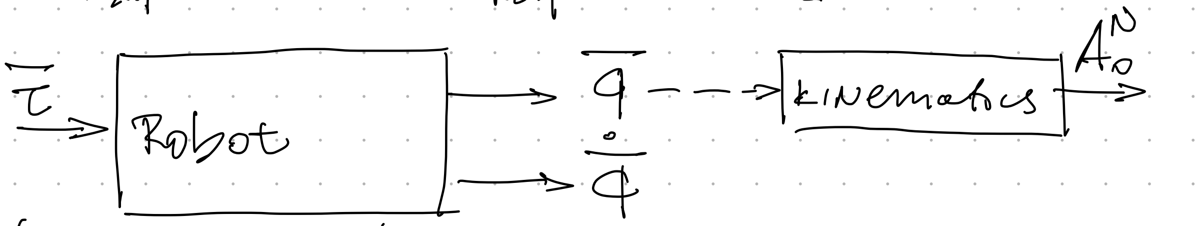
$$\underbrace{\bar{\tau}}_{N\times 1} = \underbrace{D(\bar{q})}_{N\times N}\,\underbrace{\ddot{\bar{q}}}_{N\times 1} + \underbrace{C(\bar{q},\dot{\bar{q}})}_{N\times 1} + \underbrace{G(\bar{q})}_{N\times 1}$$ Dynamics



Runge-Kutta (4th order)
Adams-Moulton (6th order)

$D^{-1}(q)$ always exist
($D(q)$ is positive definite)

$$\dot{\bar{q}} = \dot{\bar{q}}$$

$$\ddot{\bar{q}} = D^{-1}(\bar{q})\left(\bar{\tau} - C(\bar{q},\dot{\bar{q}}) - G(\bar{q})\right)$$

$$\bar{z} = \begin{bmatrix} \bar{q} \\ \hline \dot{\bar{q}} \end{bmatrix}$$

$$\boxed{\dot{\bar{z}} = H(\bar{z}, \bar{\tau})} = \begin{bmatrix} \dot{\bar{q}} \\ \hline D^{-1}(\bar{q})\left(\bar{\tau} - C(\bar{q},\dot{\bar{q}}) - G(\bar{q})\right) \end{bmatrix}$$
$2N\times 1$

Given $\bar{z}(\underbrace{kT_s}_{\text{time}})$, $\bar{\tau}(kT_s) \longrightarrow$

$$\boxed{\bar{z}((k+1)T_s) = \bar{z}(kT_s) + H(\bar{z}(kT_s), \bar{\tau}(kT_s))\cdot T_s}$$

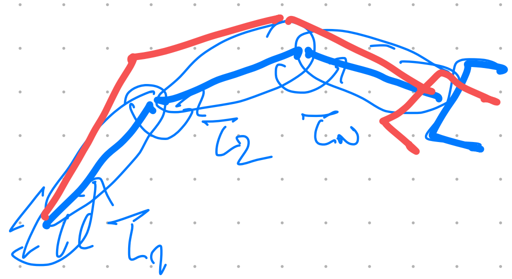
$k \in \mathbb{Z}^+$

T_s - sampling period

Euler forward integration

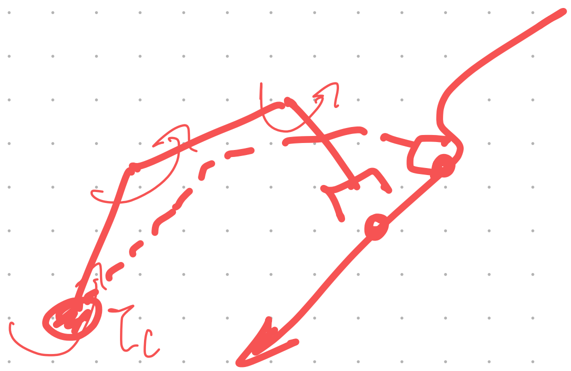
$T_s = 1\,\mathrm{msec}$

$\bar{\tau}(kT_s)$



$\bar{z}^d(t)$ (from inverse kinematics)

