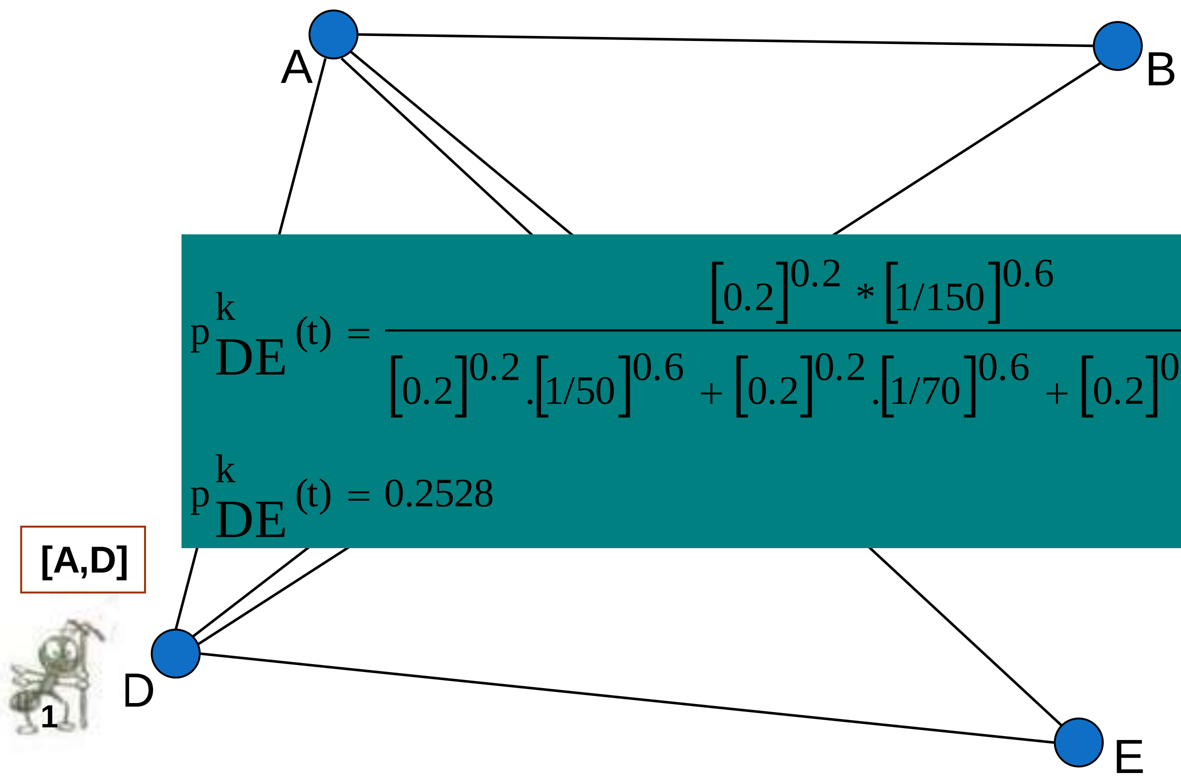
Depending on a random number.

How to build next sub-solution?

The amount of pheromone on **edge(D,B) ,edge(D,C)** and **edge(D,E)** are $\boldsymbol{\tau_{D,B}(t)}$,$\boldsymbol{\tau_{D,C}(t)}$ and $\boldsymbol{\tau_{D,E}(t)}$ respectively with initial value 0.2.

The pheromone on path AD after first iteration will be updated by.

$$\Delta\tau_{ij}^{k} = Q / L^{k}(t) \quad if\ (i, j) \in T^{k}(t)\ else\ 0.$$



$$p_{DE}^{k}(t) = \frac{[0.2]^{0.2} * [1/150]^{0.6}}{[0.2]^{0.2}.[1/50]^{0.6} + [0.2]^{0.2}.[1/70]^{0.6} + [0.2]^{0.2}.[1/70]^{0.6}}$$

$$p_{DE}^{k}(t) = 0.2528$$

50

Where,
Q=100 and
L^K =50

.2 and **β=0.6**

And the probabilities $p_{DB}^{A}(t)$, $p_{DC}^{A}(t)$ and $p_{DE}^{A}(t)$ are as shown here respectively.

Assuming a random number = 0.32, A_1 choose DC as path.

