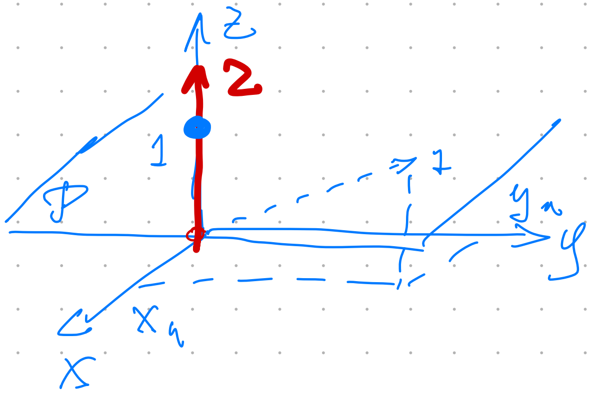
VECTOR $\bar{V} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ PLANE $\mathcal{P} = [a, b, c, -1]$

$[a, b, c, d]$ $m = \sqrt{a^2 + b^2 + c^2}$

Point $\bar{V} \in \mathcal{P}$ if $\mathcal{P} \cdot \bar{V} = \emptyset$ $ax + by + cz = 1$

$= -\frac{d}{m}$



$\mathcal{P} = [0, 0, 1, -1]$

PLANE // $(x-y$ PLANE & intersecting z-axis @ 1

any $\bar{V} = \begin{bmatrix} x_u \\ y_u \\ 1 \\ 1 \end{bmatrix}$ $\forall\, x_u\, y_u$ ⟶ is in $\mathcal{P}$

i.e $\bar{V} = \begin{bmatrix} 10 \\ 20 \\ 1 \\ 1 \end{bmatrix}$ $\mathcal{P} \cdot V = \emptyset$

Point $\bar{V} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ lies above the plane $\mathcal{P} \cdot V = 1$

while $\bar{V} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ lies below the plane $\mathcal{P} V = -1$

$$\bar{V} = \mathbb{H}_{4\times4}\, \bar{u}_{4\times1}$$

$$H = \mathrm{Trans}(a,b,c) = \left[\begin{array}{ccc|c} 1 & & & a \\ & 1 & & b \\ & & 1 & c \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$V = \mathrm{Trans}(a,b,c)\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ z+c \\ 1 \end{bmatrix}$$

$$\mathrm{Rot}(x,\theta) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & C\theta & -S\theta & 0 \\ 0 & S\theta & C\theta & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Rot}(y,\theta) = \left[\begin{array}{ccc|c} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Rot}(z,\theta) = \left[\begin{array}{ccc|c} C\theta & -S\theta & 0 & 0 \\ S\theta & C\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline & & & 1 \end{array}\right]$$

$\bar{u} = \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ rotate $\bar{u}$ about the z-axis to $\bar{V}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 90°$

$$\underbrace{\begin{bmatrix} -3 \\ 7 \\ 2 \\ 1 \end{bmatrix}}_{\bar{V}} = \left[\begin{array}{ccc|c} 0 & -1 & 0 & \\ 1 & 0 & 0 & \\ 0 & 0 & 1 & \\ \hline & & & 1 \end{array}\right] \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\bar{V} = Rot(z, 90°)\,\bar{u}$$

