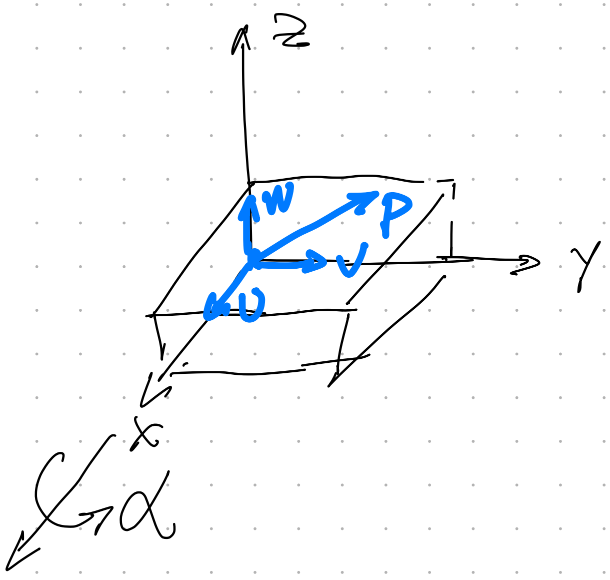
ROBOT KINEMATICS
(ROTATION MATRICES)



$$P_{uvw} = (p_u, p_v, p_w)^T$$

$$P_{xyz} = (p_x, p_y, p_z)^T$$

$$P_{xyz} = R_{3\times3} \, P_{uvw}$$

$$P_{uvw} = p_u \, \bar{i}_u + p_v \, \bar{j}_v + p_w \, \bar{k}_w$$

$$p_x = \bar{i}_x \cdot \bar{P} = \bar{i}_x \cdot \bar{i}_u \, p_u + \bar{i}_x \cdot \bar{j}_v \, p_v + \bar{i}_x \cdot \bar{k}_w \, p_w$$

$$\vdots$$

$$p_y$$

$$p_z$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \bar{i}_x \cdot \bar{i}_u & \bar{i}_x \cdot \bar{j}_v & \bar{i}_x \cdot \bar{k}_w \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \bar{k}_z \cdot \bar{k}_w \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

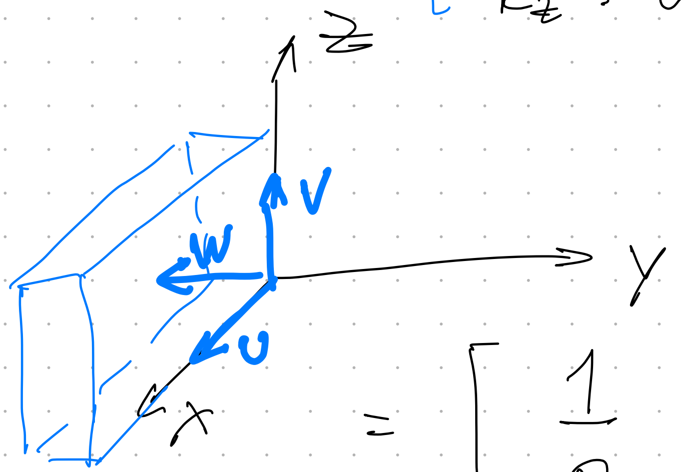
$$= R \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