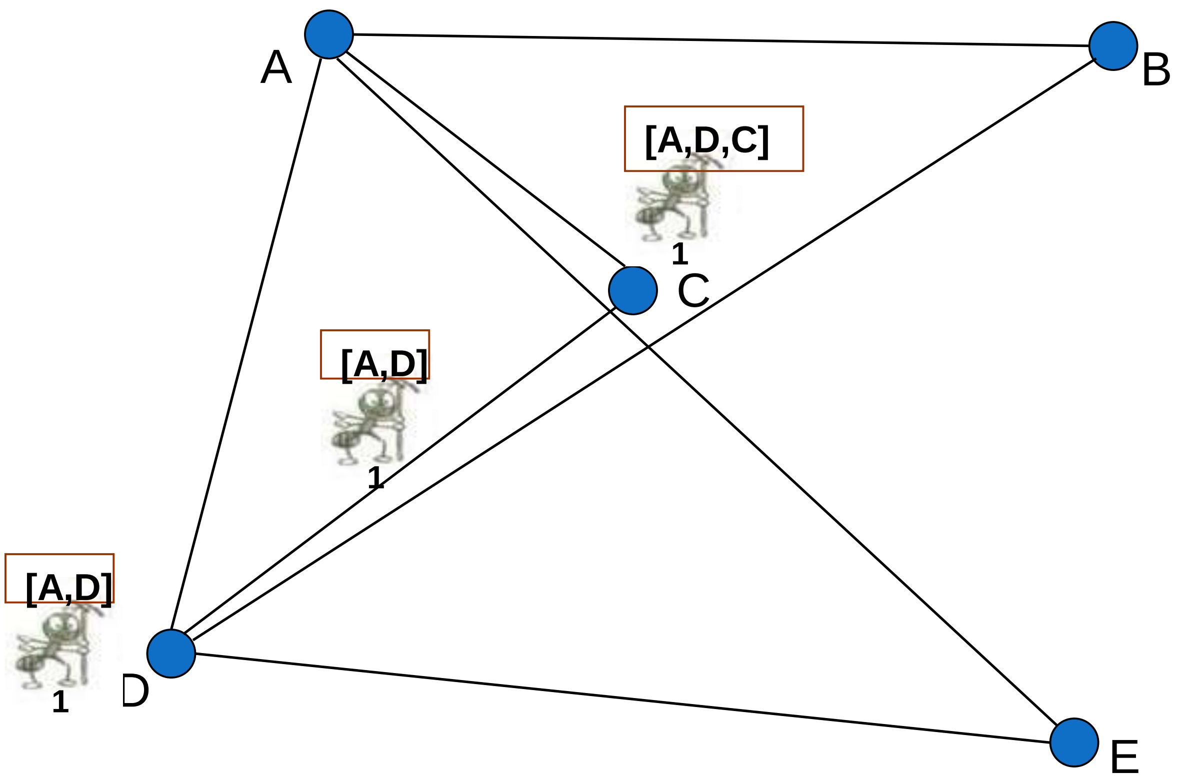
EVAPORATION RATE IN ITERATION 2

Pheromone decay: $\tau_{ij}(t) = (1-\rho).\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}(t)$

Assume that ρ =0.6

Iteration	Path	Decay Rate
2	AB	0.032
	AC	0.032
	AD	0.832
	AE	0.032
	BC	0.032
	BD	0.032
	BE	0.032
	CD	0.032
	CE	0.032
	DE	0.032

ITERATION 3



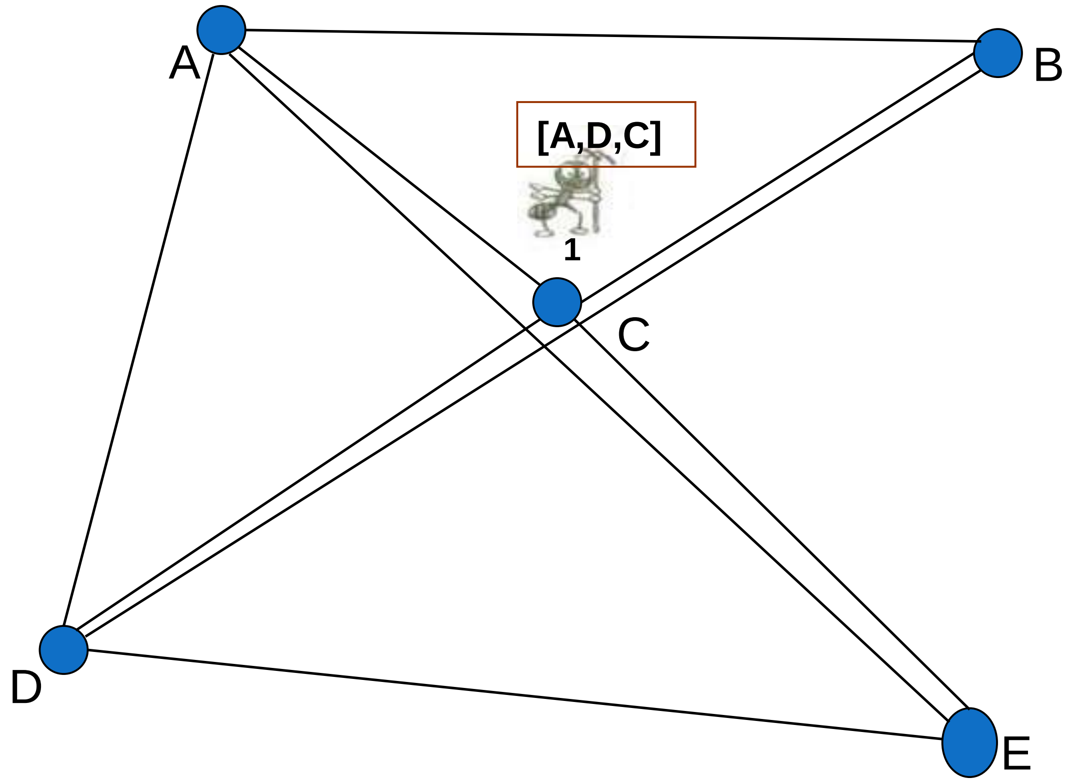
How to build next sub-solution?