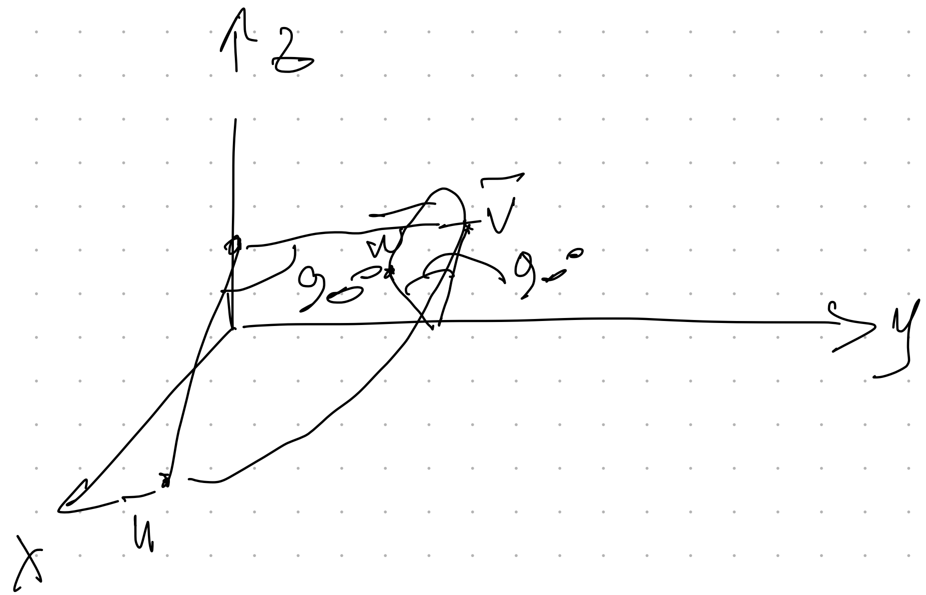
$$\bar{W} = Rot(y, 90°)\,\bar{V}$$

$$\bar{W} = \begin{bmatrix} 2 \\ 7 \\ 3 \\ 1 \end{bmatrix}$$

$$\bar{W} = \underbrace{Rot(y, 90°)}\,\underbrace{Rot(z, 90°)}\,\bar{u}$$

look @ Fig. 1.2

$$Rot(z, 90°)\,Rot(y, 90°) \neq$$



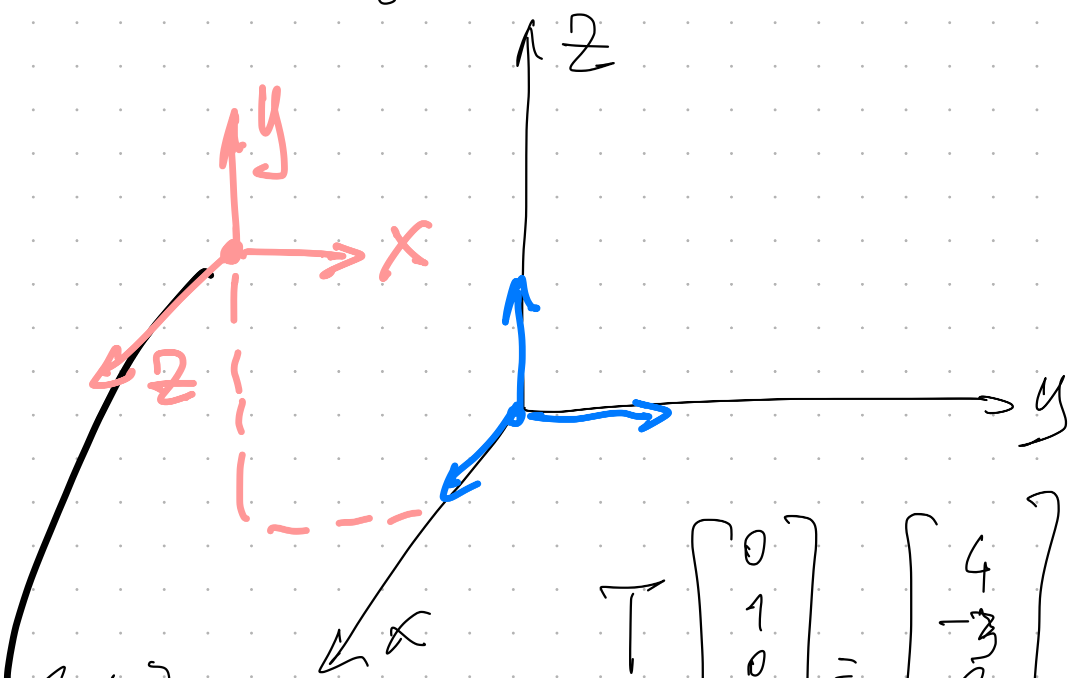
$$\mathrm{Trans}(4,-3,7)\;\mathrm{Rot}(y,90)\;\mathrm{Rot}(z,90) = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T$$

Coordinate Frames

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Unit axis vector along x-axis $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ z-axis $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