$$Q = R^{-1} = R^T$$

orthonormal

$$R^T R = R R^T = I$$

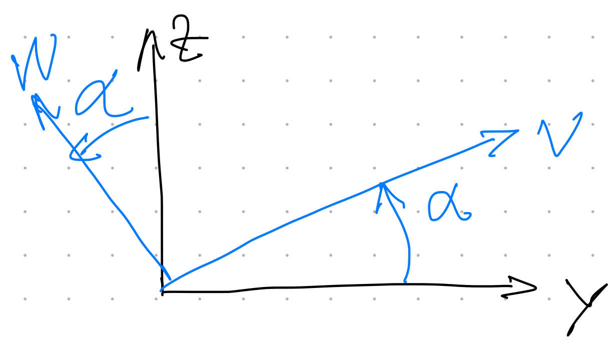
$$\mathbb{R}_{x,\alpha} = \begin{bmatrix} \bar{i}_x \cdot \bar{i}_u & \bar{i}_x \cdot \bar{j}_v & \bar{i}_x \cdot \bar{k}_w \\ \bar{j}_y \cdot \bar{i}_u & \bar{j}_y \cdot \bar{j}_v & \bar{j}_y \cdot \bar{k}_w \\ \bar{k}_z \cdot \bar{i}_u & \bar{k}_z \cdot \bar{j}_v & \bar{k}_z \cdot \bar{k}_w \end{bmatrix}$$



$$\bar{i}_x = \bar{i}_u \Rightarrow \bar{i}_x \cdot \bar{i}_y =$$

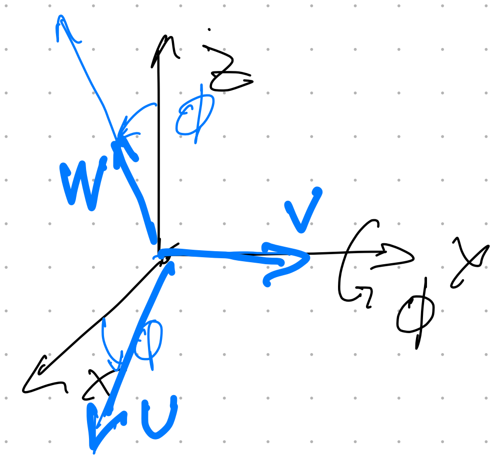
$$\|\bar{i}_x\| = 1$$

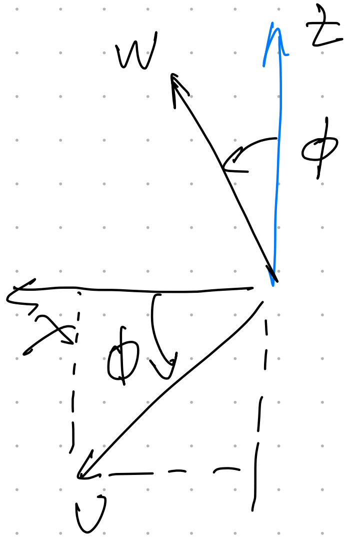
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$