Now A_1 has another two option i.e. **CE=40**,**CB=60**

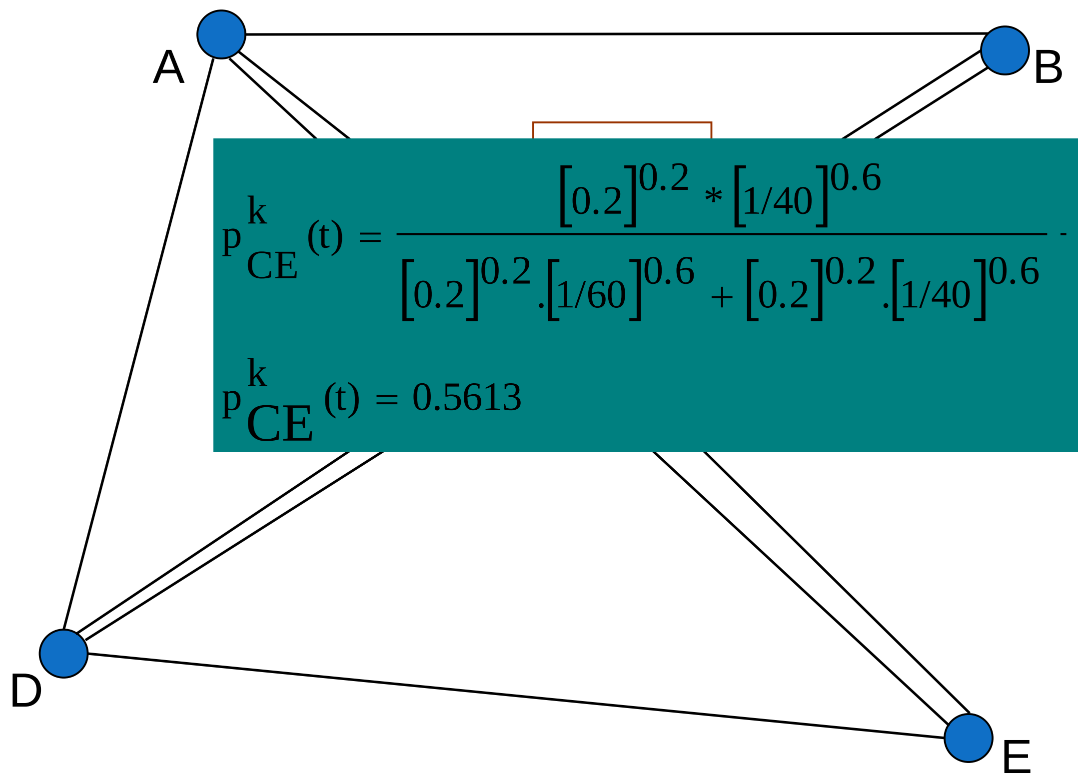
Again, A_1 will choose path by the probabilistic value

$$p_{ij}^{k}(t) = \frac{\left[\tau_{ij}(t)\right]^{\alpha} . \left[\eta_{ij}\right]^{\beta}}{\sum_{\ell \in J_i^k} \left[\tau_{il}(t)\right]^{\alpha} . \left[\eta_{il}\right]^{\beta}}$$

Depending on a random number.



How to build next sub-solution?



$$p^{k}_{CE}(t) = \frac{[0.2]^{0.2} * [1/40]^{0.6}}{[0.2]^{0.2}.[1/60]^{0.6} + [0.2]^{0.2}.[1/40]^{0.6}}$$

$$p^{k}_{CE}(t) = 0.5613$$

The amount of pheromone on **Edge(C,B)** and **edge(C,E)** are $\tau_{C,B}(t)$ and $\tau_{C,E}(t)$ respectively with initial value 0.2.
The pheromone on path DC after second iteration will be updated by.

$$\Delta\tau^{k}_{ij} = Q / L^{k}(t) \quad if\ (i, j) \in T^{k}(t)\ else\ 0.$$

SO,

$$\Delta\tau^{A}_{DC} = 100/140$$

$$\Delta\tau^{A}_{DC} = 0.7142$$

Where,
Q=100 and
L^{K} =50+90
=140

Value of **α=0.2** and **β=0.6**

And the probabilities $p^{A}_{CB}(t)$ and $p^{A}_{CE}(t)$ are as shown here respectively.

Assuming a random number =0.5, A_1 choose CE as path.