y-axis $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$



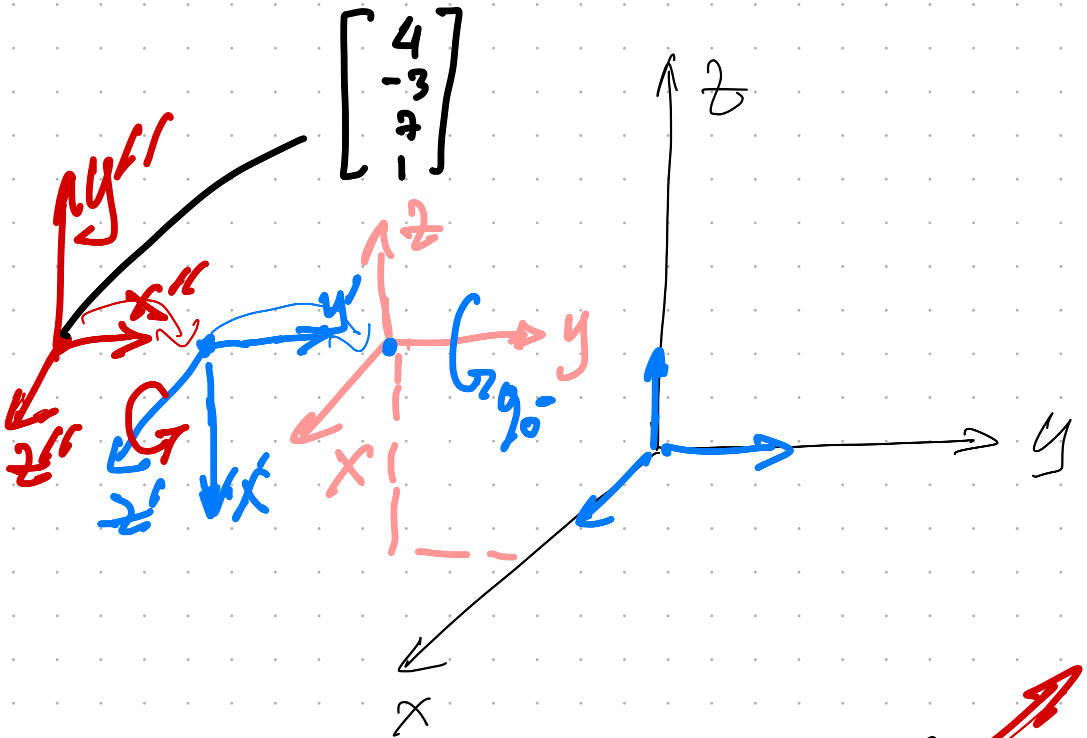
$$\begin{bmatrix} 4 \\ -3 \\ 7 \\ 1 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 7 \\ 1 \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 7 \\ 1 \end{bmatrix}$$

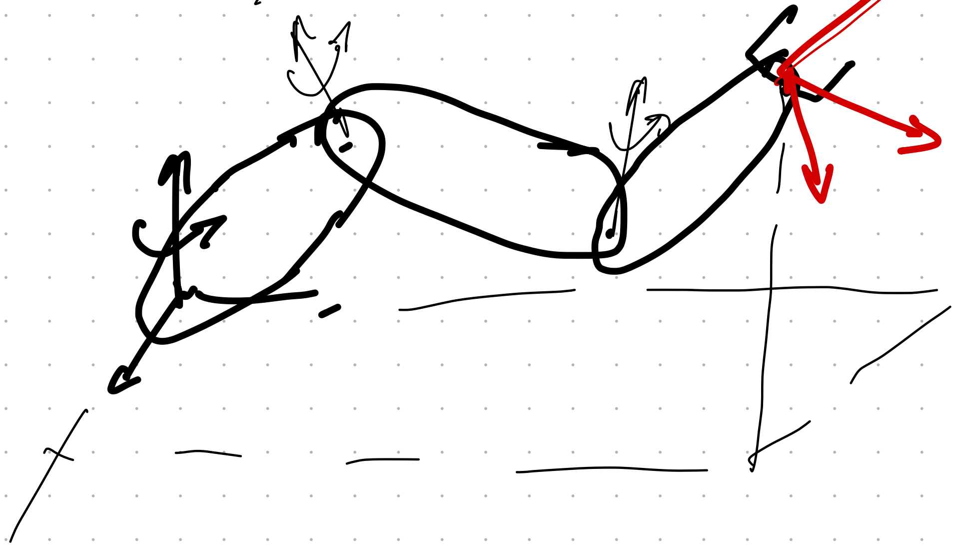
$$T\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 8 \\ 1 \end{bmatrix}$$