$$\sqrt{0^2 + \cos^2\alpha + \sin^2\alpha} = 1$$

$$\mathbb{R}_{y,\phi} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

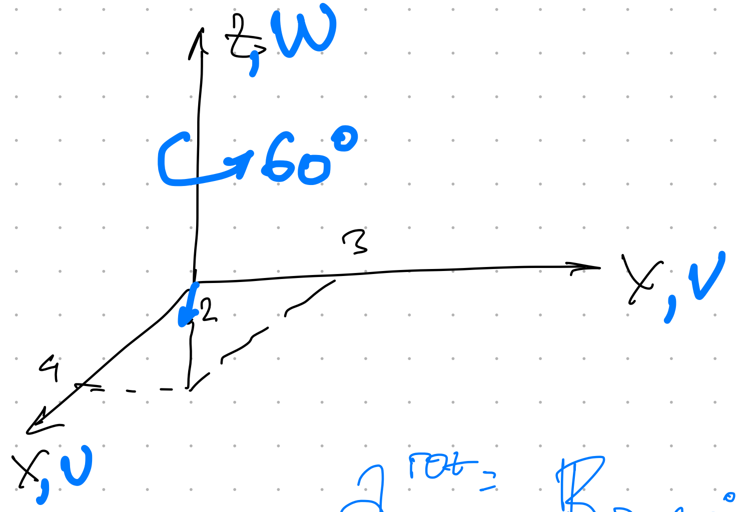


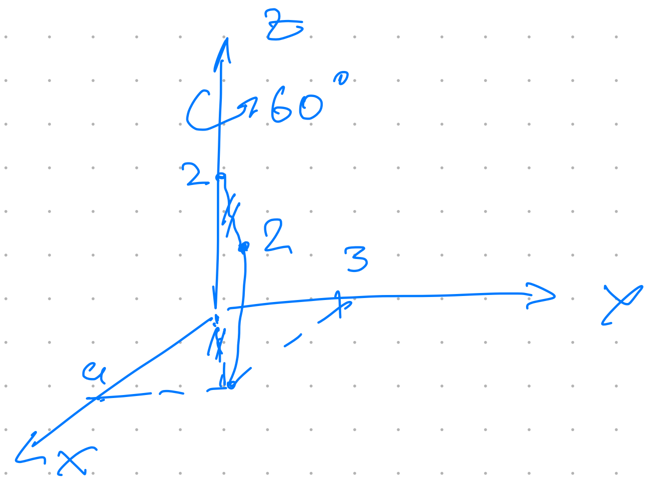


$$\mathbb{R}_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_{xyz} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \qquad \|a\|_{xyz} = \sqrt{4^2 + 3^2 + 2^2}$$

Compute a_{xyz} such that it has rotated 60° about Oz-axis





$$a^{rot}_{uvw} = \mathbb{R}_{z,60^\circ} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.598 \\ 4.964 \\ 2 \end{bmatrix} \qquad \|a\|_{uvw} = \sqrt{(-0.598)^2 + (4.964)^2 + 2^2}$$

Composite Rotation Matrix

$$R = R_{y,\phi} \cdot R_{z,\theta} \cdot R_{x,\alpha}$$

1) rotation around OX-axis α

2) rotation OZ-axis θ

3) OY-axis ϕ

$\underbrace{\qquad\qquad}_{\text{inertial axes}}$

$$= \begin{bmatrix} C\phi & 0 & S\phi \\ 0 & 1 & 0 \\ -S\phi & 0 & C\phi \end{bmatrix} \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix}$$

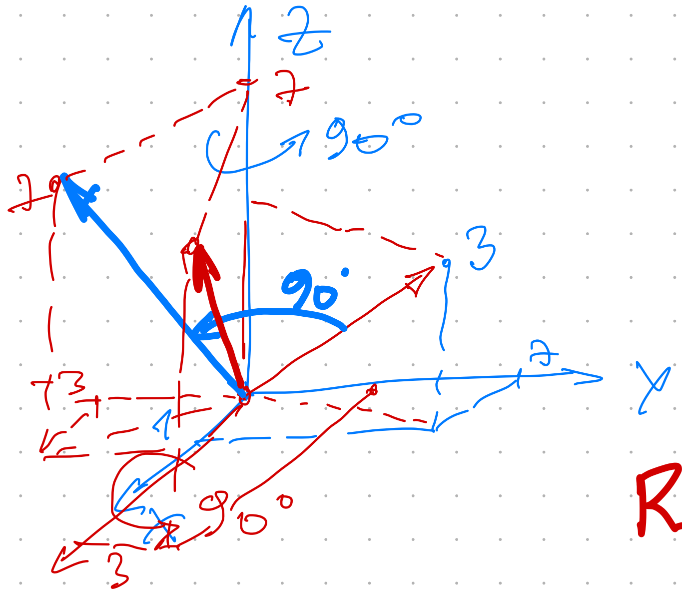
$$= \begin{bmatrix} C\phi\, C\theta & S\phi\, S\alpha - C\phi\, S\theta\, C\alpha & C\phi\, S\theta\, S\alpha + S\phi\, C\alpha \\ S\theta & C\theta\, C\alpha & -C\theta\, S\alpha \\ -S\phi\, C\theta & S\phi\, S\theta\, C\alpha + C\phi\, S\alpha & C\phi\, C\alpha - S\phi\, S\theta\, S\alpha \end{bmatrix}$$