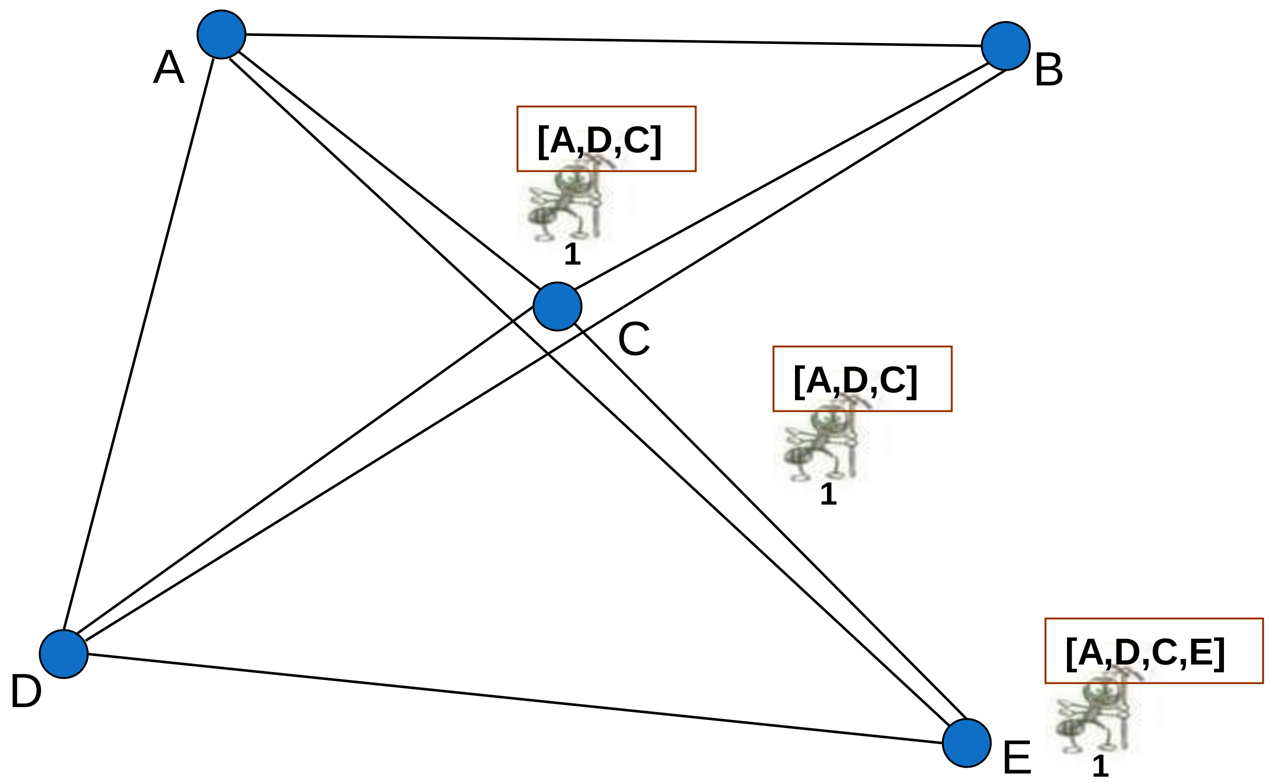
EVAPORATION RATE IN ITERATION 3

Pheromone decay: $\tau_{ij}(t) = (1-\rho).\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}(t)$

Assume that ρ =0.6

Iteration	Path	Decay Rate
3	AB	0.0128
	AC	0.0128
	AD	0.3328
	AE	0.0128
	BC	0.0128
	BD	0.0128
	BE	0.0128
	CD	0.2984
	CE	0.0128
	DE	0.0128

ITERATION 4



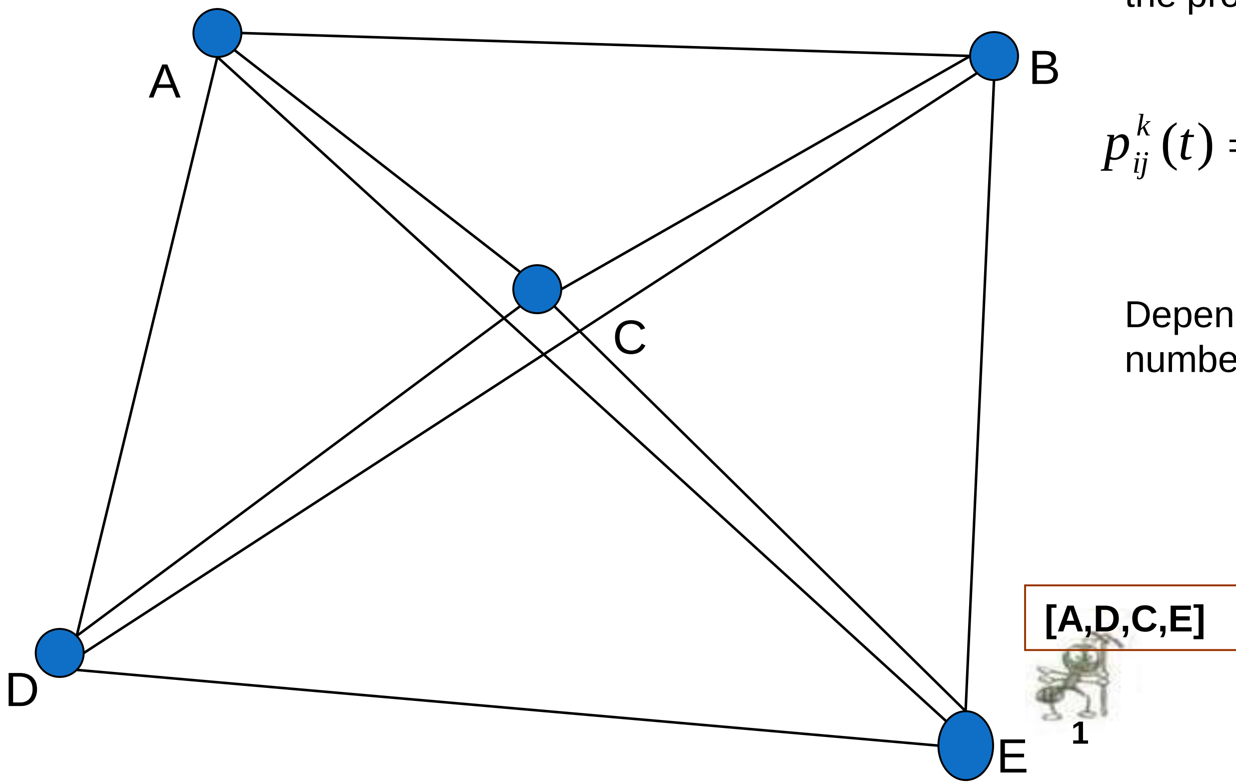
How to build next sub-solution?

Now A_1 has only one path i.e. **EB=60** as A_1 has already visited city A,C and D.

