Inverse Simulation

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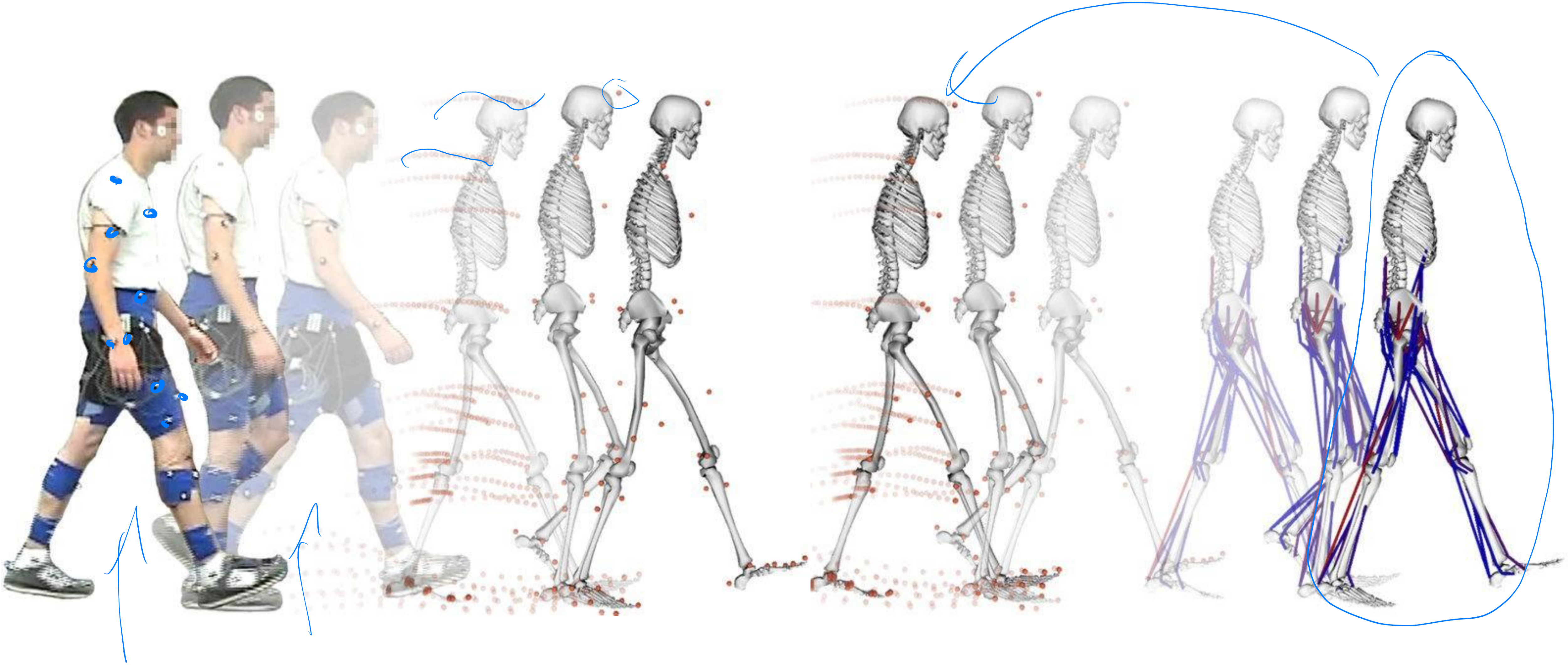
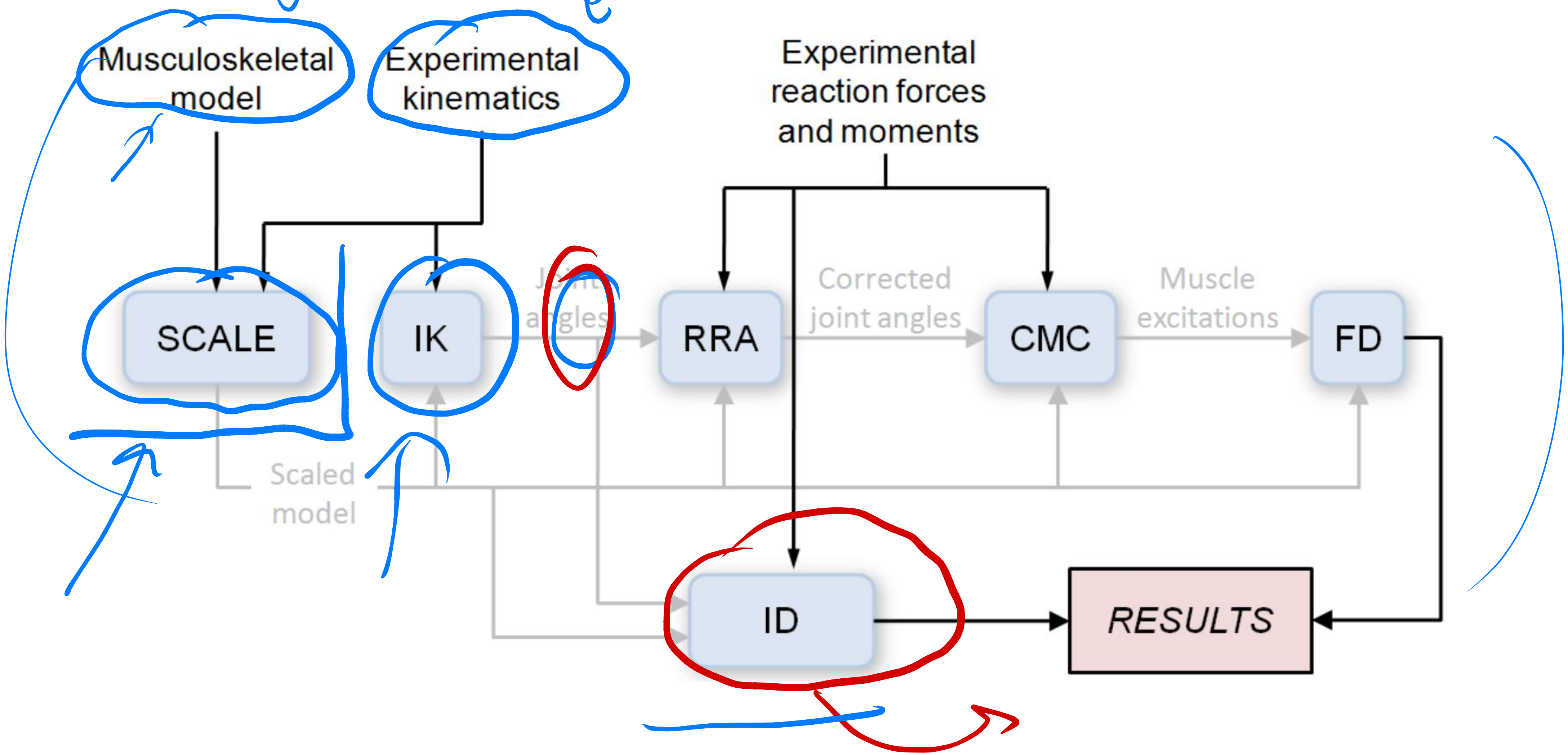
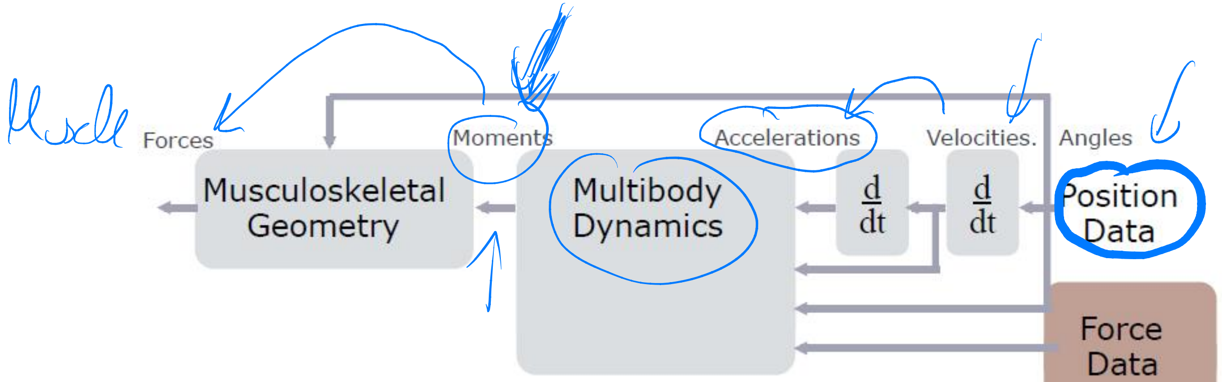


Inverse Simulation



The Inverse Problem



Muscle

Forces

Moments

Accelerations

Velocities.

