HMM1

Wednesday, May 27, 2020 12:15 PM

$$\boxed{p(\underline{\beta}_1,\dots,\underline{\beta}_T)} = \underbrace{\sum_{A_m} \left(\prod_{t=1}^{T} p(\underline{\beta}_t|\alpha_t)\right) p(\alpha_1)\cdot\left(\prod_{t=2}^{T} p(\alpha_t|\alpha_{t-1})\right)}_{(*)}$$

$0,1,1$

$$\lambda = (\overset{24}{\Pi}, \overset{24^2}{A}, \overset{24^2}{B})$$

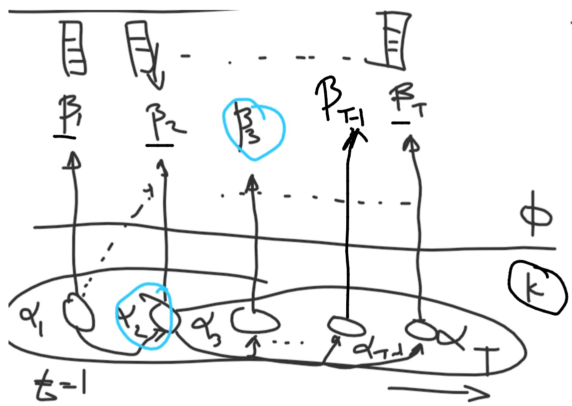
$10\times$

$2^3 = 8$ $\#A_m$ $\times 6 = \underline{\underline{48}}$ $(2T)\times \boxed{N^T \ \#A_m}$

1): ΚΡΥΦΟΥ: ΜΑΡΚΟΒ-Ι

2): ΔΕΝ ΧΑΝΩ ΚΡΥΦΑ ΣΥΝ

3): β_t ΕΞΑΡ ΑΠΟ α_t



ΕΞΟΔ. ΑΥΤΟ ΑΝ-ΧΑΡ

(Π, A) $(B)_t$

$\alpha_t \in Q = \{q_1, q_2, \dots, q_N\}$ $(\emptyset, 1)$

$\beta_t \in S = \{s_1, \dots, s_M\}$ $(0,1)$, $\beta_t \in \mathbb{R}^Q$

$1 \le i \le N$

$$\boxed{p(B) = \sum_{\forall A_i} p(B, A_i)}$$

$$P(A,B) = P(B|A)\cdot P(A)$$

$$\boxed{q_t(i) \equiv p(\beta_1,\dots,\beta_t, \alpha_t = q_i)} = \boxed{\sum_{\forall \alpha_t} q_t = p(\beta_1,\dots,\beta_t)}$$

$$= p(\beta_t | \cancel{\beta_1,\dots,\beta_{t-1}}, \alpha_t)\, p(\beta_1,\dots,\beta_{t-1},\alpha_t) =$$

$$= \boxed{p(\beta_t|\alpha_t)} \left[\sum_{q_j (j=1,N)} p(\beta_1,\dots,\alpha_{t-1}=q_j, \alpha_t = q_i)\right] =$$

$$= p(\beta_t|\alpha_t=q_i) = \sum_j p(\alpha_t = q_j | \cancel{\beta_1 \dots \beta_{t-1}}, \alpha_{t-1}=q_j)$$

$$= p(\beta_t \mid \alpha_t = q_i) = \sum_j p(\alpha_t = q_j \mid \cancel{\beta_1, \ldots, \beta_{t-1}}, \alpha_{t-1} = q_j) \cdot p(\beta_1, \ldots, \beta_{t-1}, \alpha_{t-1} = q_j) =$$

$$= p(\beta_t \mid \alpha_t = q_i) \sum_j \underbrace{p(\alpha_t = q_i \mid \alpha_{t-1} = q_j)} \boxed{p(\beta_1, \ldots, \beta_{t-1}, \alpha_{t-1} = q_j)} =$$

$$a_t(i) = \underbrace{p(\beta_t \mid \alpha_t = q_i)}_{B} \underbrace{\sum_j \underbrace{p(q_i \mid q_j)}_{A} \, a_{t-1}(j)}_{a_{t-1}(j)}$$