Again, A_1 will choose path by the probabilistic value

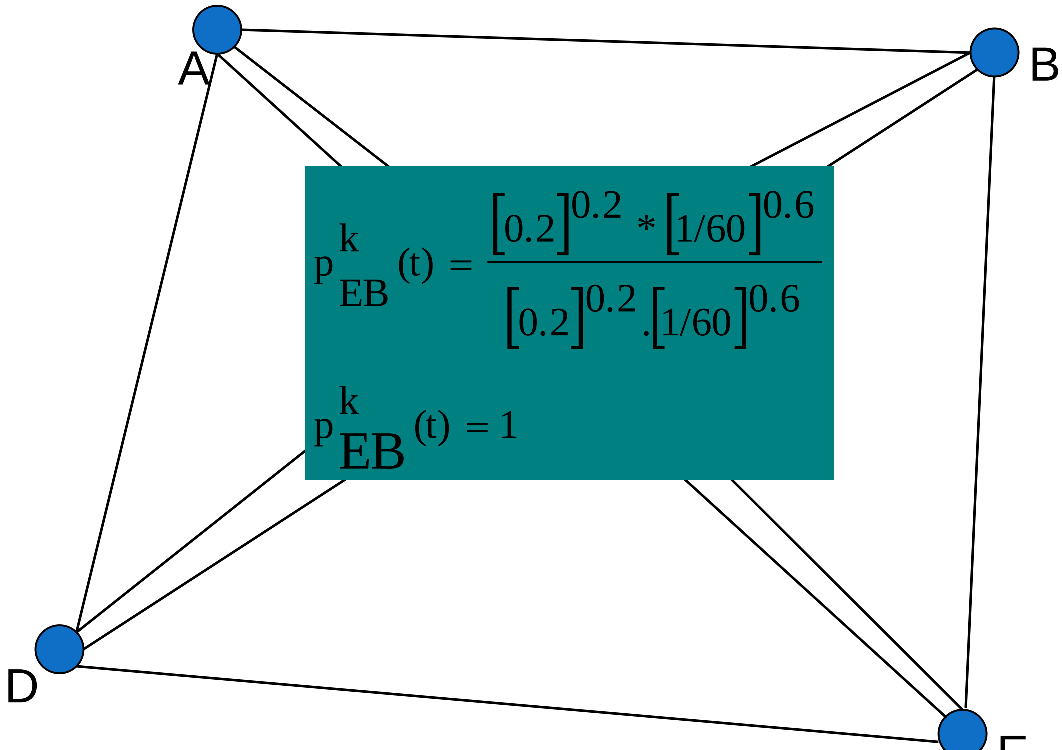
$$p_{ij}^{k}(t) = \frac{\left[\tau_{ij}(t)\right]^{\alpha} . \left[\eta_{ij}\right]^{\beta}}{\sum_{\ell \in J_i^k} \left[\tau_{il}(t)\right]^{\alpha} . \left[\eta_{il}\right]^{\beta}}$$

Depending on a random number.



How to build next sub-solution?

The amount of pheromone on **Edge(E,B)** is $\boldsymbol{\tau_{E,B}(t)}$ with initial value 0.2.
The pheromone on path CE after fourth iteration will be updated by.



$$p^{k}_{EB}(t) = \frac{[0.2]^{0.2} * [1/60]^{0.6}}{[0.2]^{0.2} . [1/60]^{0.6}}$$

$$p^{k}_{EB}(t) = 1$$

[A,D,C,E]

$$\Delta\tau^{k}_{ij} = Q / L^{k}(t) \quad if\ (i, j) \in T^{k}(t)\ else\ 0.$$

$SO,$

$$\Delta\tau^{A}_{CE} = 100/180$$

$$\Delta\tau^{A}_{CE} = 0.5555$$

Where,
Q=100 and
L^K =50+90+40
=180

Value of **α=0.2** and **β=0.6**

And the probabilities $p^{A}_{EB}(t)$ is as shown here,

Assuming a random number =0.5, A_1 choose EB as path.