Angles

Musculoskeletal Geometry

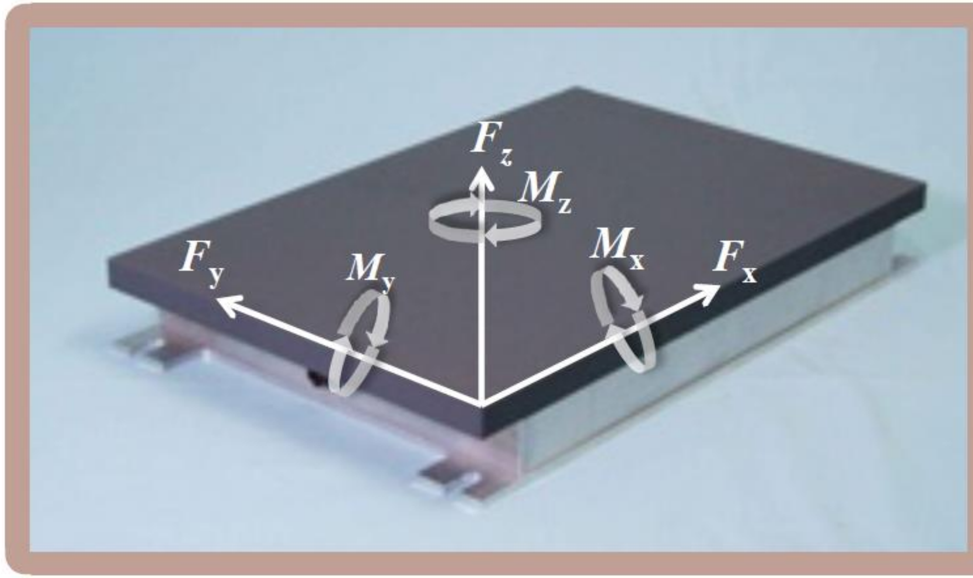
Multibody Dynamics

$\frac{d}{dt}$

$\frac{d}{dt}$

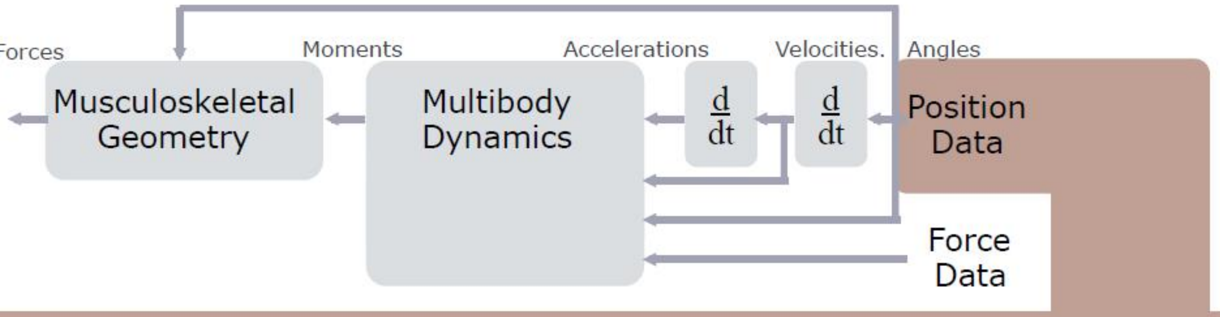
Position Data

Force Data

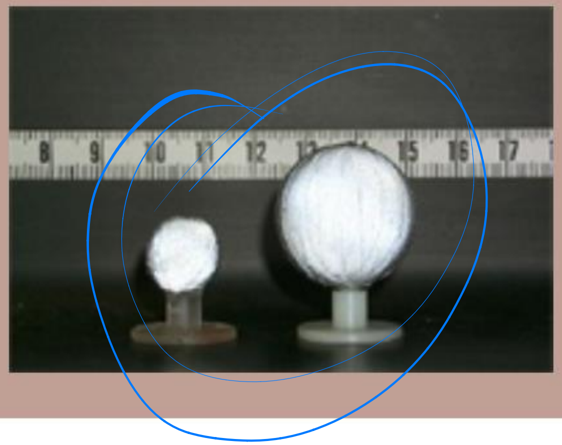


3 Forces
3 Moments

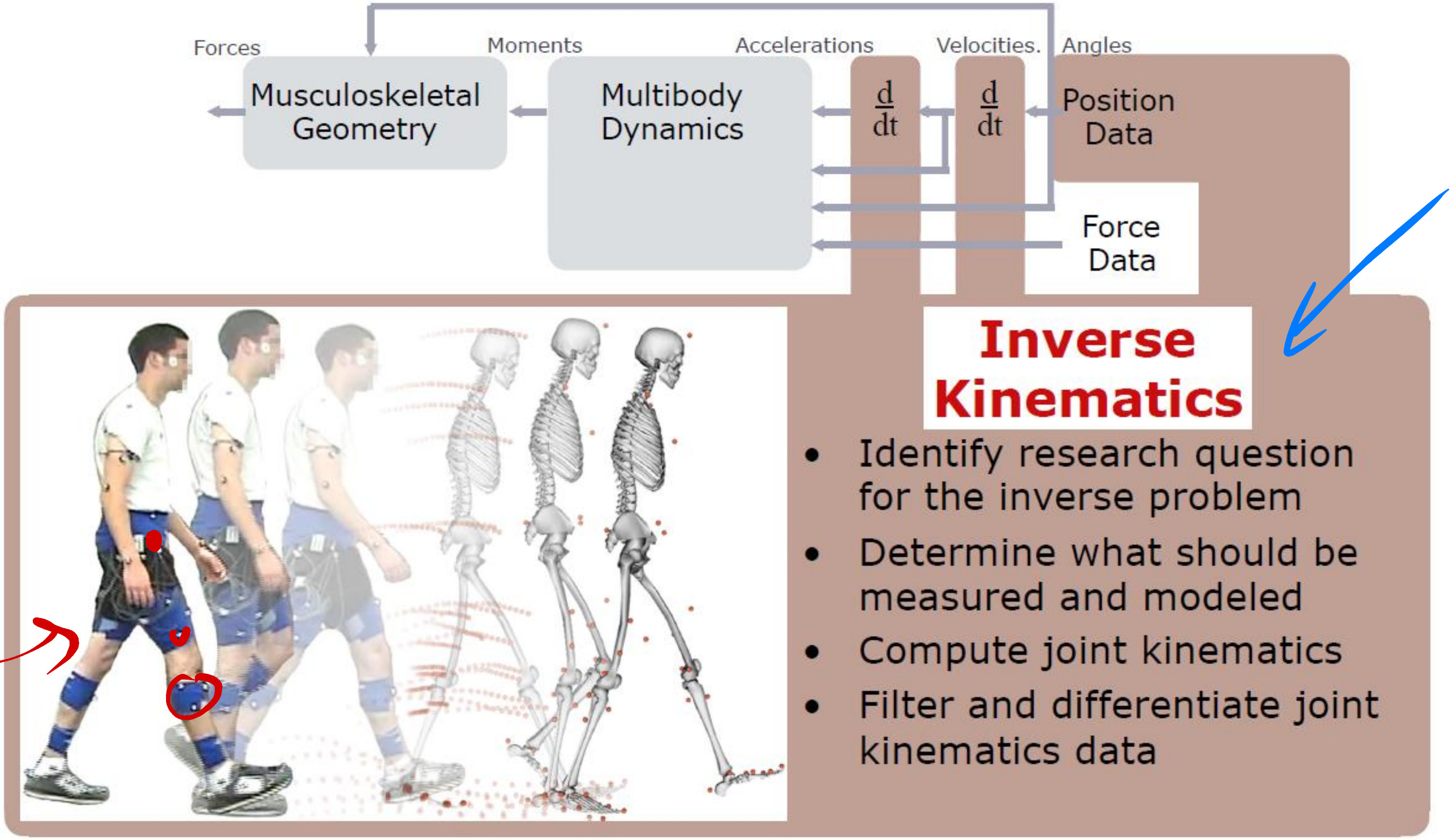
The Inverse Problem



Video Cameras
Reflective Markers



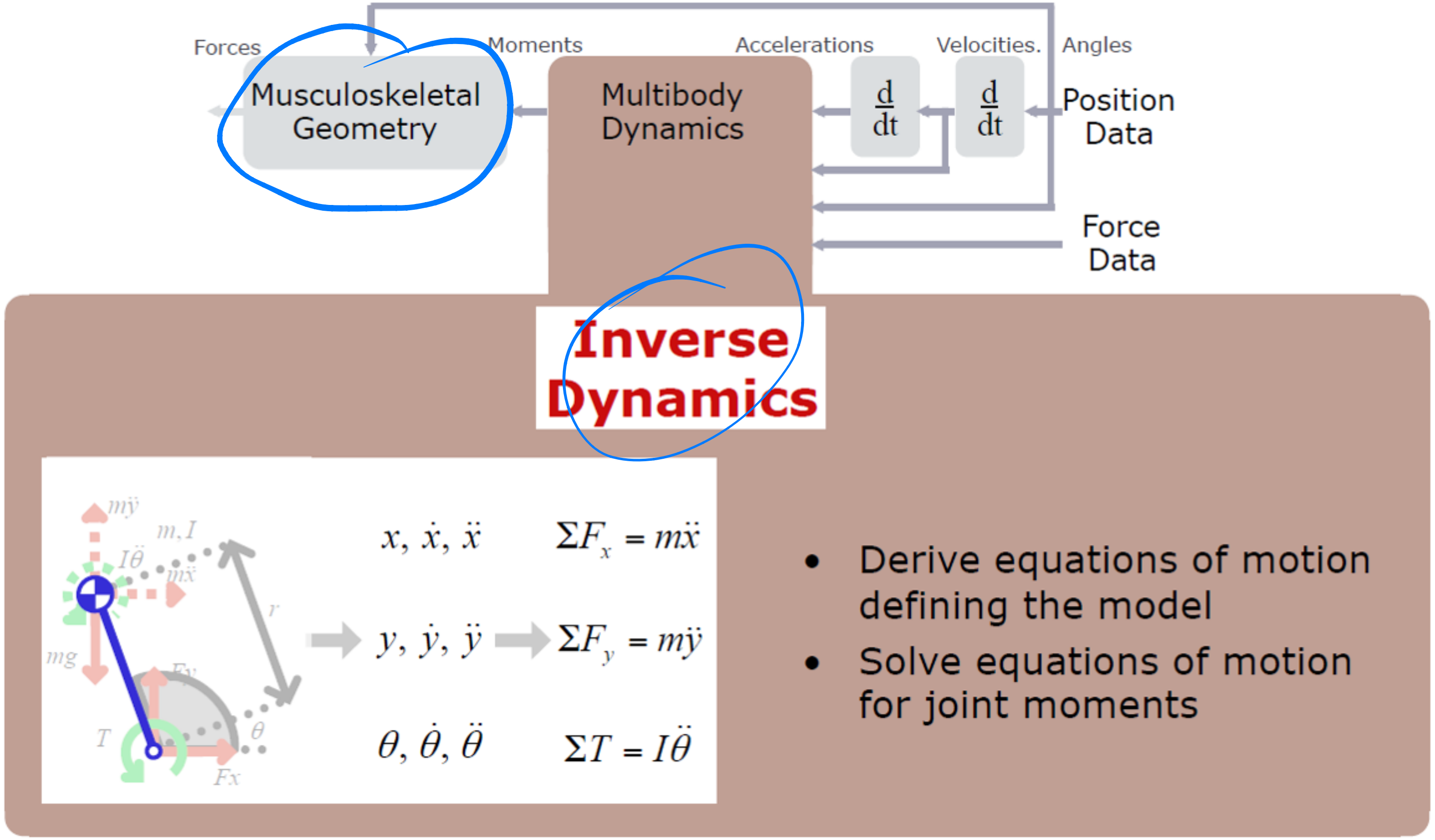
The Inverse Problem



Inverse Kinematics

- Identify research question for the inverse problem
- Determine what should be measured and modeled
- Compute joint kinematics
- Filter and differentiate joint kinematics data