$$p(\beta_1, \ldots, \beta_T)$$

$q \in (0,1)$ $\qquad$ $(0,1)$

$$p(\beta_1 = \phi, \beta_2 = 1, \beta_3 = 1)$$

$$\boxed{\phi, 1, 1}$$

$$\Pi = \begin{array}{c} \begin{pmatrix} 0.1 \\ 0.9 \end{pmatrix} \end{array} \begin{array}{c} \phi \\ 1 \end{array} \qquad \lambda = (\Pi, A, B)$$

$$A = \begin{array}{c c} & \begin{array}{cc} \phi & 1 \end{array} \\ \begin{array}{c} \phi \\ 1 \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{pmatrix} \end{array} \begin{array}{l} = 1 \\ = 1 \end{array} \longrightarrow B = \begin{array}{c c} & \begin{array}{cc} \phi & 1 \end{array} \\ \begin{array}{c} \phi \\ 1 \end{array} & \begin{pmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{pmatrix} \end{array} \begin{array}{l} = 1 \\ = 1 \end{array}$$

$t=1$ $\qquad \longrightarrow t=2$ $\qquad t=3$

$$p(\phi, 1)$$

ΠΗΓΗΣ	ϕ	1	1
ϕ	$0.4 \times 0.1 = 0.04$	$a_2(\phi) = R_\phi$ 0.1704	$0.6 \times (0.8 \times R_0 + 0.4 \times R_1) = R_2$
1	$0.7 \times 0.9 = 0.63$	$0.3(0.2 \times 0.04 + 0.6 \times 0.63) = R_1$	$0.3 \times (0.2 \times R_0 + 0.6 \times R_1) = R_3$

$$(0.8 \times 0.04 + 0.4 \times 0.63) \times 0.6$$

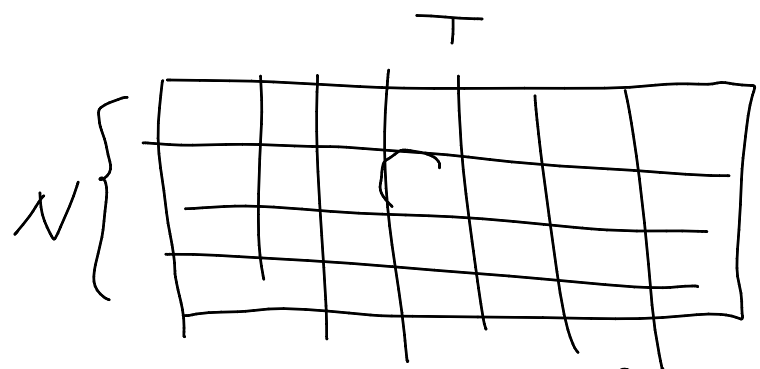
$q_1(0) = p(B_1=0, \alpha_1=0) = p(\underset{B}{0}, \underset{\pi}{0}) = p(\underset{B}{0}|0)\cdot \underset{\pi}{p(0|}$