$S\alpha = \sin(\alpha)$

$C\alpha = \cos(\alpha)$

$\mathbb{R}^* = \mathbb{R}_{x,\alpha}\ \mathbb{R}_{z,\theta}\ \mathbb{R}_{y,\theta}$

$\longrightarrow$

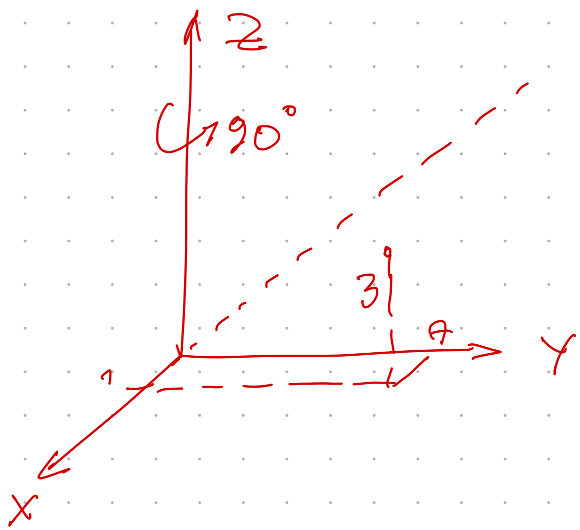


$$Rot_{x,90°}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}1\\-3\\2\end{bmatrix}$$

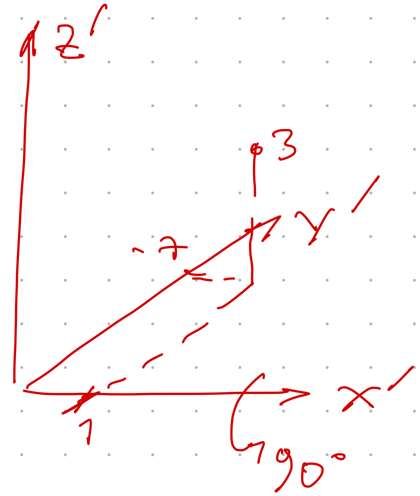
$$Rot_{z,90°}\begin{bmatrix}1\\-3\\2\end{bmatrix}=\begin{bmatrix}3\\1\\2\end{bmatrix}$$

$$Rot_{z,90°}\ Rot_{x,90°}$$

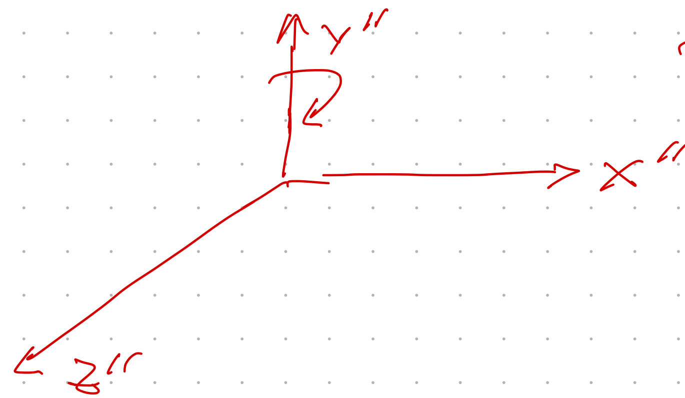
$\longleftarrow$



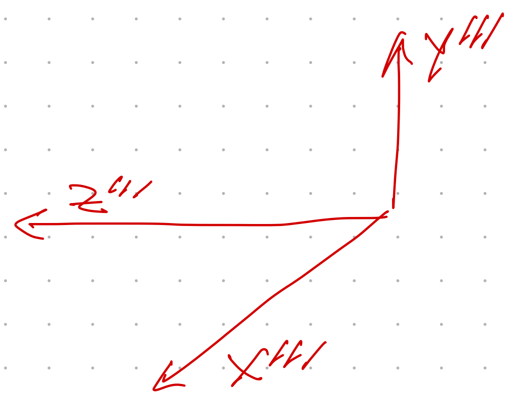
Rot(z, 90°)