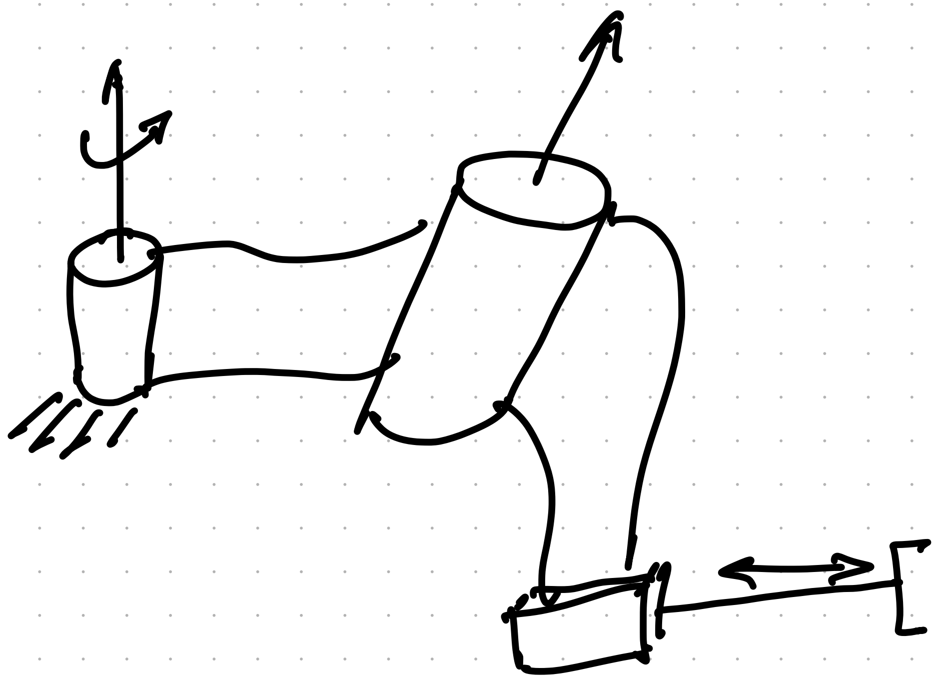
$$T\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 7 \\ 1 \end{bmatrix}$$



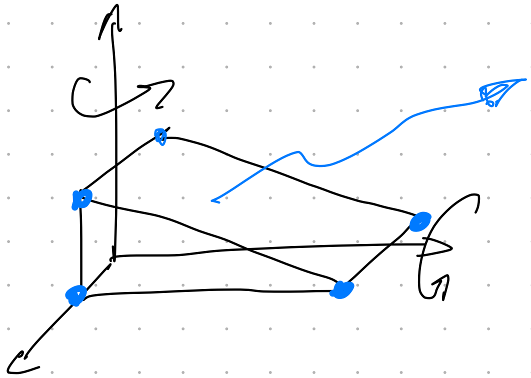
$Trans(4, -3, 7)\ Rot(y, 90°)$.

$Rot(z, 90°)$





$$\cos x = 1 + \frac{x^2}{2!} + \dots$$

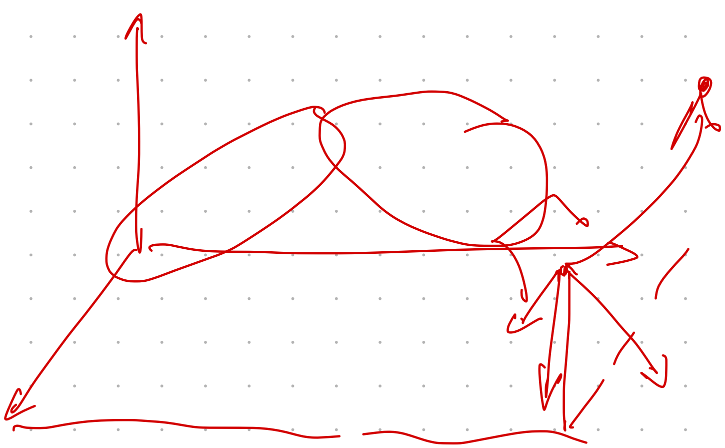


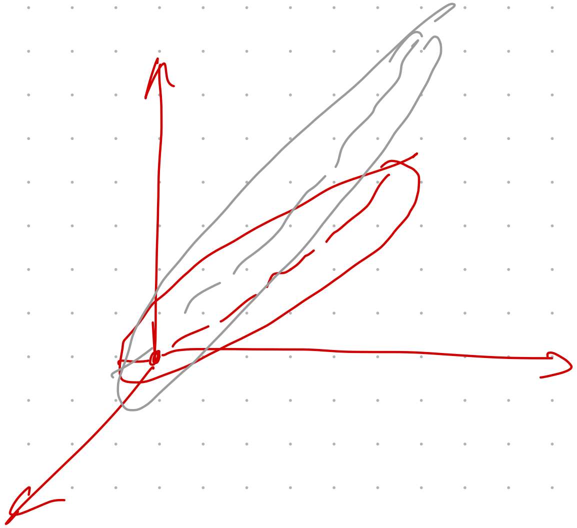
$$T = \left[\begin{array}{ccc|c} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} R & \bar{p} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$$R^{-1} = R^{T}$$