The Inverse Problem



Inverse Dynamics

$x, \dot{x}, \ddot{x}$ $\quad \Sigma F_x = m\ddot{x}$

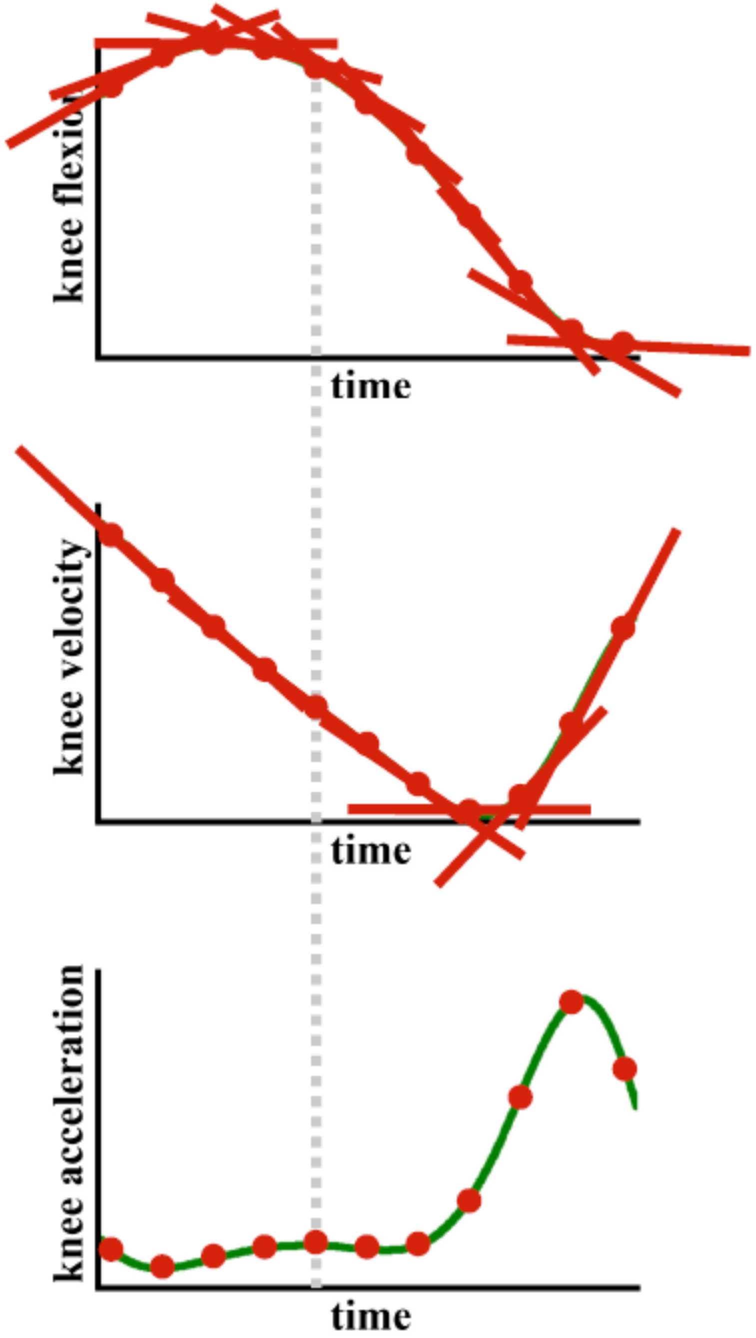
$y, \dot{y}, \ddot{y}$ $\quad \Sigma F_y = m\ddot{y}$

$\theta, \dot{\theta}, \ddot{\theta}$ $\quad \Sigma T = I\ddot{\theta}$

- Derive equations of motion defining the model
- Solve equations of motion for joint moments

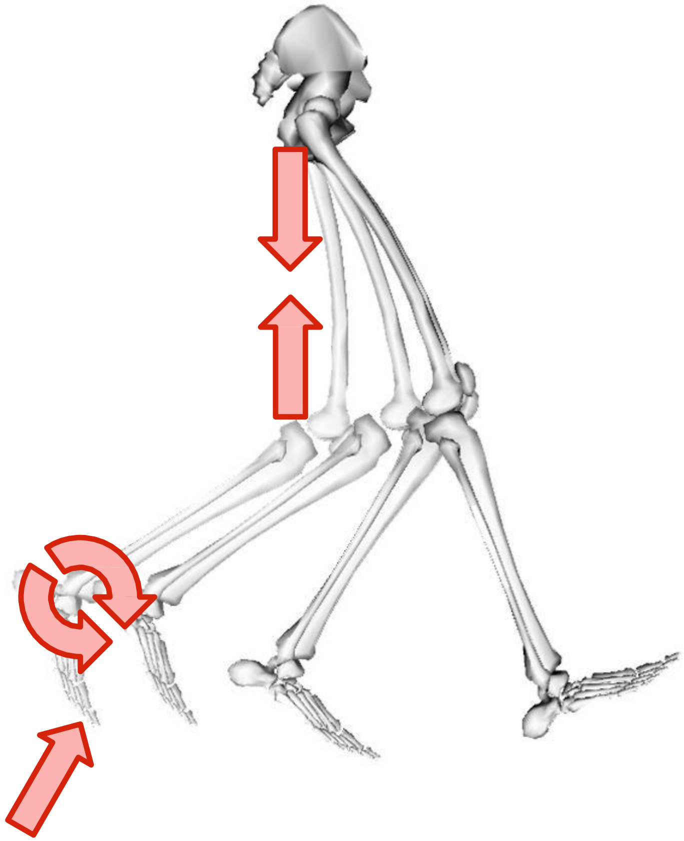
Kinematics

- Coordinate
  - Joint angle or distance specifying relative orientation or location of two body segments
- Coordinate velocity
  - Derivative (rate of change) of a coordinate with respect to time
- Coordinate acceleration
  - Time derivative of a coordinate velocity with respect to time
- Kinematics
  - Set of all coordinates and their velocities and accelerations



Kinetics

- Kinetics
  - Forces and torques cause the model to accelerate
- Force
  - Applied to points (e.g., ground reactions) or between points (e.g., muscles)
- Torque
  - Applied to a coordinate (e.g., joint torque)