$q_1(1) = p(B_1=0, \alpha_1=1) = p(0,1) = p(0|1)\cdot p(1)$

$\underline{p(0)} = q_1(0) + q_1(1) = p(0,0) + p(0,1) = p(0)$

$0.04 + 0.63 = 0.69 = p(0)$

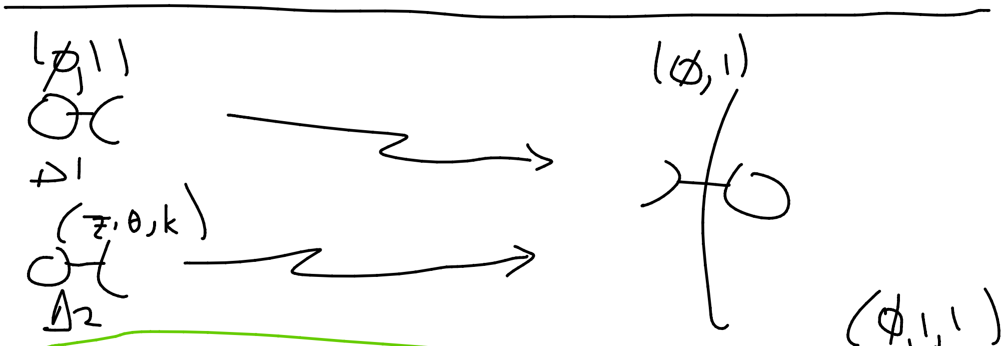
$\boxed{P(0,1,1) = R_2 + R_3}$



$O(N\cdot T(2N+1)) = O(2\underline{N^2T} + NT)$

$\boxed{O(N^2T)}$

$O(2TN^T) = O(\underline{T\cdot N^T})$



$\lambda = (\pi, A, B) = ($

$$\lambda_1 = (\pi_1, A_1, B_1) = \left(\underbrace{\begin{array}{c|c} & \\ 0 & 0.7 \\ 1 & 0.3 \end{array}}_{\pi_1},\ \underbrace{\begin{array}{c|cc} & 0 & 1 \\ 0 & 0.2 & 0.8 \\ 1 & 0.4 & 0.6 \end{array}}_{A_1},\ \underbrace{\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}}_{B_1} \right)$$

$$\lambda_2 = \left(\pi_2 = \begin{array}{c} z \\ \theta \\ k \end{array}\begin{pmatrix} 0.3 \\ 0.5 \\ 0.2 \end{pmatrix},\ A_2 = \begin{array}{c|ccc} & z & \theta & k \\ z & 0.1 & 0.3 & 0.6 \\ \theta & 0.6 & 0.2 & 0.2 \\ k & 0.1 & 0.1 & 0.8 \end{array},\ B_2 = \begin{array}{c|cc} & 0 & 1 \\ z & 0.7 & 0.3 \\ \theta & 0.2 & 0.8 \\ k & 0.5 & 0.5 \end{array} \right)$$

<u>ΕΡΩΤΗΣΗ:</u>

ΕΛΑΒΑ (0,1,1).

ΠΟΙΟΣ ΔΟΡΥΦΟΡΟΣ (Δ1, Δ2) ΕΙΝΑΙ ΠΙΘΑΝ.

ΝΑ ΕΣΤΕΙΛΕ ΤΟ ΜΗΝΥΜΑ;

ΚΡΙΤΗΡΙΟ: ΕΛΑΧΙΣΤΟΠΟΙΗΣΗ ΠΙΘΑΝ. ΛΑΘΟΥΣ

$$P(\Delta 1 \mid 0,1,1) > P(\Delta 2 \mid 0,1,1) \iff$$

BAYES

$$\frac{P(0,1,1 \mid \Delta 1)\cdot \cancel{P(\Delta 1)}^{\,0.5}}{\cancel{P(0,1,1)}} > \frac{P(0,1,1 \mid \Delta 2)\cdot \cancel{P(\Delta 2)}^{\,0.5}}{\cancel{P(0,1,1)}}$$

$$\Updownarrow$$

$$\underline{P(0,1,1 \mid \Delta 1)} > P(0,1,1 \mid \Delta 2) \longrightarrow \text{ΕΣΤΕΙΛΕ Ο Δ1}$$

ΑΑΙ

0 1 1

n →

$\Delta_1 \Rightarrow$

AA1

	0	1	1
ϕ	$\pi_0 \beta(\phi\|\phi)$ 0.56	0.0248 ~~0.0552~~	0.03454
1	0.03	0.4194 ~~0.0466~~	0.244

$+ \rightarrow 0.2788$

$[0.56 \quad 0.03] \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \cdot 0.2 =$

$[0.56 \quad 0.03] \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} \cdot 0.9 =$