$$\det(R) = \pm 1$$

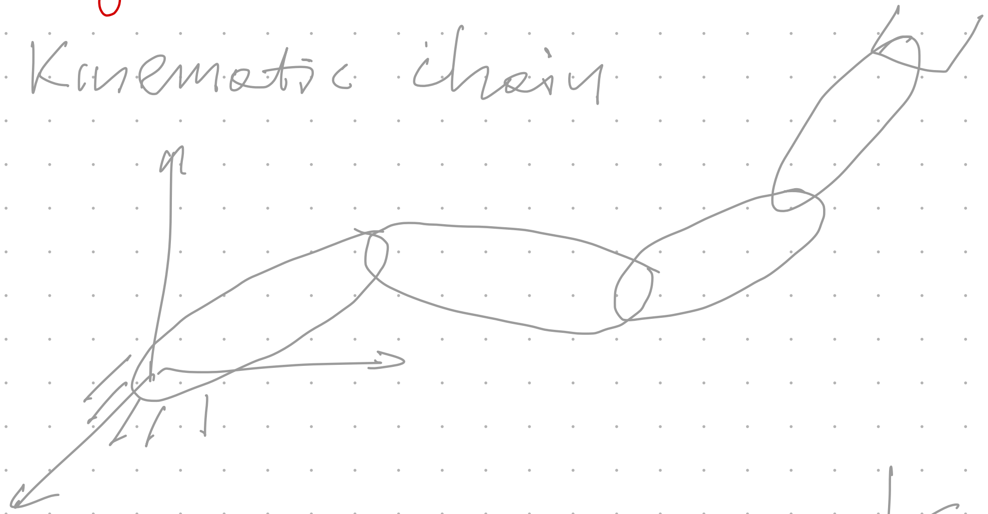
$$T^{-1} = \left[\begin{array}{c|c} R^{T} & \begin{array}{c} -\bar{p}\cdot\bar{n} \\ -\bar{p}\cdot\bar{o} \\ -\bar{p}\cdot\bar{a} \end{array} \\ \hline 0\quad 0\quad 0 & 1 \end{array}\right]$$