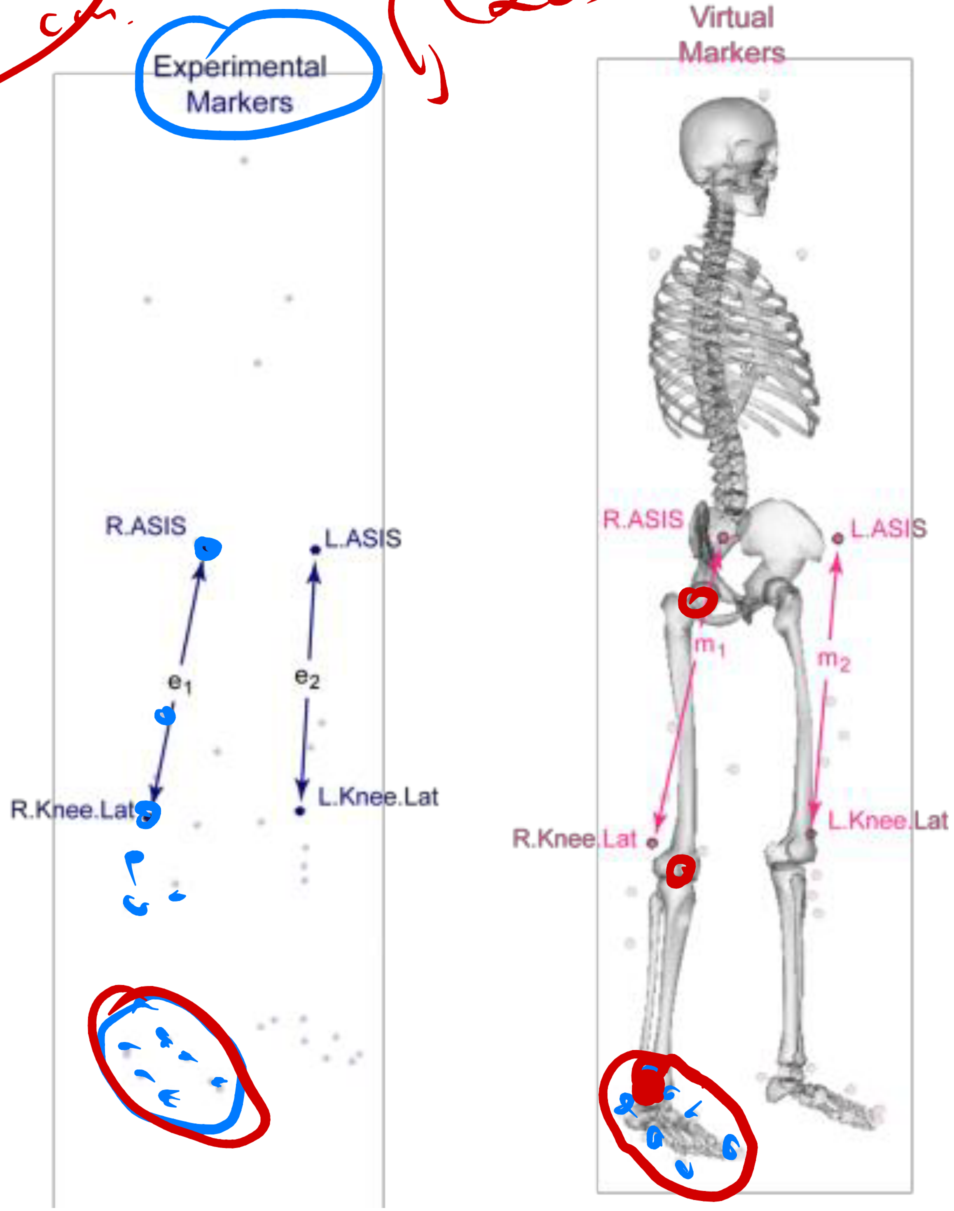
Scaling

$$\alpha = \frac{S_r}{S_s}$$

$$\frac{200\ \text{cm}}{178\ \text{cm}}$$

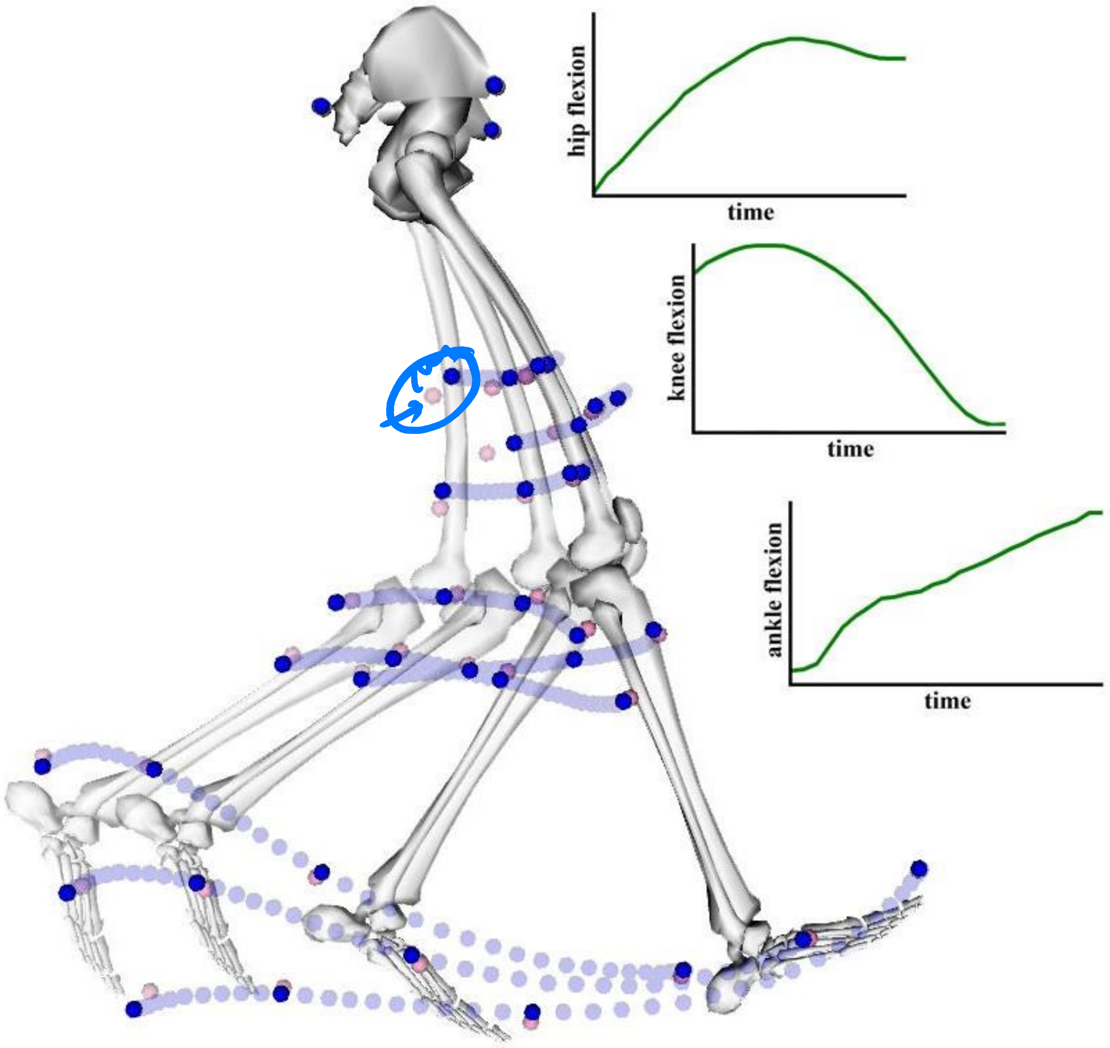
$$\frac{30}{20} = 1,5$$

- Improves transition errors form recorded to model motion
- Uniform vs non-uniform scaling
- What about mass parameters?



Inverse Kinematics

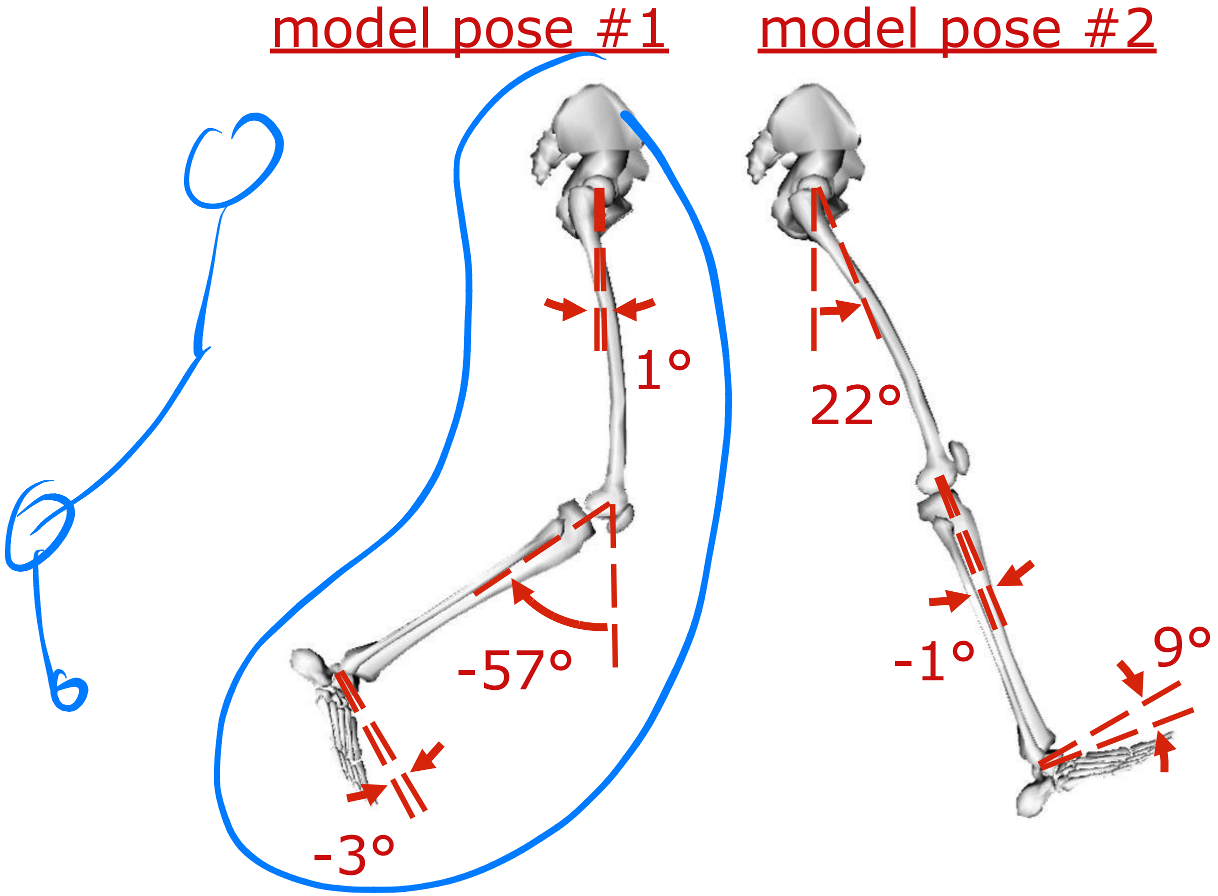
- Model pose and coordinates
- Marker error
- Coordinate error
- Weighted least squares minimization