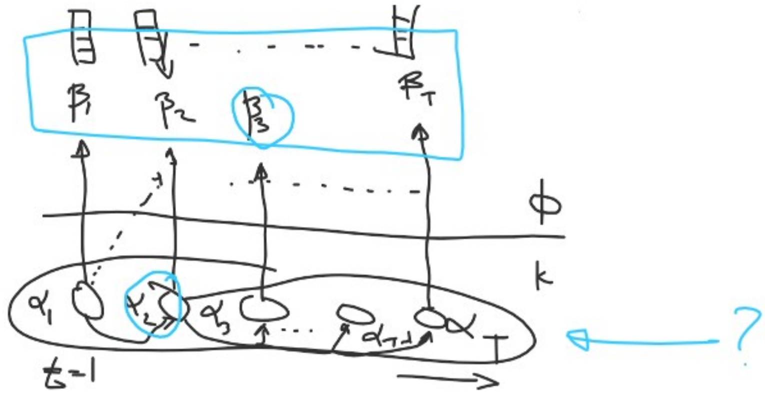
$P(0,1,1|\Delta 1) = 0.2788$

$P(0,1,1|\Delta 2) = 0.1059$

	0	1	1
Z			
Θ			
K			

0.210000	0.027300	0.017601
0.100000	0.074400	0.027496
0.100000	0.113000	0.060830

ΠΙΘΑΝΟΝ ΕΣΤΕΙΛΕ Ο Δ1



$\phi, 1$

$q_t \in Q = \{q_1, q_2, \dots, q_N\}$

$\beta_t \in S = \{s_1, \dots, s_M\}$ | P(A)

$$\lambda_1 = (\Pi_1, A_1, B_1) = \left(\underbrace{\begin{array}{c|c} \phi & 0.7 \\ 1 & 0.3 \end{array}}_{\Pi_1}, \underbrace{\begin{array}{c|cc} & \phi & 1 \\ \phi & 0.2 & 0.8 \\ 1 & 0.4 & 0.6 \end{array}}_{A_1}, \underbrace{\begin{array}{c|cc} & \phi & 1 \\ \phi & 0.8 & 0.2 \\ 1 & 0.1 & 0.9 \end{array}}_{B_1} \right)$$

0,1,1 $\xrightarrow{t}$ Viterbi

T

	ϕ	1	1
ϕ	0.56 ×	0.0224 1	0.032256 ~~0.03056~~ 2
1	0.03 ×	0.4032 1	0.24192 1

$\boxed{\phi, \phi, 1}$

(1) $0.56 * 0.2 = 0.112 \xrightarrow{\times 0.2} 0.0224$

(2) $0.03 * 0.4 = 0.012 \rightarrow$

---

(1) $0.56 * 0.8 = 0.448 \xrightarrow{\times 0.9} 0.04032$

(2) $0.03 * 0.6 = 0.018$

$0.0224 \times 0.2 = \cancel{0.0448}\ 0.00448$

$0.432 \times 0.4 = 0.1528 \xrightarrow{\times 0.2} 0.03056$

$0.16128 \rightarrow$

0.1926 → 0.03056

0.1728 →

$0.0224 \times 0.8 = 0.01792 \xrightarrow{\times 0.9}$

$0.432 \times 0.6 = 0.2592$

α	0.1 / y	1	2	5	0.014 / 5
β	0.01 / x	2	3	4	0.32 / 1
γ	0.2 / x	3	1	2	0.61 / 1
δ	0.5 / x	3	1	2	0.01 / 3
ε	0.4 / x	1	4	3	0.7 / 4

$\gamma \quad \gamma \quad \beta \quad \delta \quad \varepsilon$

$\gamma \quad \delta \quad \varepsilon \quad \alpha \quad \varepsilon$

$O(N^2 T)$

$\lambda = (\Pi, A, B)$

①

$\theta_t \longrightarrow$

④

A'

Δ	1	0	1	1
Π	1	1	0	0

$\Pi = \begin{array}{c|c} & \\ \hline 0 & 0 \\ 1 & 1 \end{array}$

$A = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 4 & 0 \\ 1 & 3 & 3 \end{array}$