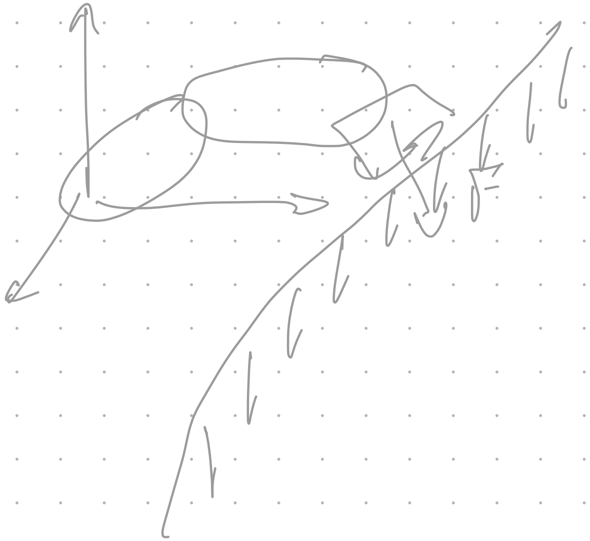


✓ Rigid ≠ flexible

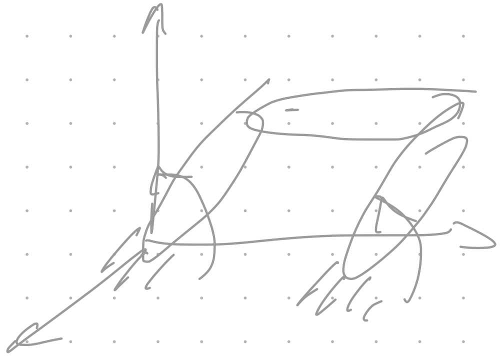
Kinematic chain

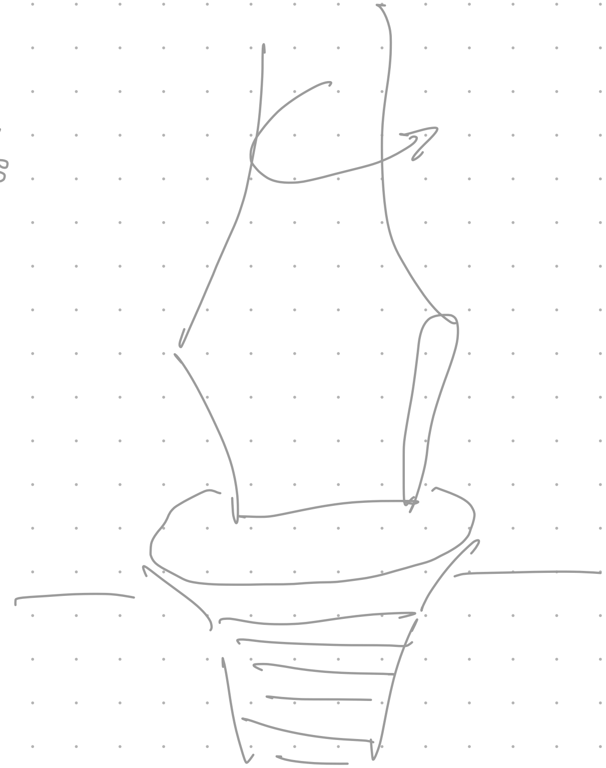


Does not exert force

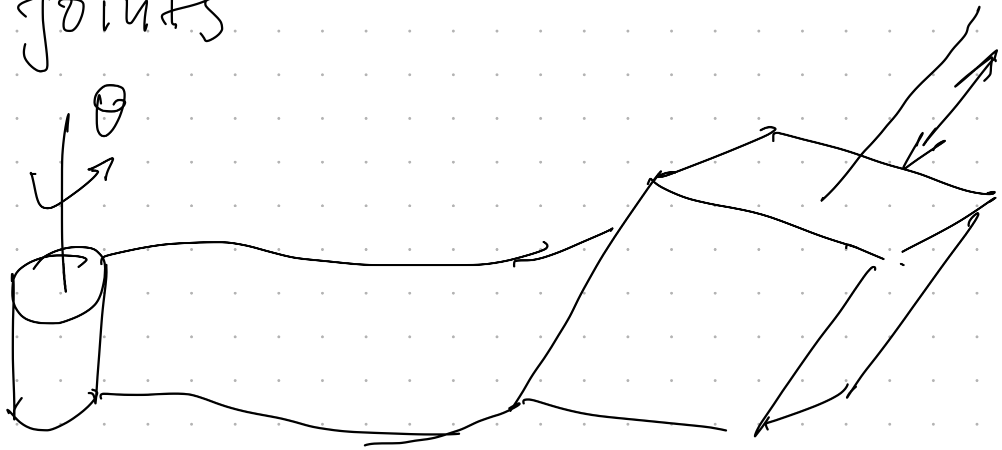


open ≠ closed