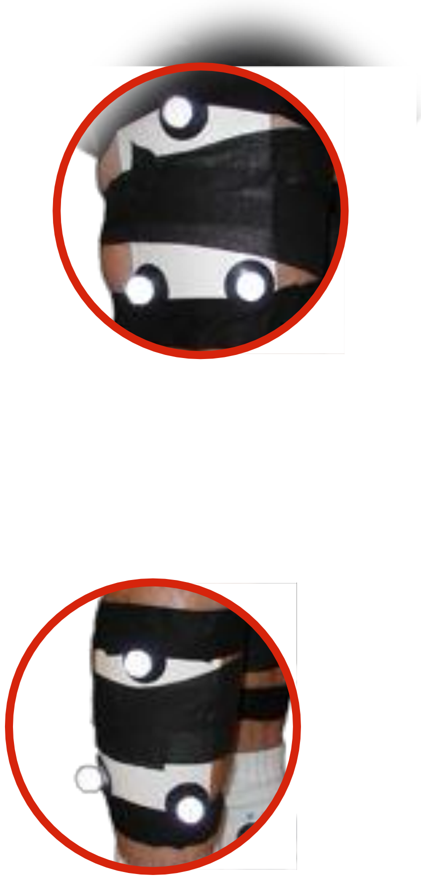
Model Pose and Coordinates

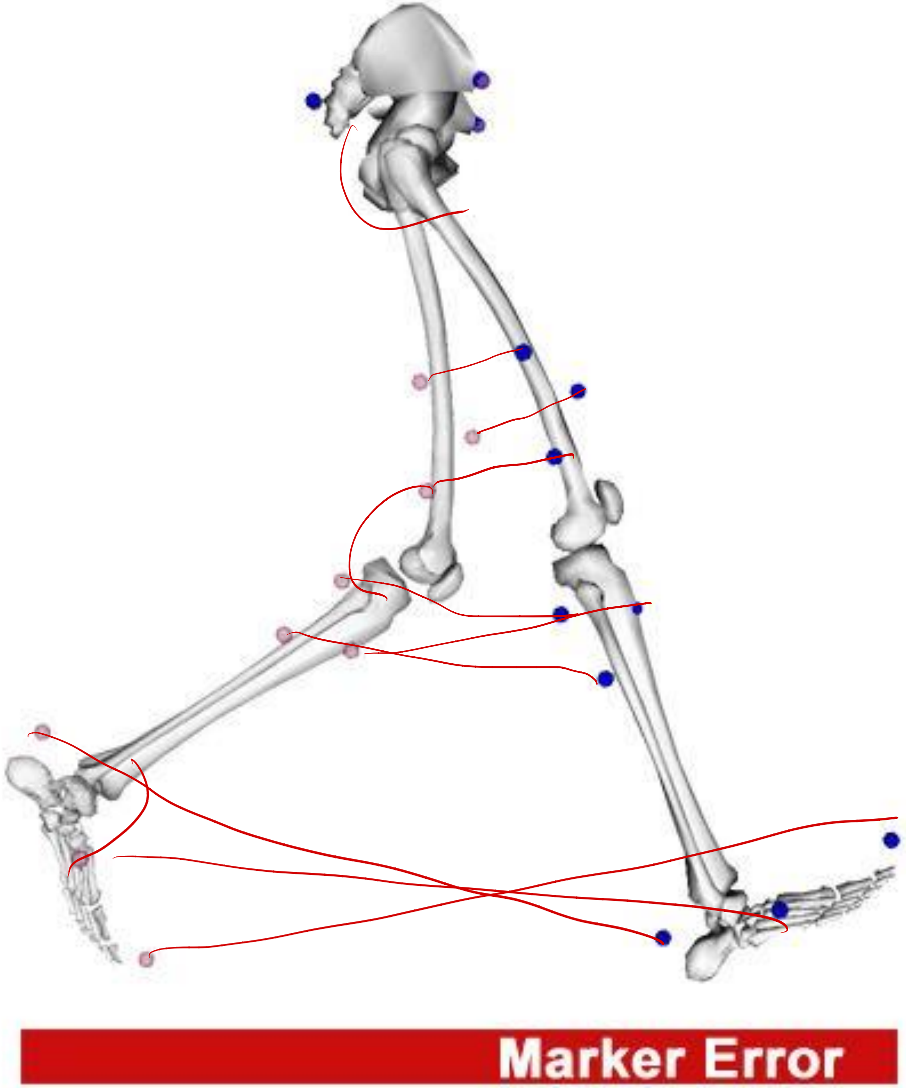
- Model Pose
  - Orientations and locations of body segments in the model
  - Defined by set of model coordinates

- Coordinate
  - Joint angle or distance specifying relative orientation or location of two body segments

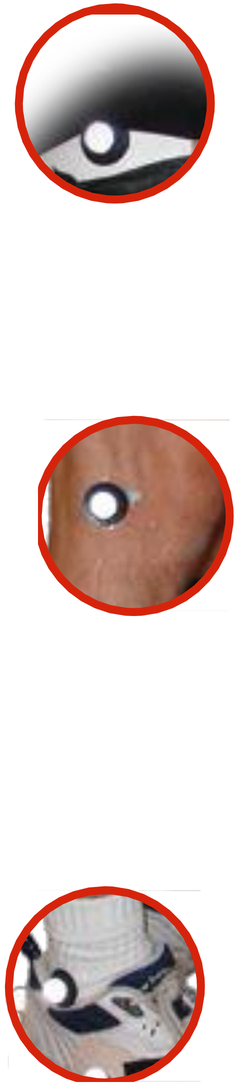


Marker Error





Coordinate Error



S2

Weighted Least Squares Minimization

$$\frac{\partial e}{\partial q} = \min_{q}\left[\sum_{m=1}^{\#\,\text{markers}} w_m \left\| \boldsymbol{x}_m^{\text{exp}} - \boldsymbol{x}_m(\boldsymbol{q}) \right\|^2 + \sum_{c=1}^{\#\,\text{coordinates}} \omega_c \left(q_c^{\text{exp}} - q_c\right)^2\right]$$

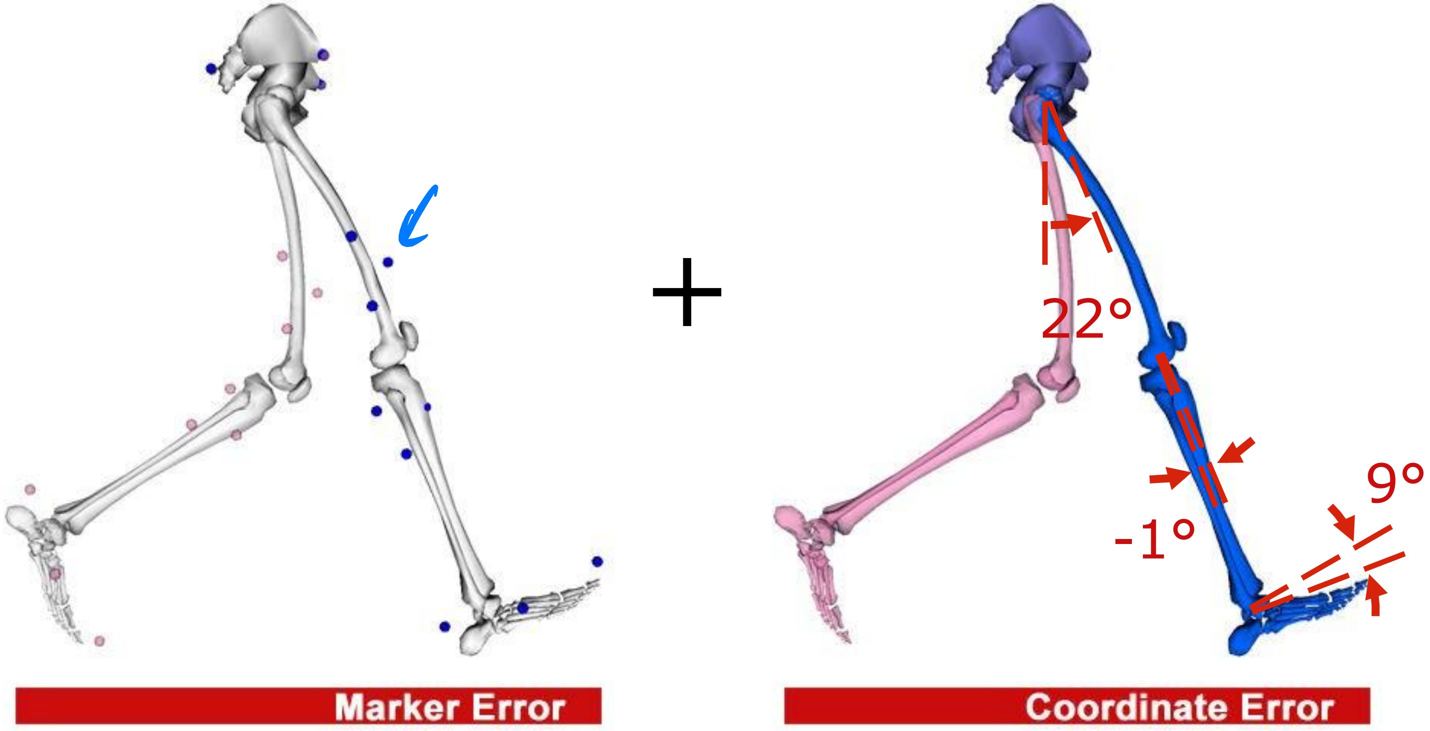
$q_s = 30$

50

150

$$[\;] - [\;] =$$

$$[\;]$$



Inverse Kinematics Exercise

- For the model shown on the right, which coordinate(s) need to be adjusted to create a model pose best matches the experimental markers as shown at the beginning swing phase?

