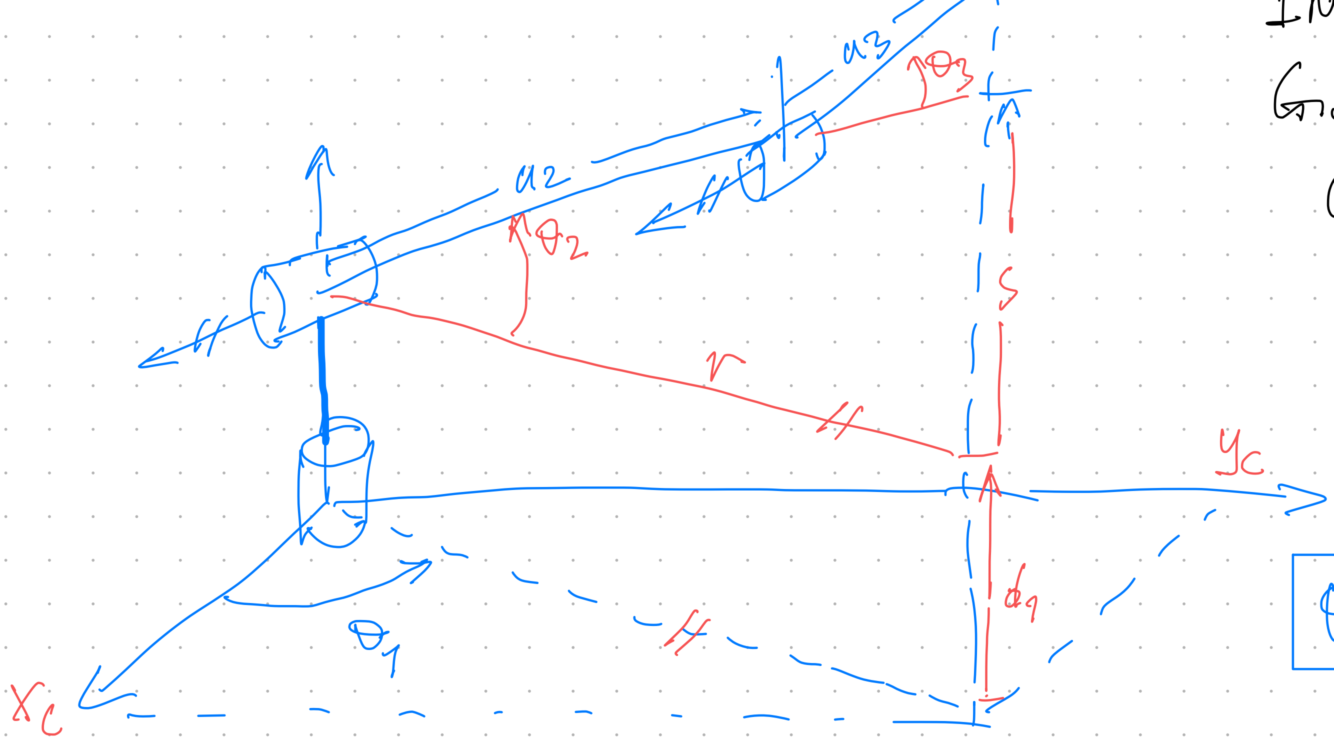
MATLAB Symbolic sym
- subst
- $\cos(th_1) \to c1$ $th_1: \theta_1$
- simplify $\cos^2 x + \sin^2 x \to 1$

Robotics Toolbox by Corke

by Lee, Gonzales & Fu (for PUMA 560)

Generate pdf for HW (or jpeg)   (Word → pdf) Latex

First three DoF of PUMA 560



INVERSE KINEMATICS

Given x_c, y_c, z_c

Compute $\theta_1, \theta_2, \theta_3$

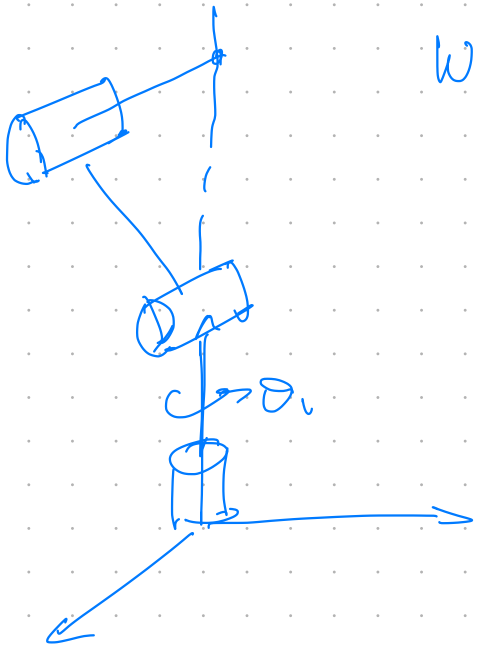
$$\theta_1 = \cancel{\operatorname{atan}\left(\frac{y_c}{x_c}\right)}$$