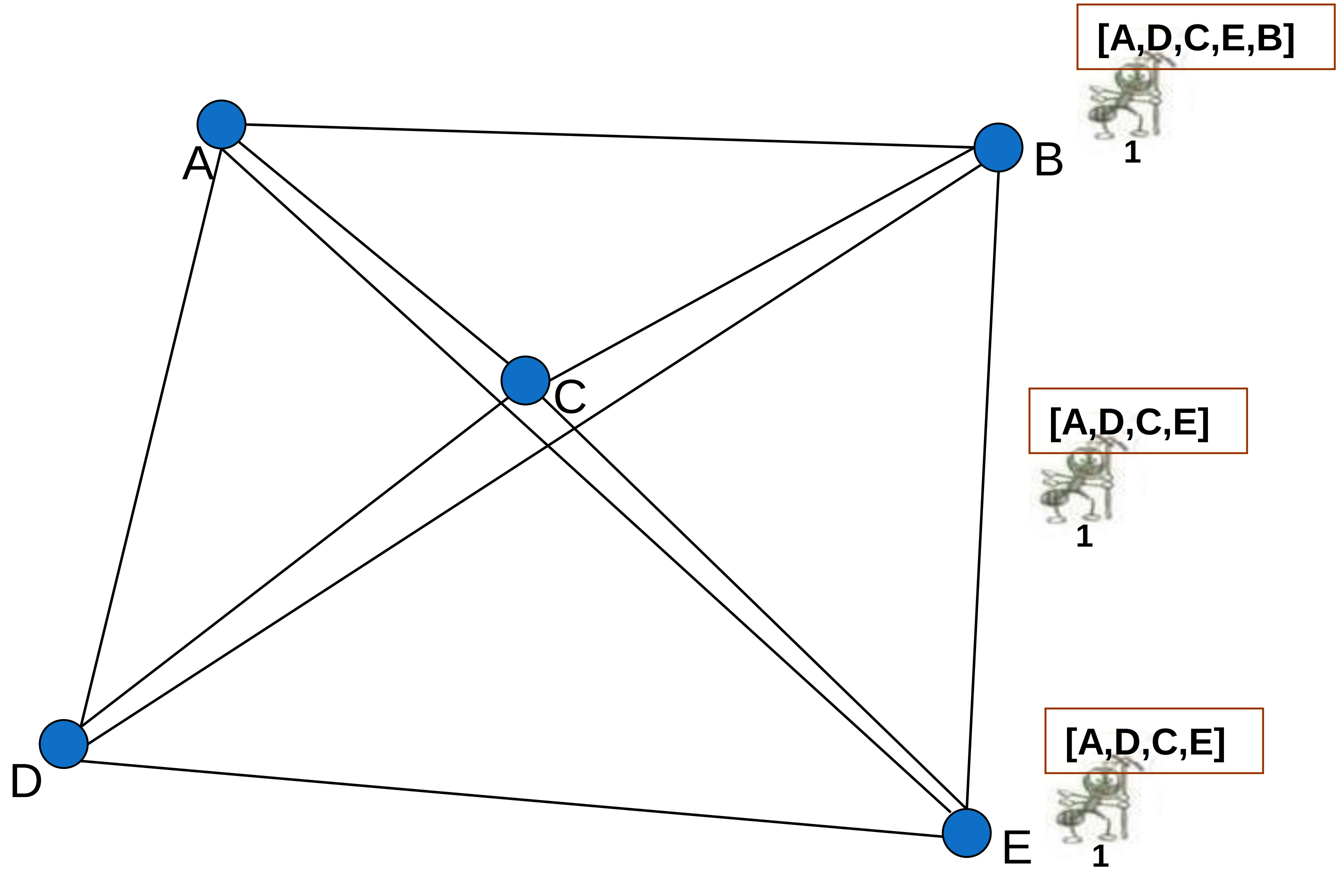
Evaporation rate in Iteration 4

Pheromone decay: $\tau_{ij}(t) = (1-\rho).\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}(t)$

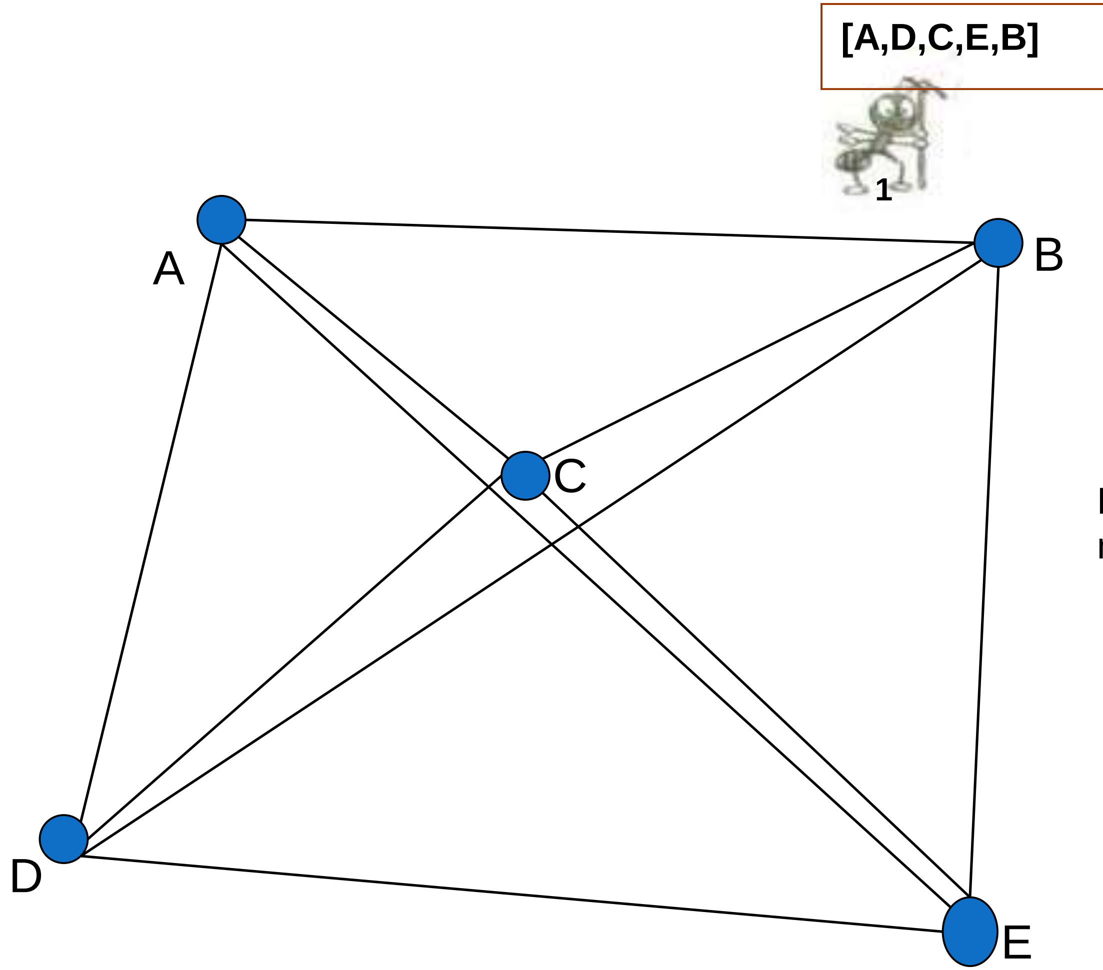
Assume that ρ =0.6

Iteration	Path	Decay Rate
4	AB	0.00512
	AC	0.00512
	AD	0.13312
	AE	0.00512
	BC	0.00512
	BD	0.00512
	BE	0.00512
	CD	0.11936
	CE	0.22732
	DE	0.00512

ITERATION 5



How to build next sub-solution?