desired

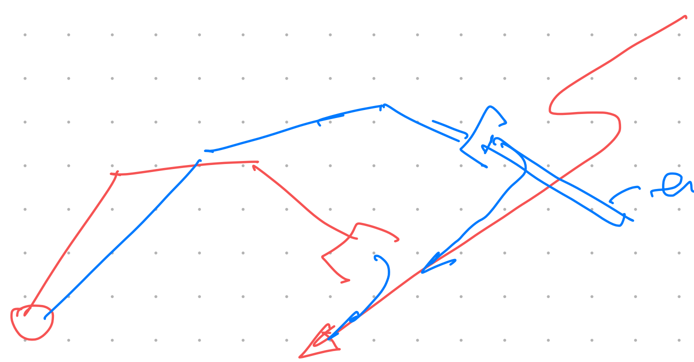


$$\bar{z}(t) \xrightarrow[\bar{\tau}(t)]{?} \bar{z}^d(t)$$

Derivation of $\bar{\tau}(t)$ is a matter of control

$$\bar{\tau}^d(t) = D(\bar{q}^d)\,\ddot{\bar{q}}^d + C(\bar{q}^d, \dot{\bar{q}}^d) + G(\bar{q}^d)$$

$$\forall\ \bar{q}, \dot{\bar{q}}$$



Computed Torque Controller

$$\tau(t) = D(\bar{q})\Big[\ddot{\bar{q}}^d + \underset{n\times n}{\mathbb{K}_d}\,\underset{n\times 1}{(\dot{\bar{q}}^d - \dot{\bar{q}})} + \underset{n\times n}{\mathbb{K}_p}\,(\bar{q}^d - \bar{q})\Big]_{n\times 1} +$$

$$+ C(\bar{q}, \dot{\bar{q}}) + G(\bar{q}) \quad (2)$$

$$\tau(t) = D(\bar{q})\ddot{\bar{q}} + C(\bar{q}, \dot{\bar{q}}) + G(\bar{q}) \quad (1)$$

$$(1) \xrightarrow{(2)} D(\bar{q})\ddot{\bar{q}} = D(\bar{q})\Big[\ddot{\bar{q}}^d + K_d(\dot{\bar{q}}^d - \dot{\bar{q}}) + K_p(\bar{q}^d - \bar{q})\Big]$$

$$\underset{n\times 1}{0} = D(\bar{q})\Big[(\ddot{\bar{q}}^d - \ddot{\bar{q}}) + K_d(\dot{\bar{q}}^d - \dot{\bar{q}}) + K_p(\bar{q}^d - \bar{q})\Big]$$

$$\varepsilon = \bar{q}^d - \bar{q} = \begin{bmatrix} \varepsilon_1(t) \\ \vdots \\ \varepsilon_N(t) \end{bmatrix}$$

error

$$0 = D(\bar{q})\Big[\ddot{\bar{\varepsilon}} + K_d\dot{\bar{\varepsilon}} + K_p\bar{\varepsilon}\Big]$$

$$\ddot{\bar{\varepsilon}} + K_d\dot{\bar{\varepsilon}} + K_p\bar{\varepsilon} = 0$$ Linear Ordinary Equation

$$\lim_{t\to\infty} \varepsilon(t) = 0 \quad (\text{Select } \mathbb{K}_d, \mathbb{K}_p)$$

$$K_d = \begin{bmatrix} K_{d_1} & & \\ & \ddots & \\ & & K_{d_N} \end{bmatrix} \qquad K_p = \begin{bmatrix} K_{p_1} & & \\ & \ddots & \\ & & K_{p_N} \end{bmatrix}$$

K_{d_i}, K_{p_i}: positive

$$\ddot{\varepsilon}_i(t) + K_{d_i}\dot{\varepsilon}_i(t) + K_{p_i}\varepsilon_i(t) = 0$$

$$\lim_{t \to \infty} \varepsilon_i(t) \longrightarrow \emptyset$$

Measurement requirements $\bar{q}, \dot{\bar{q}}$

Computation requirements $D(\bar{q})$, $\underbrace{C(\bar{q}, \dot{\bar{q}})}_{\text{time consuming}}$, $G(\bar{q})$

Trajectory requirements $\ddot{\bar{q}}^d, \dot{\bar{q}}^d, \bar{q}^d$

$\bar{\tau}(t)$ is continuous $\longrightarrow$ @ discrete time $\bar{\tau}(kT_s)$

1. $C(\bar{q}, \dot{\bar{q}}) = O_{N \times N}$
2. $C(\bar{q}^d, \dot{\bar{q}}^d)$ use instead (off-line)
   if $\bar{q} \to \bar{q}^d$ $\dot{\bar{q}} \to \dot{\bar{q}}^d$ then $C(\bar{q}^d, \dot{\bar{q}}^d) \simeq C(\bar{q}, \dot{\bar{q}})$