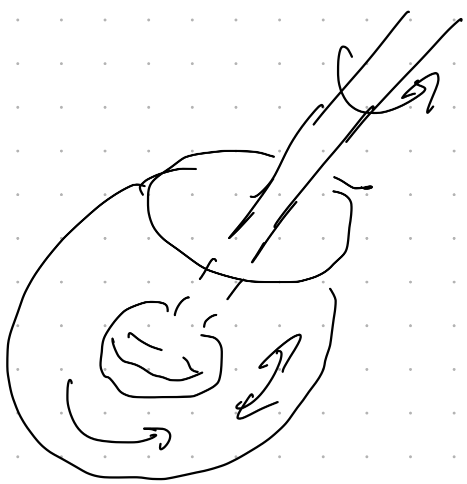
revolute joints

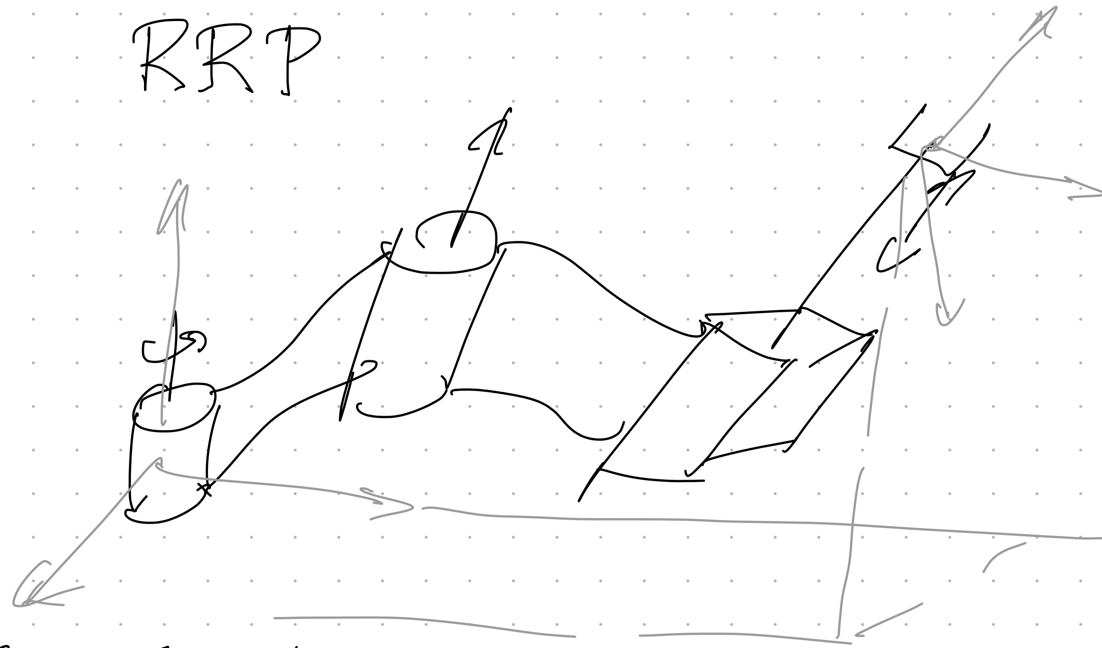


prismatic joints

~~spherical joints~~

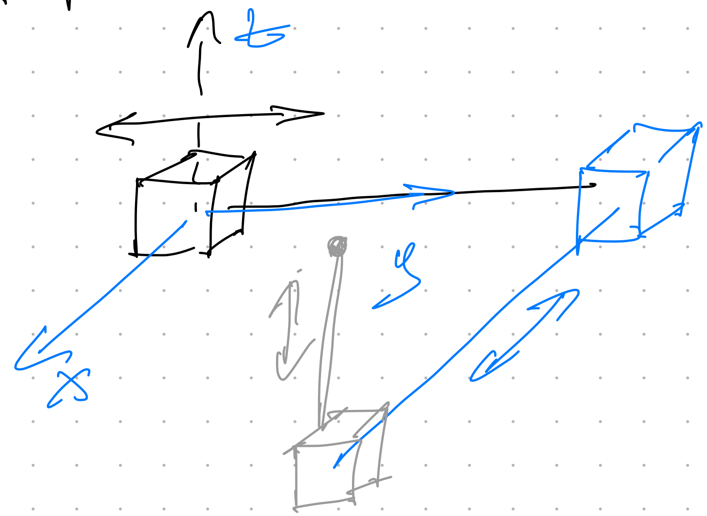
RRP

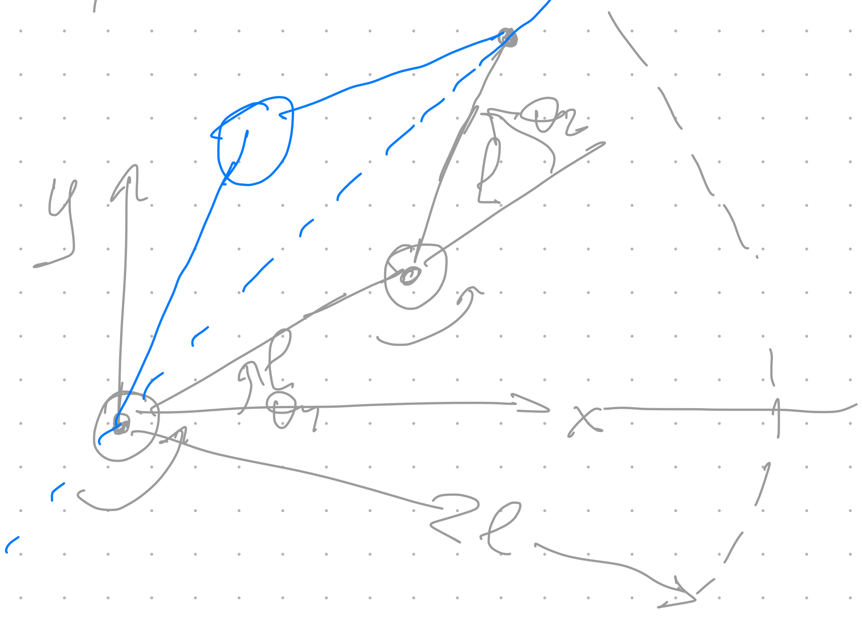