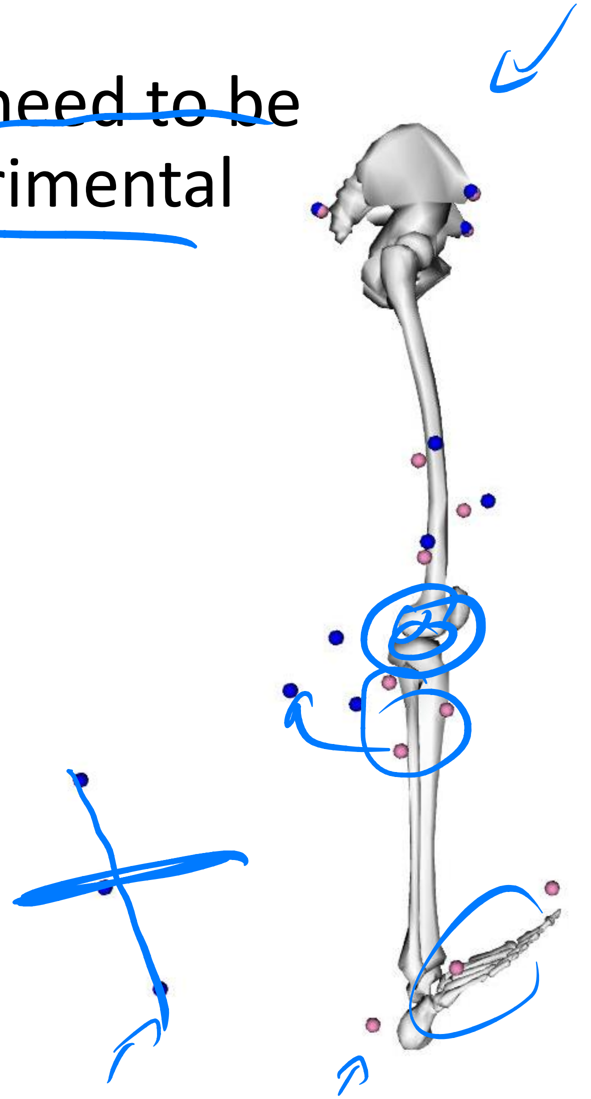
A. Hip

B. Knee

C. Ankle

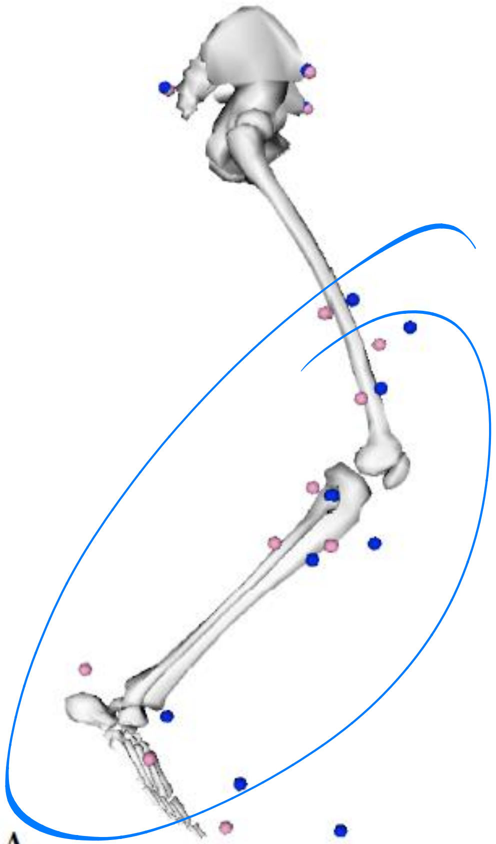
D. Hip and ankle

[✓ E.] Knee and ankle

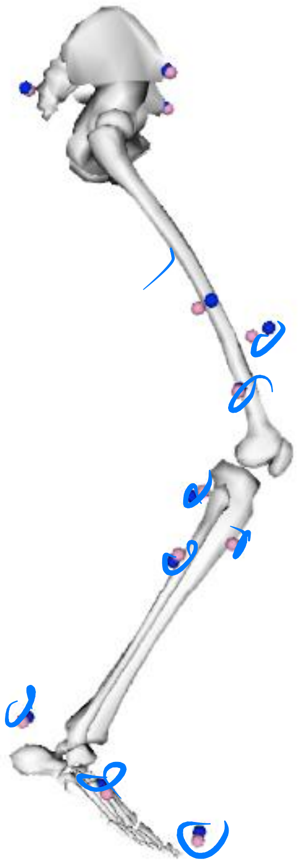


Inverse Kinematics Exercise

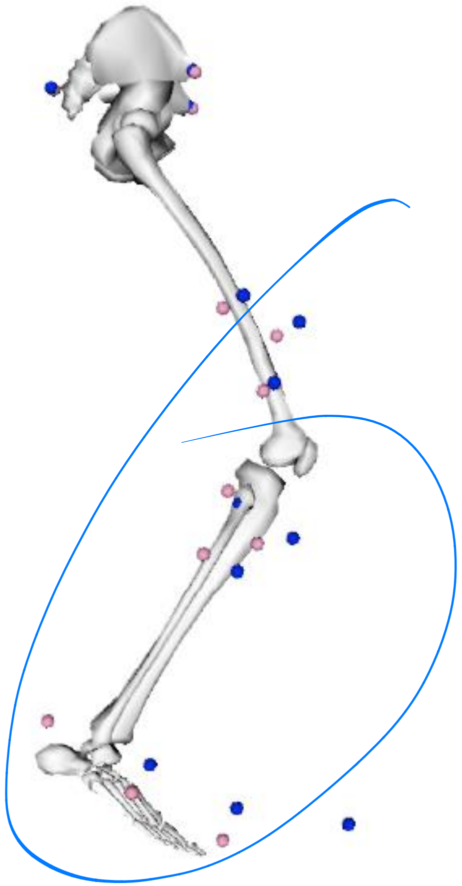
For the model poses and experimental markers shown below, which combination of pose and markers has the minimum marker errors?



A.



[✓ B.]



C.

Inverse Kinematics Exercise

In theory, experimental markers on the thigh and shank could have more skin movement artifacts compared with the foot markers; which of the following scenarios would be most appropriate for the weighted least squares minimization solved by the Inverse Kinematics Tool?

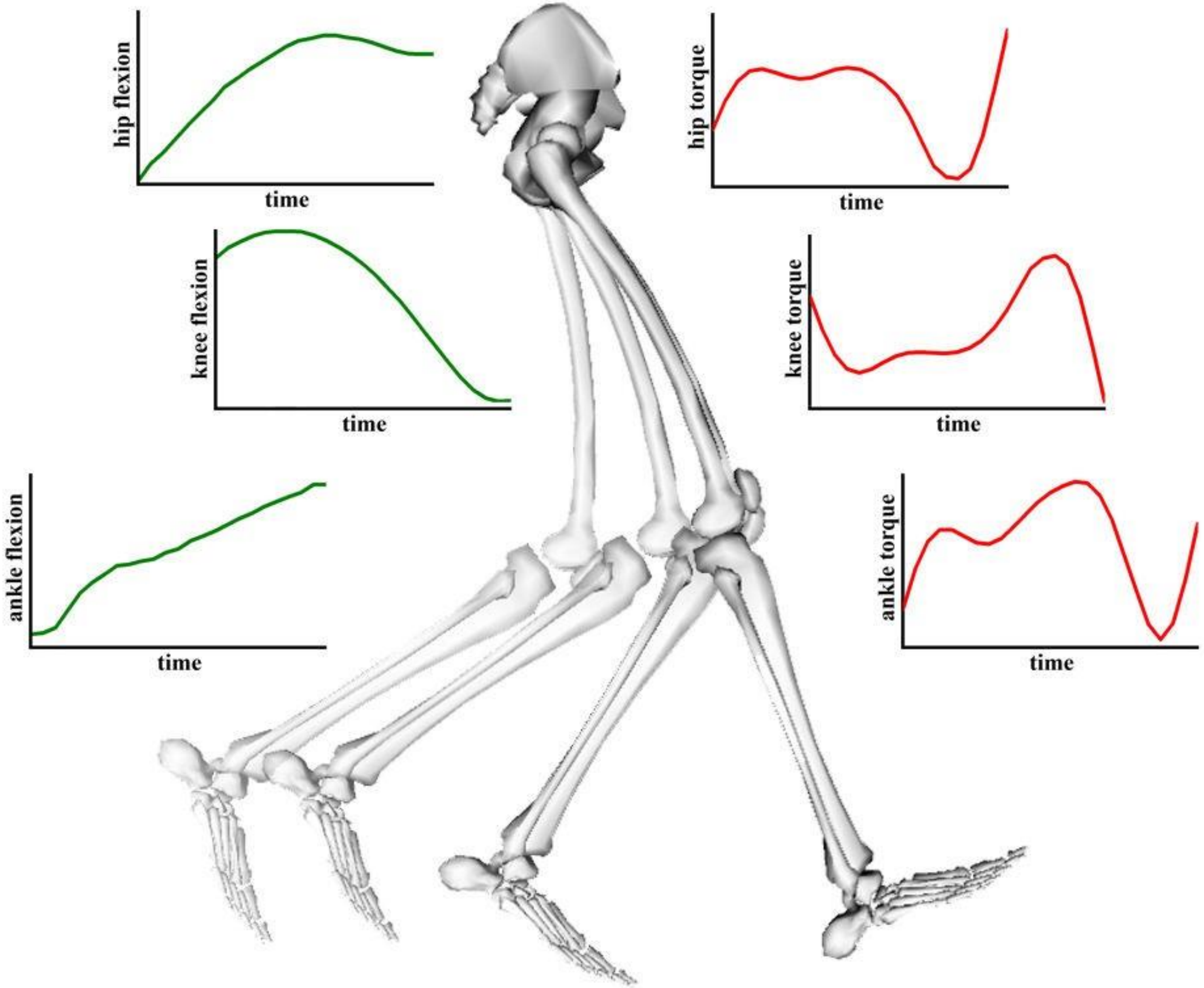
A. Decrease tracking weights on thigh markers
B. Decrease tracking weights on shank markers
C. Increase tracking weights on foot markers
D. All of the above

Tips and Tricks

- Marker weights are relative
- Check max and RMS marker errors in messages window
- Weight “motion” marker triads on body segments higher than anatomical markers
- Max marker error should be < 2 cm with RMS error <1 cm