3. $G(\bar{q}^d)$ instead of $G(\bar{q})$

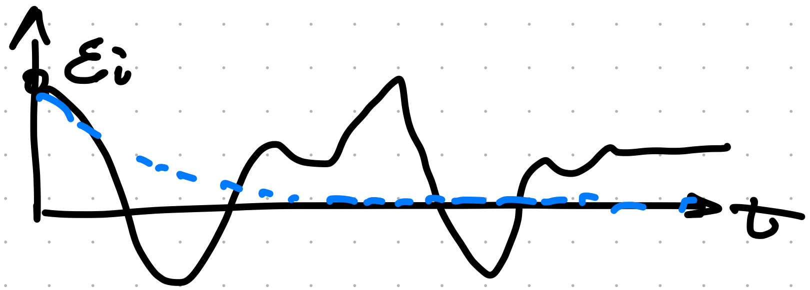
4. $D(\bar{q}^d)$ instead of $D(\bar{q})$

5. $I_{N\times N}$ instead of $D(\bar{q})$

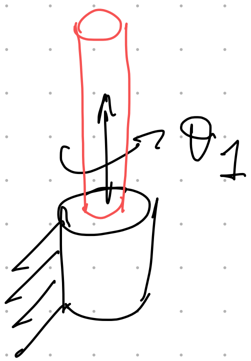
6. $I_{N\times N} \rightarrow D(\bar{q})$