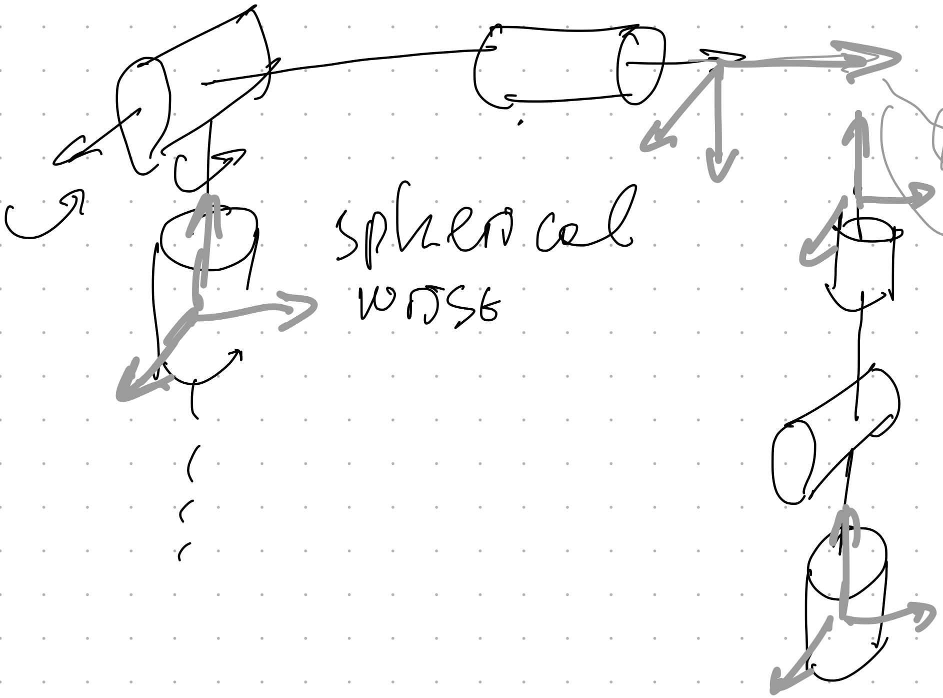


6 DoF Degree of Freedom

PPP

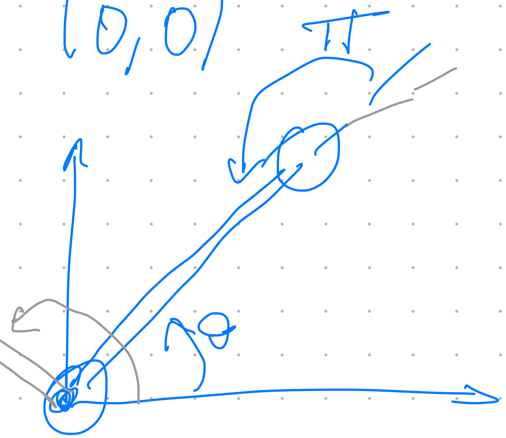




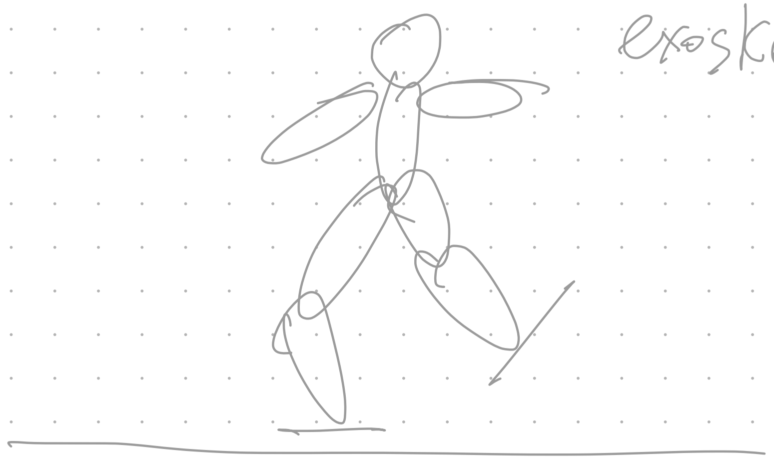


spherical wrist

(0,0)



singularities



exoskeleton

Relative Transformations