$$\cancel{\operatorname{atan}\left(\frac{-y_c}{-x_c}\right)}$$

$$\boxed{\theta_1 = \operatorname{atan2}(x_c, y_c)}$$

$$\theta_1 = \pi + \operatorname{atan2}(x_c, y_c)$$

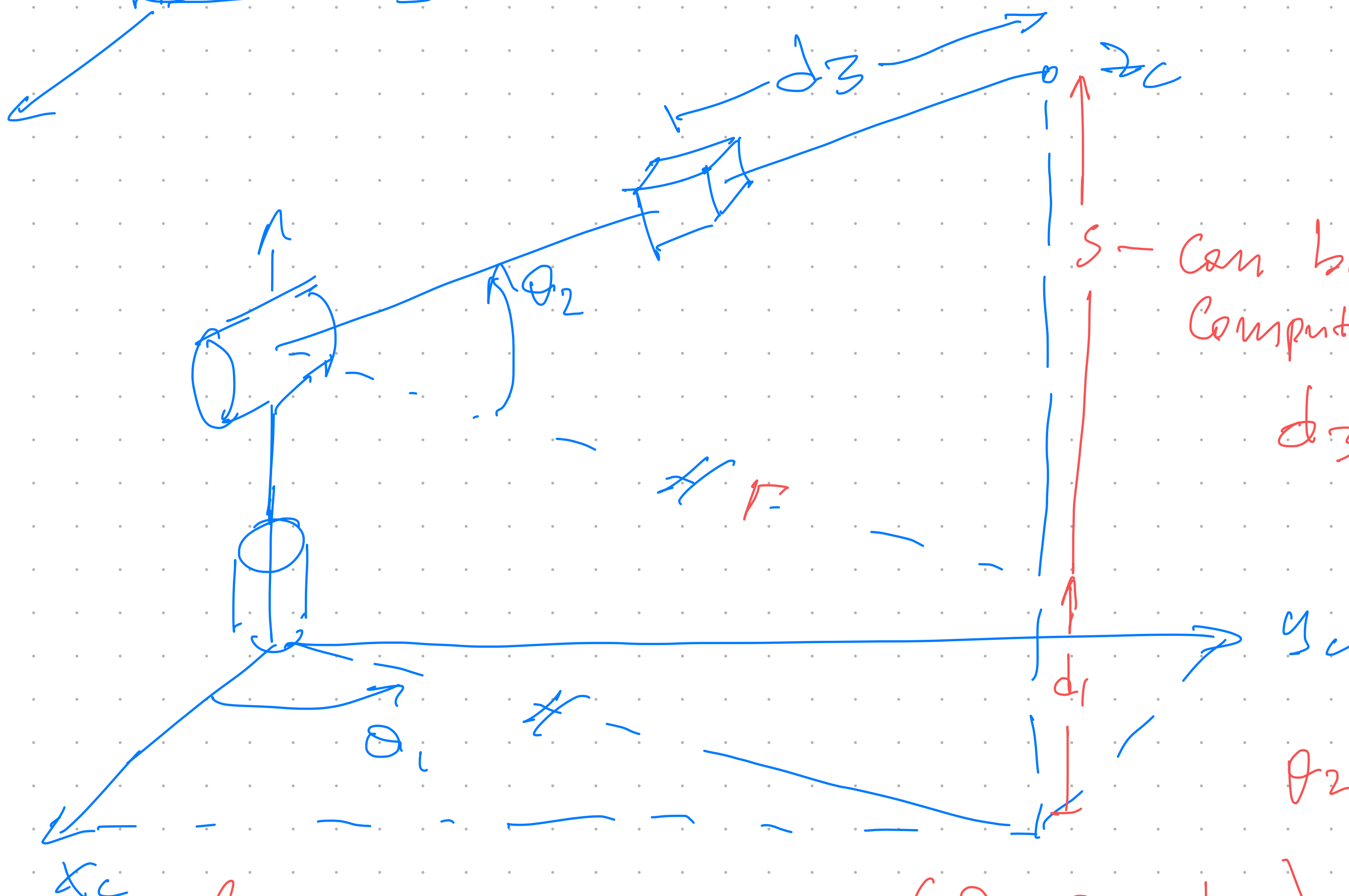
$$r = \sqrt{x_c^2 + y_c^2}$$



What if $x_c = y_c = 0$ $\exists$ singularity

Any θ_1 - value is ok

θ_1, θ_2, d_3



s - can be computed

$$r = \sqrt{x_c^2 + y_c^2}$$

$$s = z_c - d_1$$

given

$$d_3 = \sqrt{r^2 + s^2} = \sqrt{x_c^2 + y_c^2 + (z_c - d_1)^2}$$

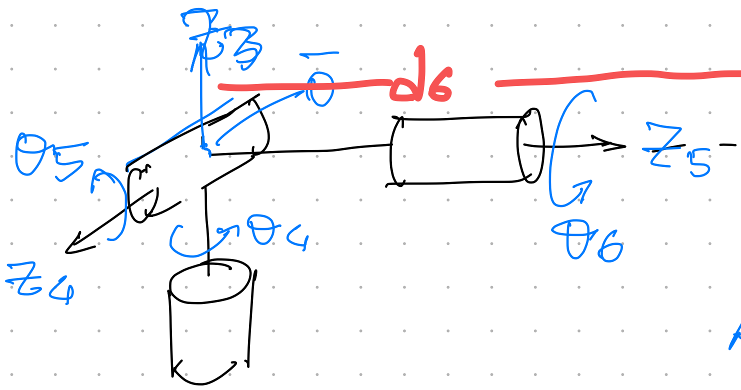
$$\theta_2 = \text{atan2}(r, s)$$

$$(x_c, y_c, z_c) \longrightarrow (\theta_1, \theta_2, d_3)$$

Spherical wrist (axes z_3, z_4, z_5 intersect @ pt. $\bar{o}$)

ϕ, θ, ψ roll, pitch, yaw

$\theta_4, \theta_5, \theta_6$



$S5 = \sin(\theta_5)$

$C5 = \cos(\theta_5)$

$$A_3^4 = \left[\begin{array}{ccc|c} C4 & 0 & -S4 & 0 \\ S4 & 0 & C4 & 0 \\ 0 & -1 & 0 & 0 \\ \hline & & & 1 \end{array}\right]$$

$$A_4^5 = \left[\begin{array}{ccc|c} C5 & & S5 & 0 \\ S5 & 0 & -C5 & 0 \\ & -1 & 0 & 0 \\ \hline & & & 1 \end{array}\right]$$

$$T_3^6 = \left[\begin{array}{ccc|c} t_{11} & t_{12} & t_{13} & \\ t_{21} & t_{22} & t_{23} & \\ t_{31} & t_{32} & t_{33} & \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$A_5^6 = \left[\begin{array}{ccc|c} C6 & -S6 & 0 & 0 \\ S6 & C6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ \hline & & & 1 \end{array}\right]$$