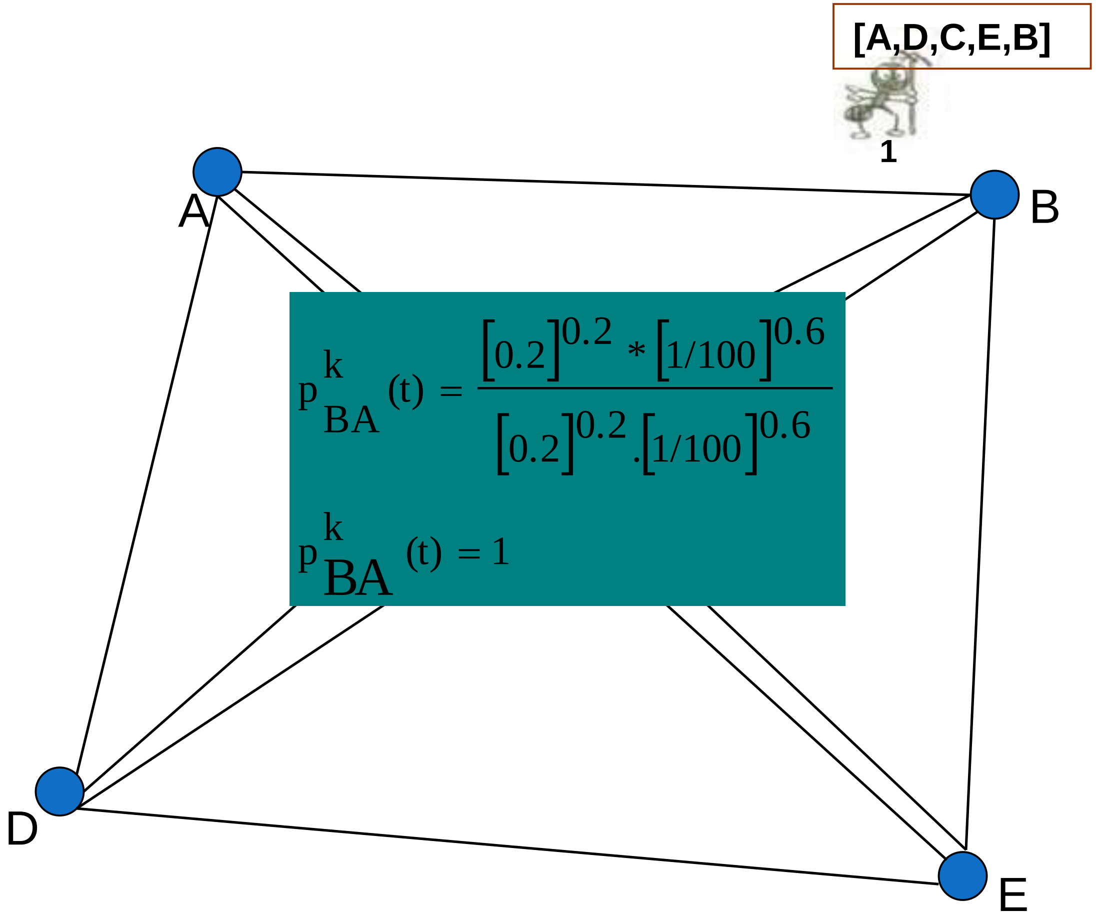
A_1 now has only one path i.e. **BA=100** as A_1 has already visited all other city and return to home city.

Again, A_1 will choose path by the probabilistic value

$$p_{ij}^{k}(t) = \frac{\left[\tau_{ij}(t)\right]^{\alpha} . \left[\eta_{ij}\right]^{\beta}}{\sum_{\ell \in J_i^k} \left[\tau_{il}(t)\right]^{\alpha} . \left[\eta_{il}\right]^{\beta}}$$

Depending on a random number.

How to build next sub-solution?



$$p^{k}_{BA}(t) = \frac{[0.2]^{0.2} * [1/100]^{0.6}}{[0.2]^{0.2} . [1/100]^{0.6}}$$

$$p^{k}_{BA}(t) = 1$$

The amount of pheromone on **Edge(B,A)** is $\boldsymbol{\tau_{BA}(t)}$ with initial value 0.2.
The pheromone on path EB after fifth iteration will be updated by.

$$\Delta\tau^{k}_{ij} = Q / L^{k}(t) \quad if\ (i, j) \in T^{k}(t)\ else\ 0.$$

$SO,$

$$\Delta\tau^{A}_{EB} = 100 / 240$$

$$\Delta\tau^{A}_{\mathbf{EB}} = 0.4166$$

Where,
Q=100 and
L^{K}=
50+90+40+60
=240

Value of **α=0.2** and **β=0.6**

And the probabilities $p^{A}_{BA}(t)$ is as shown here,

Assuming a random number=0.8, A_1 choose BA as path.