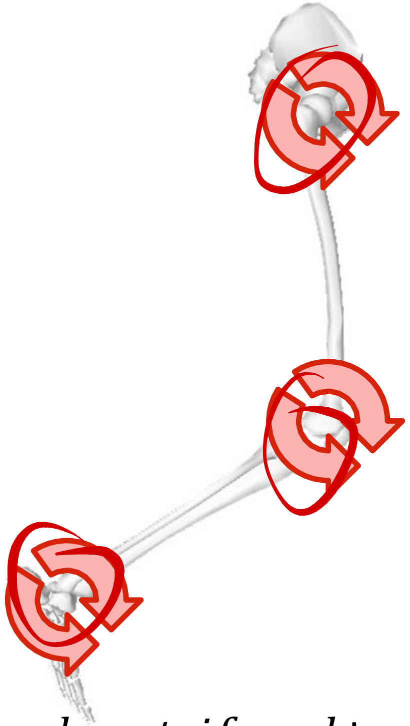
Inverse Dynamics

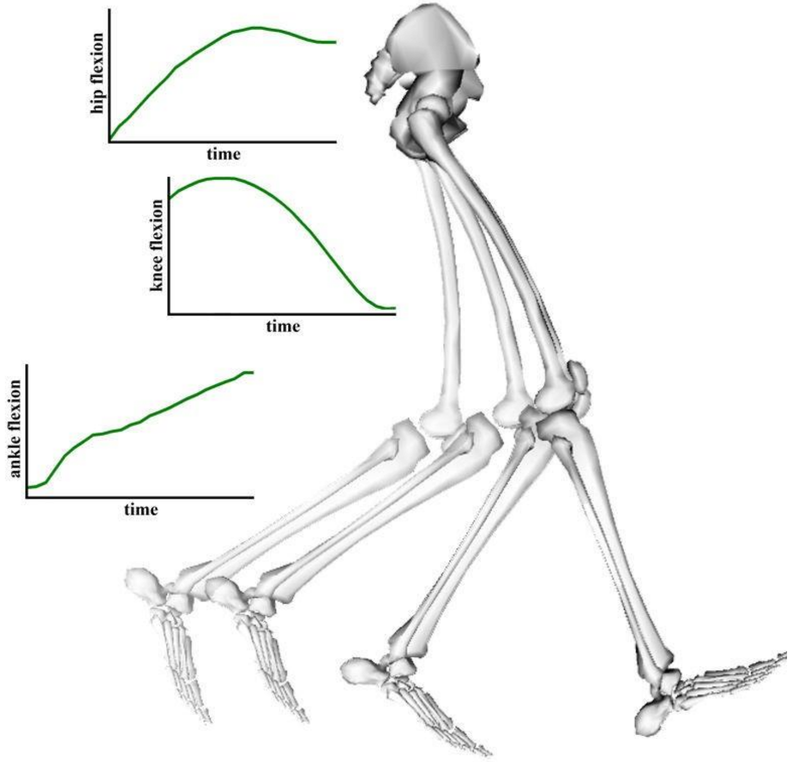
- Kinematics: coordinates and their velocities and accelerations
- Kinetics: forces and torques
- Dynamics: equation of motion



Dynamics

$$\underbrace{\tau}_{\text{unknowns}} = \underbrace{M(q)\dot{q} - C(q,\dot{q}) - G(q) - F}_{\text{knowns}}$$





M: $mass\ matrix$, Ψ: $Coriolis\ and\ centrifugal$ torques,
G: gravity torques, F: other forces (ligaments), τ: active forces (muscles)

Inverse Dynamics Exercise

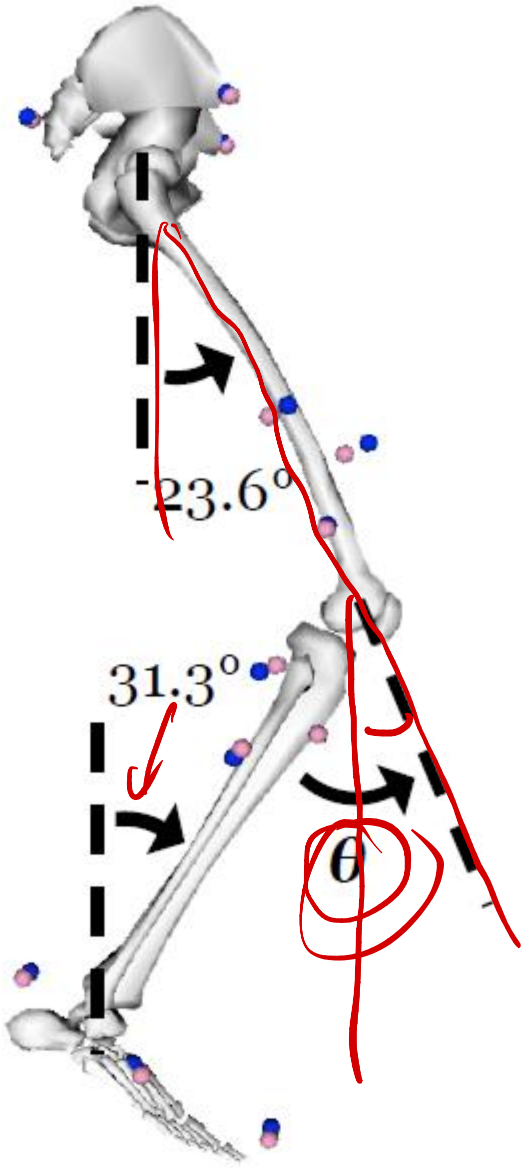
- For the model shown on the right, what is the value (*ϑ*) of the knee coordinate (*Note: extension is* ***+***)?

A. 23.6°

B. [✓ -54.9°]

C. 31.3°

D. -125.1°



Inverse Dynamics Exercise

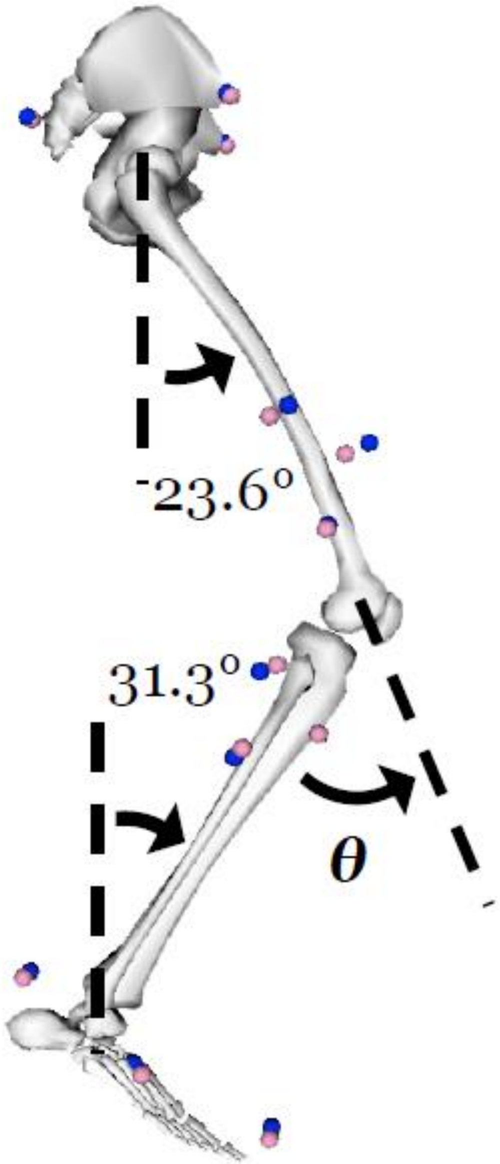
- Given that the model shown on the right is at rest, what is the velocity of the knee?

A. 23.6°/s

B. -54.9°/s

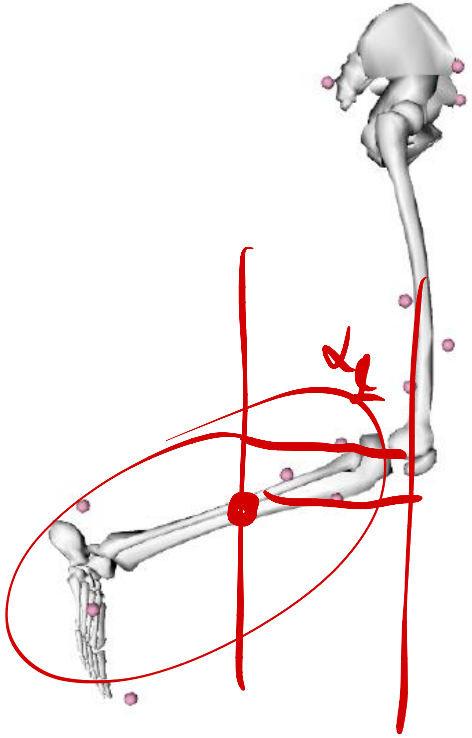
C. 3.89°/s

[✓ D. 0°/s]

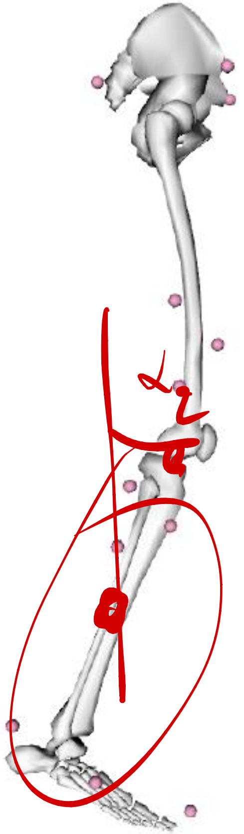


Inverse Dynamics Exercise

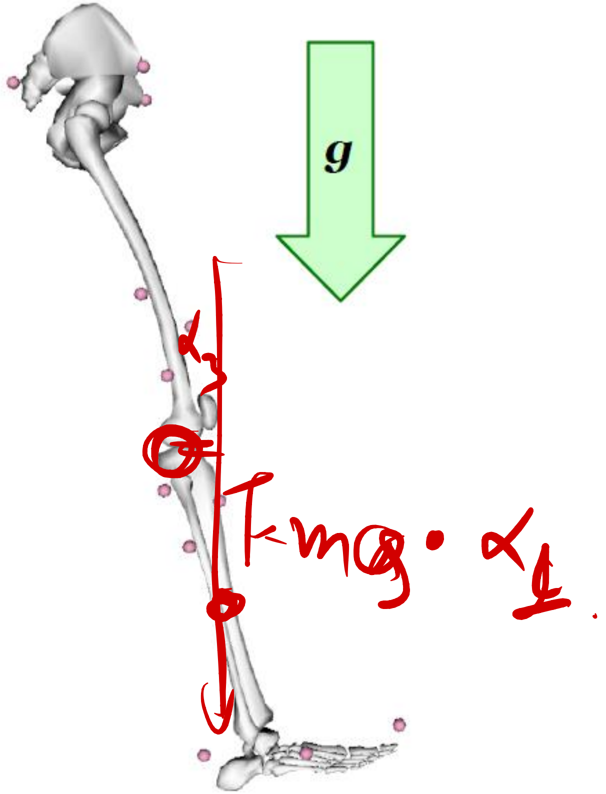
For the model poses shown below at rest and with gravity (g) as the only force acting on the model, which pose requires the largest torque at the knee joint?