$$A_3^6 = \left[\begin{array}{c|c|c|c} C4\,C5\,C6 - S4\,S6 & -C4\,C5\,S6 - S4\,C6 & C4\,S5 & d6\,C4\,S5 \\ S4\,C5\,C6 + C4\,S6 & -S4\,C5\,S6 + C4\,C6 & S4\,S5 & d6\,S4\,S5 \\ -S5\,C6 & S5\,S6 & C5 & d6\,C5 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\theta_5 = \cos^{-1}(T_{33})$$

$$d6\sqrt{(C4\,S5)^2 + (S4\,S5)^2 + (C5)^2} = d6$$

$\theta_5 = \text{atan2}\left(T_{33}, \sqrt{1-T_{33}^2}\right)$ $\theta_5 = \text{atan2}\left(T_{33}, -\sqrt{1-T_{33}^2}\right)$

Let $\sin(\theta_5) > \emptyset$

$\theta_4 = \text{atan2}(T_{13}, T_{23})$

$\theta_6 = \text{atan2}(-T_{31}, T_{32})$

if $\sin(\theta_5) < \emptyset$

$\theta_4 = \text{atan2}(-T_{13}, -T_{23})$

$\theta_6 = \text{atan2}(T_{31}, -T_{32})$

$\cos(\theta_4 + \theta_6) =$

$\cos\theta_4 \cos\theta_6 -$

$\sin\theta_4 \sin\theta_6$

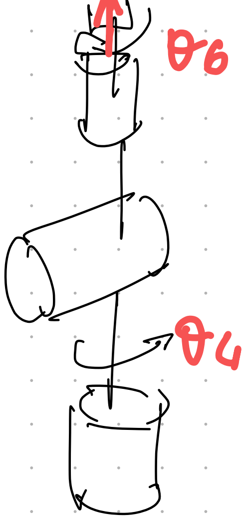
If $T_{33} = +1$ $\boxed{\theta_5 = 0^\circ}$

$$\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ 0 & 0 & \pm 1 \end{bmatrix} = \begin{bmatrix} C4C6 - S4S6 & -C4S6 - S4C6 & 0 \\ S4S6 + C4S6 & -S4S6 + C6C6 & 0 \\ 0 & 0 & +1 \end{bmatrix}$$

$$= \begin{bmatrix} C4{+}6 & -S4{+}6 & 0 \\ S4{+}6 & C4{+}6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

θ_4, θ_6

$\theta_4 + \theta_6 = \text{atan2}(t_{11}, t_{21})$



$\theta_4 + \theta_6$ - Can be computed

θ_4, θ_6 Cannot be computed

$\exists$ infinite solutions

If $T_{33} = -1$ $\quad \theta_5 = -90^\circ$

$$\theta_4 - \theta_6 = \text{atan2}(-T_{11}, -T_{12})$$