Δ	1	1	1	1	0
Π	1	1	0	0	0

$A = \begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix}$

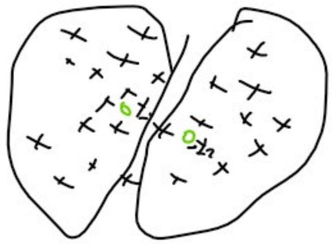
Δ	0	0	1	1

$B = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1 & 6 \end{array}$

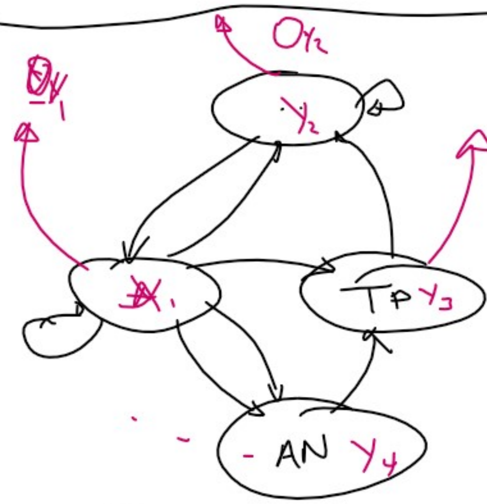
	ϕ	0	0	1	1
	π	1	1	0	0

$$B = \begin{array}{c|cc} & \phi & 1 \\ \phi & 1 & 6 \\ 1 & 3 & 3 \end{array} \Rightarrow \boxed{B = \begin{pmatrix} 1/7 & 6/7 \\ 0.5 & 0.5 \end{pmatrix}}$$

Segmental K-means



$$y = \boxed{\sum_j \min_i d(x_j, y_i)}$$



$$\lambda_y = (\pi_y, A_y, B_y)$$

$$\boxed{\lambda_{A_1} = (\; . \; .. \;)}$$

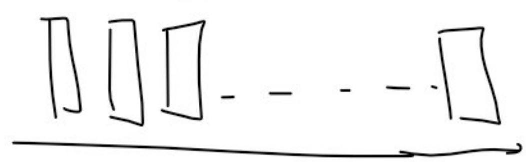
$$\lambda_{A2} = (\; ... \;)$$

$$\vdots$$

$$\lambda_{AN} = (\; .... \;)$$

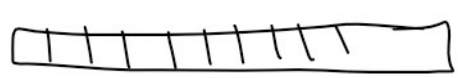
$$\lambda = (\pi, A, B)$$

1 2 3 — —. T

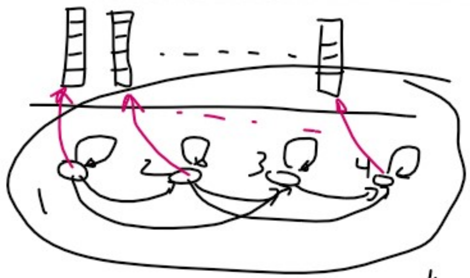


$$\boxed{\lambda_1 = (\pi_1, A_1, B_1)}$$

$$\boxed{\lambda_2 = (\pi_2, A_2, B_2)}$$



$\lambda_{\pi\epsilon} = \Pi, A, B$



$$\Pi = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

λ_{MTH}

$$A_{\pi\epsilon} = \begin{array}{cc} \begin{pmatrix} P_{11} & P_{12} & P_{13} & \emptyset \\ \emptyset & P_{22} & P_{23} & P_{24} \\ \emptyset & \emptyset & \emptyset & 1 \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

$$B = \left(p(\underline{O}|1) \cdots p(\underline{O}|4) \right)$$

4 4

$\pi \in M\Pi TH$

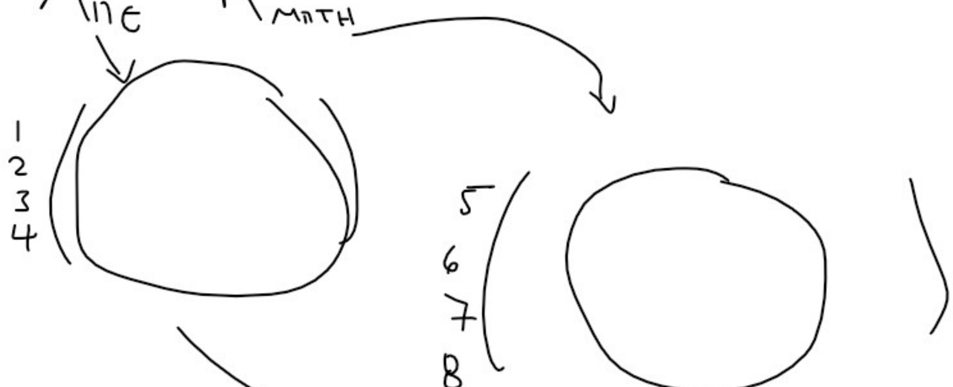
$$A_{M\Pi TH} = \begin{pmatrix} \\ \end{pmatrix}$$