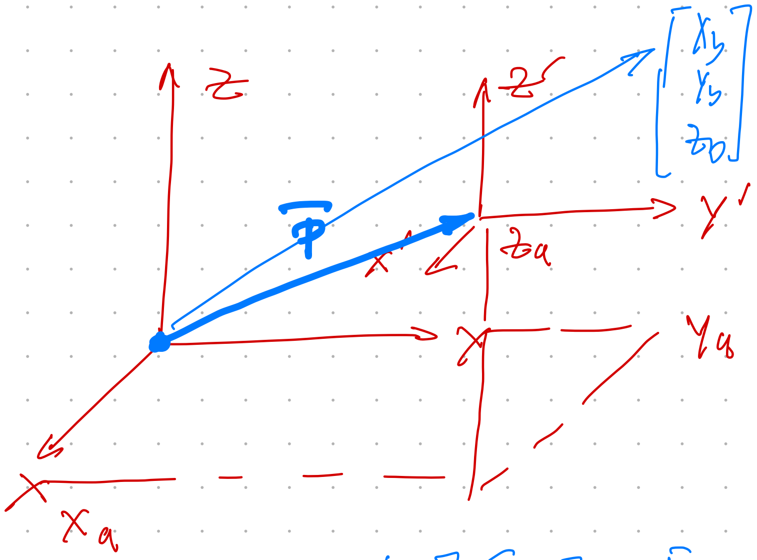
Rot (x, 90°)



$$\mathrm{Rot}(z, 90^\circ)\,\mathrm{Rot}(x, 90^\circ)\begin{bmatrix}1\\7\\3\end{bmatrix}$$



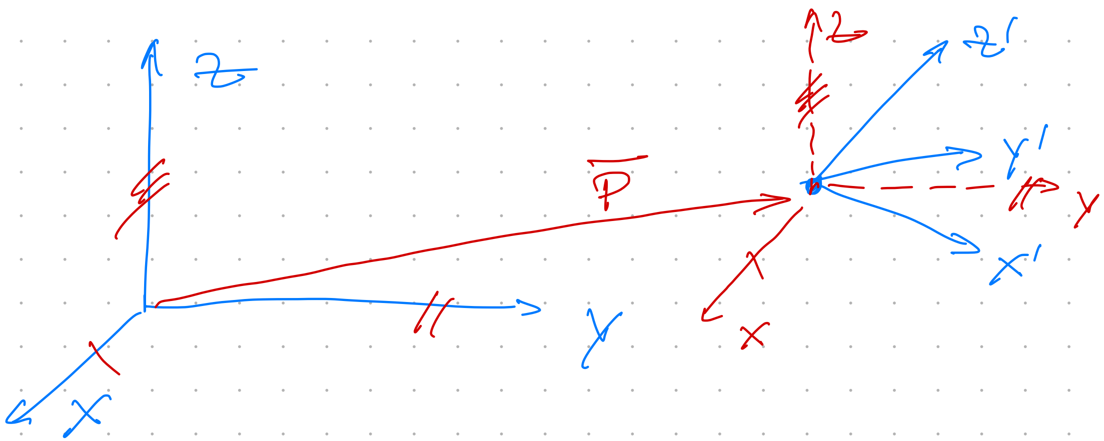
Rot (y, -90°)



$$T = \left[\begin{array}{c|c} \underbrace{R = \mathbb{I}}_{3\times 3} & \vec{p} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]_{4\times 4}$$

$$\bar{p} = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}_{3\times 1} \qquad \bar{p} \triangleq \begin{bmatrix} x_a \\ y_a \\ z_a \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & & & x_a \\ & 1 & & y_a \\ & & 1 & z_a \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x_b \\ y_b \\ z_b \\ 1 \end{bmatrix} = \begin{bmatrix} x_a + x_b \\ y_a + y_b \\ z_a + z_b \\ 1 \end{bmatrix}$$



$$T = \left[\begin{array}{c|c} R & \bar{p} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$