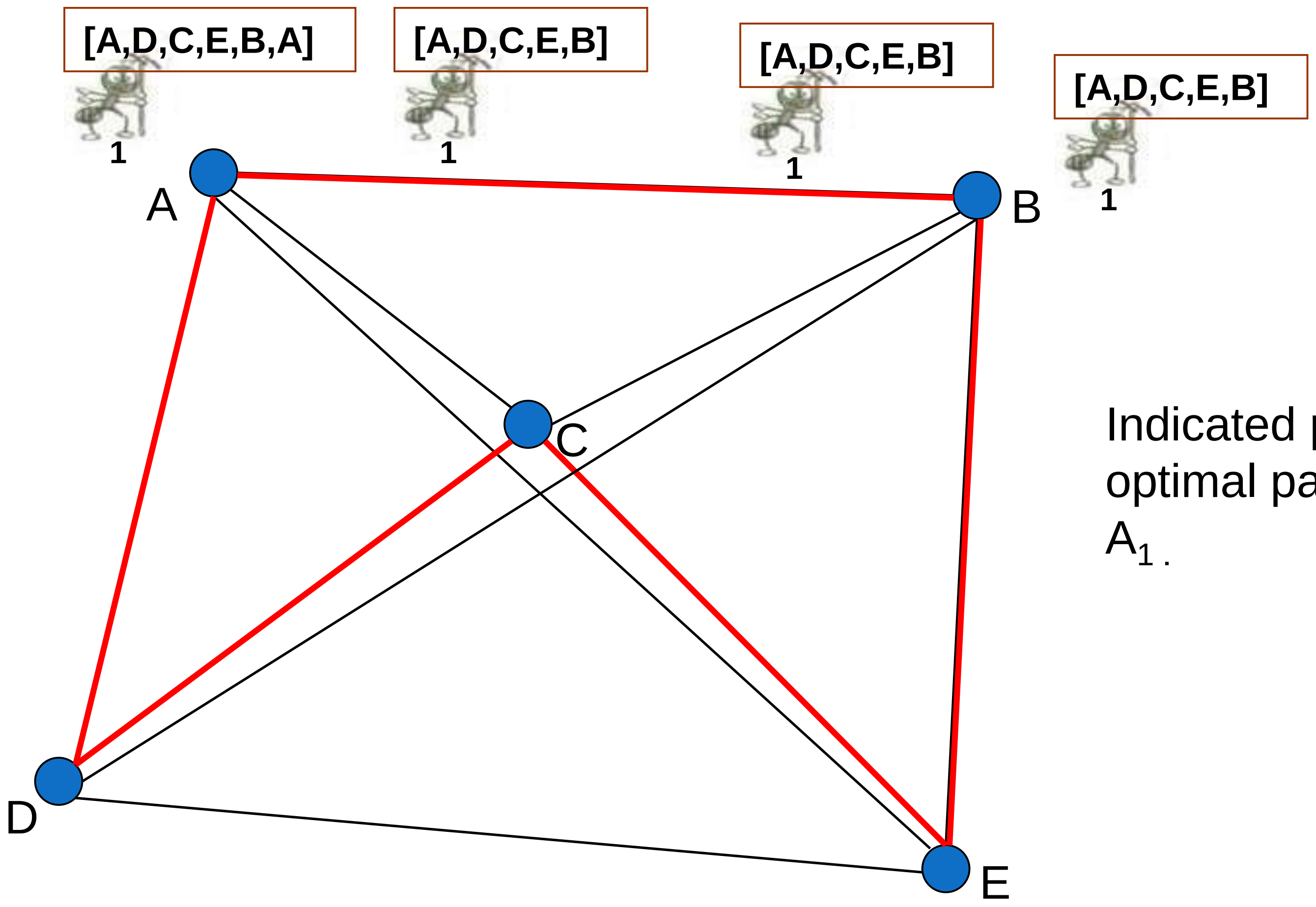
EVAPORATION RATE IN ITERATION 5

Pheromone decay: $\tau_{ij}(t) = (1-\rho).\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}(t)$

Assume that ρ =0.6

Iteration	Path	Decay Rate
4	AB	0.002048
	AC	0.002048
	AD	0.053248
	AE	0.002048
	BC	0.002048
	BD	0.002048
	BE	0.16868
	CD	0.047744
	CE	0.090928
	DE	0.002048

ITERATION 6



Indicated path is the optimal path for ant A_1 .

PATH FOR EACH ANT AND PATH LENGTH

Similarly, path for other ants are presented here with optimal path length.

[A,D,C,E,B,A]

L_1=50+90+40+60+100

L_1 =340

[B,C,D,A,E.B]

L_2 =60+90+50+70+60

L_2 =330

[C,B,E,D,A,C]

L_3 =60+60+150+50+60

L_3 =380

[D,E,A,B,C,D]

L_4 =150+70+100+60+90

L_4 =470

[E,A,B,C,D,E]

L_5 =70+100+60+90+150

L_5 = 470

References

- E. Bonabeau, M. Dorigo, G. Theraulaz. Swarm Intelligence: From Natural to Artificial Systems, 1999
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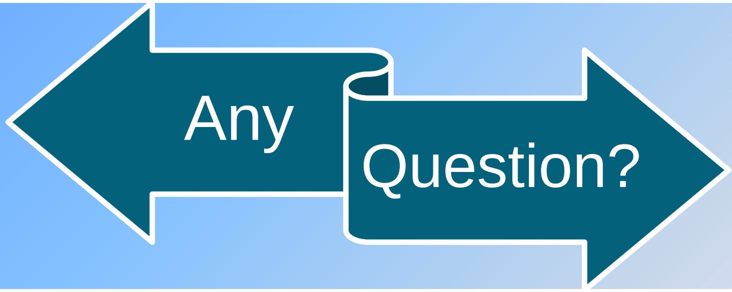
Thank you for yours attention


Any
Question?