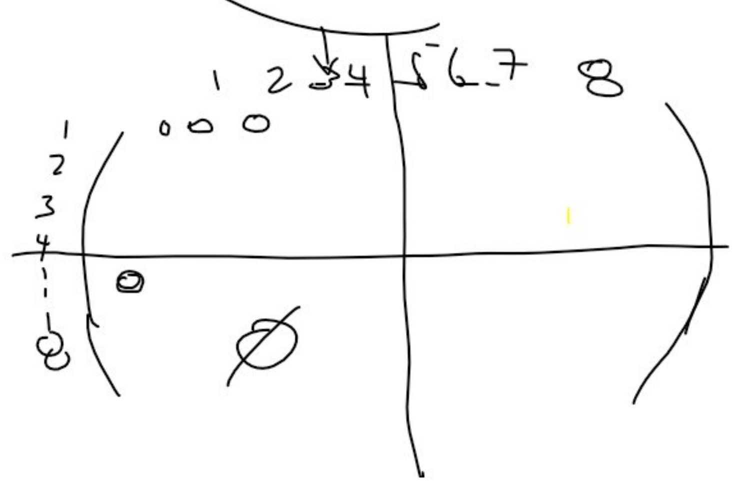
$$\Pi_{\pi\epsilon} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Pi_{MTH} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

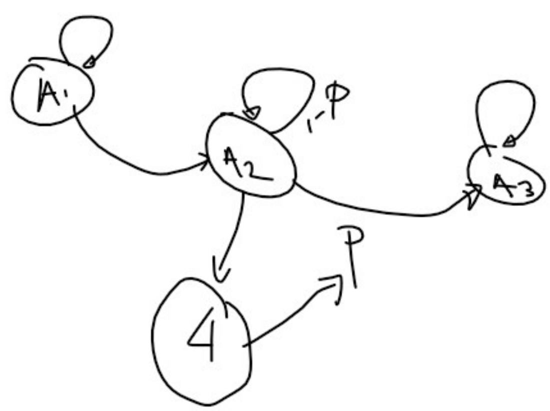
$$\Pi_{\pi\epsilon} = \begin{pmatrix} 1 & 0.5 \\ \phi & \\ \phi & \\ \phi & 0.5 \\ \vdots & \\ \phi & \end{pmatrix} \; 5$$

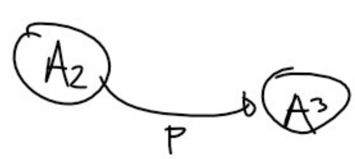
$A_{\pi\epsilon}$ $A_{M\Pi TH}$

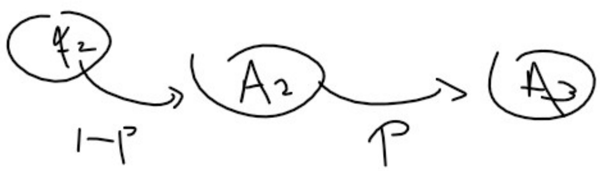




	1	2	D 3	4	5	6	D 7	8
1	0	0	0	∅	∅	∅	∅	∅
2	∅	0	0	0	∅	∅	~~4~~ 5	∅
3	∅	∅	0	0	4	∅	∅	∅
4	∅	∅	∅	3/5	1/5	~~∅~~ 1/5	∅	∅
5	∅	∅	∅	∅	0	0	0	0
6	∅	∅	∅	∅	∅	0	0	0
7	0	∅	∅	∅	∅	∅	0	0
8	∅	∅	∅	∅	∅	∅	∅	1







$$A_2 \xrightarrow{1-p} A_2 \xrightarrow{1-p} A_2 \xrightarrow{p} A_3$$

$$\underline{p(N \leq y \cap A_2 \to A_3)} = (1-p)^{N-1} \cdot p$$

$$\sum_{k=1}^{\infty} k\, p(k) =$$

$$\boxed{= \sum_{k=1}^{\infty} k(1-p)^{k-1} = 4 \Rightarrow}$$

$$\Rightarrow p = ?$$

$x, x_1, x_2, \dots, x_9$

$$\sum_{i=1}^{9} x_i\, p(x_i)$$

$$\boxed{p = ? \leftarrow \sum_{k=1}^{\infty} k p^{k-1} = \mu}$$