$O_{N\times 1} \rightarrow C(\bar{q}, \dot{\bar{q}})$

$O_{N\times 1} \rightarrow G(\bar{q})$

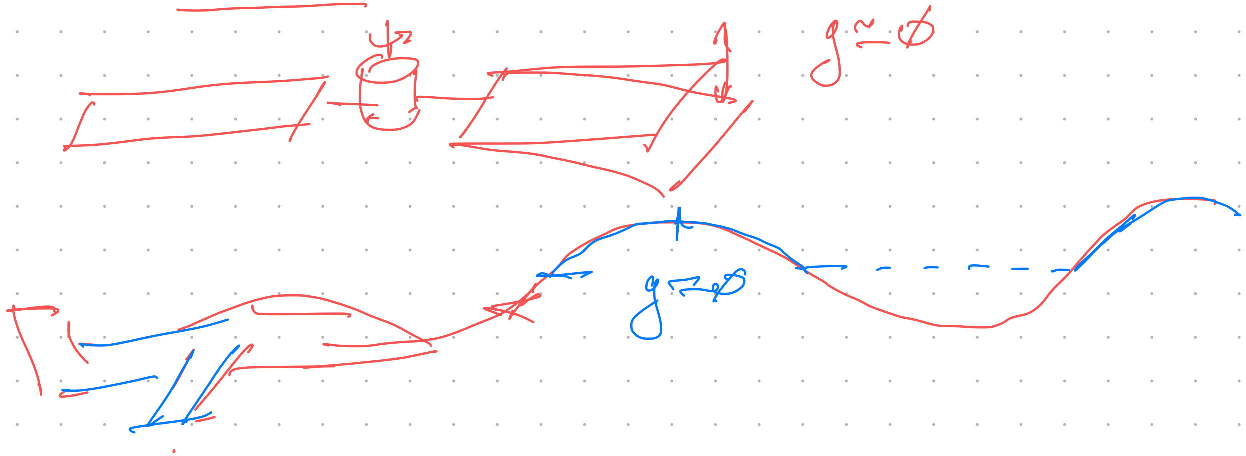


$\tau(t)$ – should be applied to joints

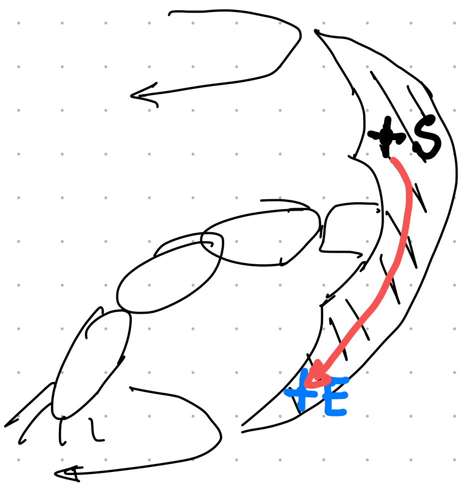


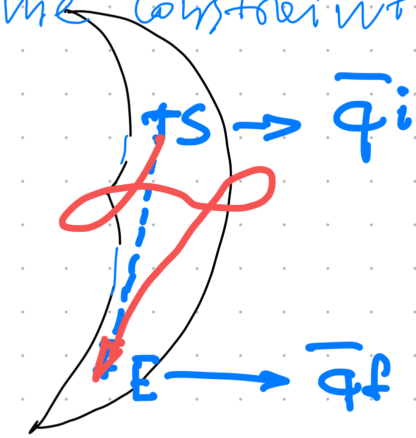
$G_1(\theta_1) = \emptyset$

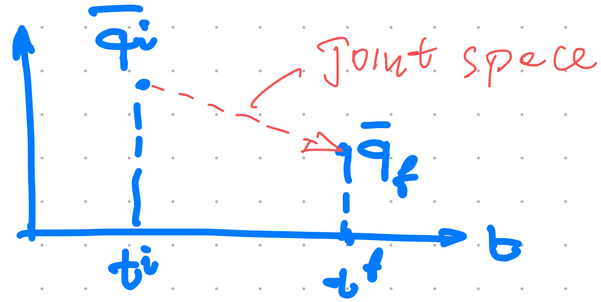
space structures

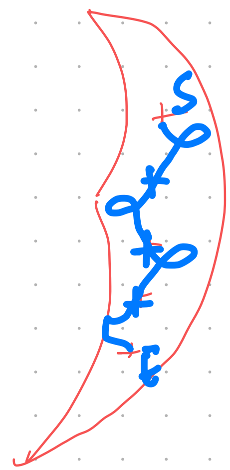


PATH PLANNING (∄ time constraints)









TRAJECTORY PLANNING

$$\bar{q}_i(t_i) \longrightarrow \bar{q}_f(t_f)$$

∃ time as a parameter

$$q_i(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

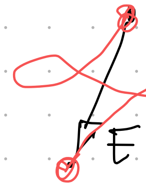
Given $q_i(t_i) = q_0 \qquad q_i(t_f) = q_f$

$\dot{q}_i(t_i) = v_0 \qquad \dot{q}_i(t_f) = v_f$

$$q_0 = a_0 + a_1 t_i + a_2 t_i^2 + a_3 t_i^3$$
$$v_0 = a_1 + 2a_2 t_i + 3a_3 t_i^2$$
$$q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$
$$v_f = a_1 + 3a_2 t_f + 3a_3 t_f^2$$

$$\begin{bmatrix} 1 & t_i & t_i^2 & t_i^3 \\ 0 & 1 & 2t_i & 3t_i^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix}$$

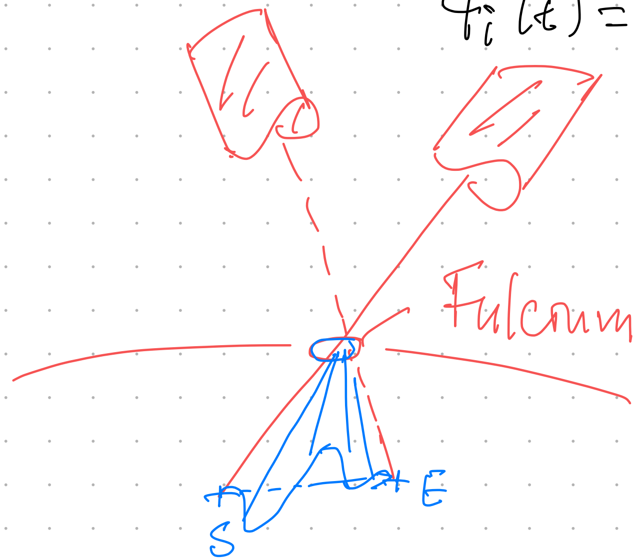
$$\det(\downarrow) = (t_f - t_i)^4$$



$S \longrightarrow$ Inverse Kinematics $q_i(t_i)$, $i = 1, .., N$

$E \longrightarrow$ $q_i(t_f)$

$$q_i^d(t) = a_0^d + a_1^d t + a_2^d t^2 + a_3^d t^3, \quad i = 1, .. N$$



Fulcrum point

Aurora NDI

Problem 4 of Project 2