A.



B.

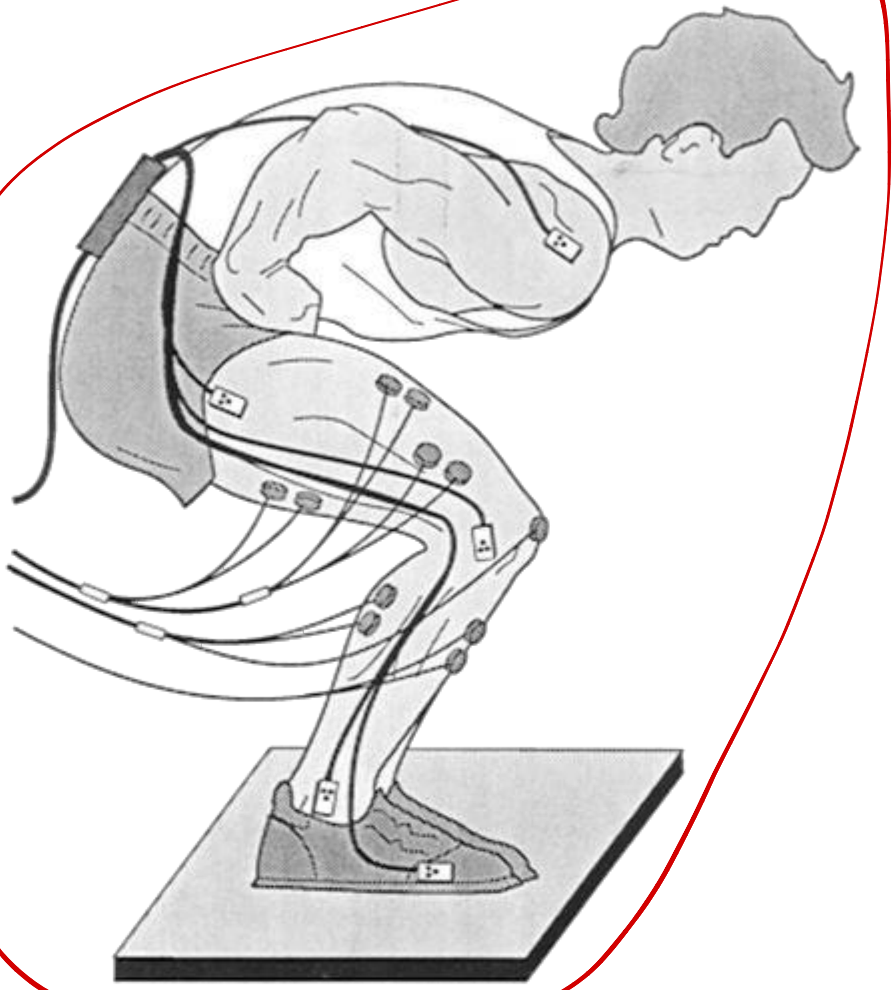


$T = mg \cdot \alpha_1$

C.

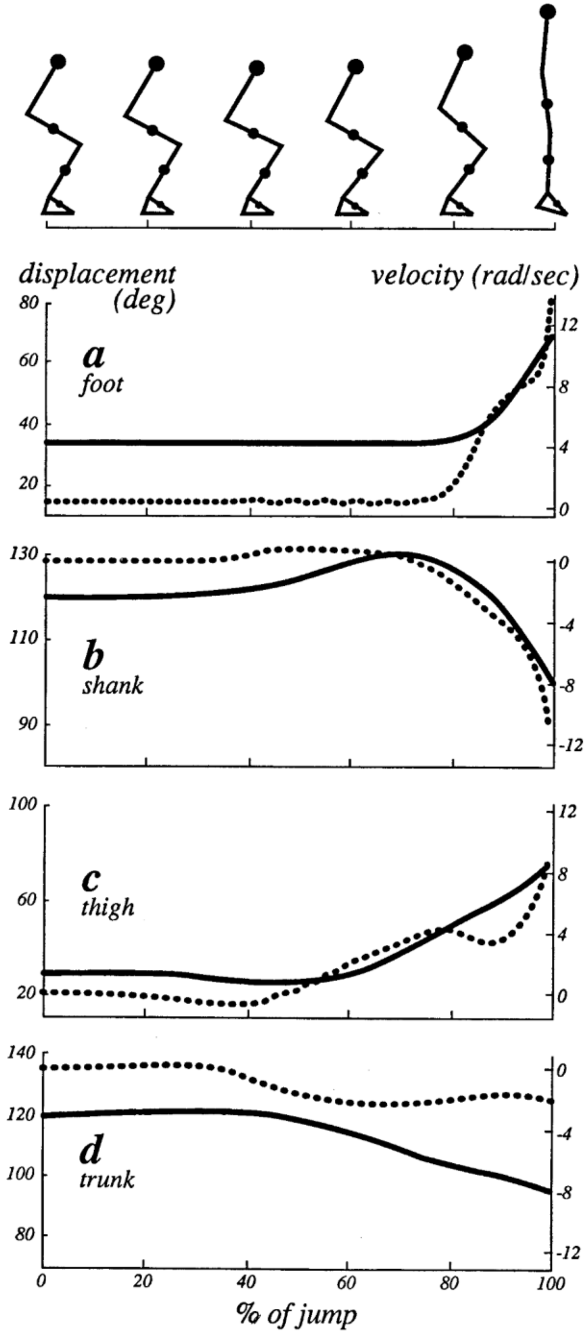
A Possible Inverse Dynamics Question

- What are the sagittal plane moments about the ankle, knee, and hip during a maximum height jump?



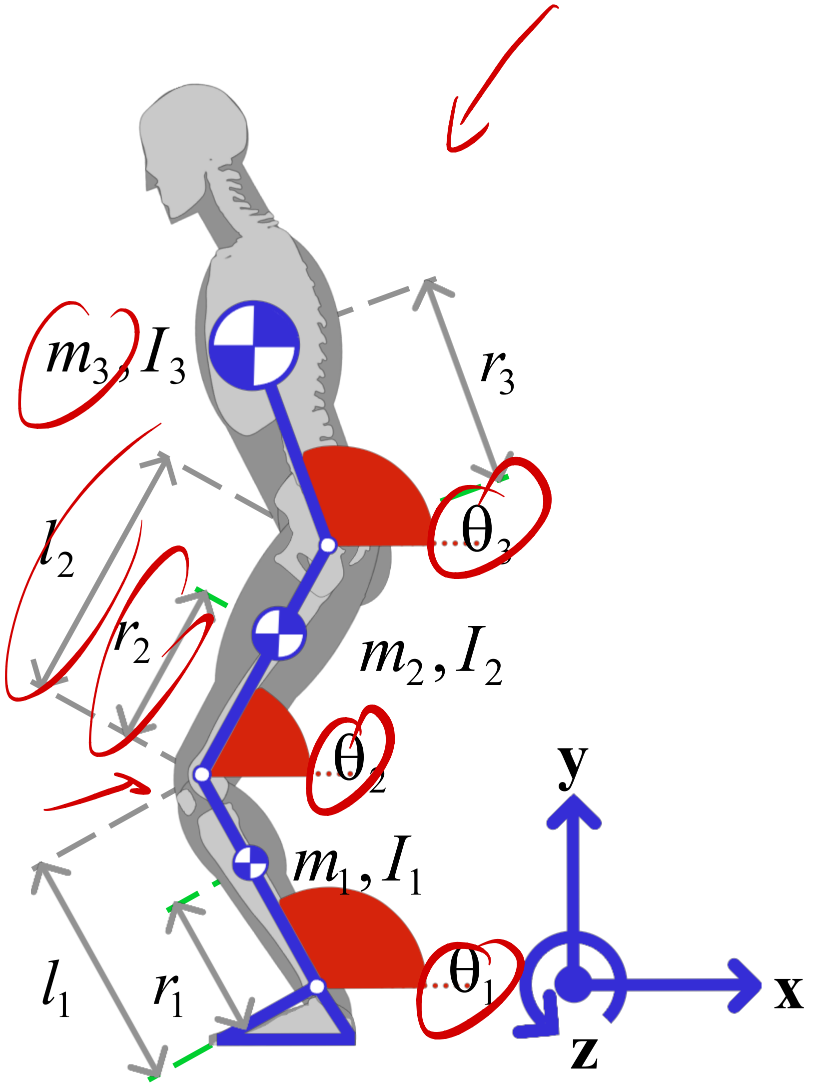
Inverse Dynamics Input

- Joint angles
- Angular velocities
- Angular accelerations
- Ground reaction forces

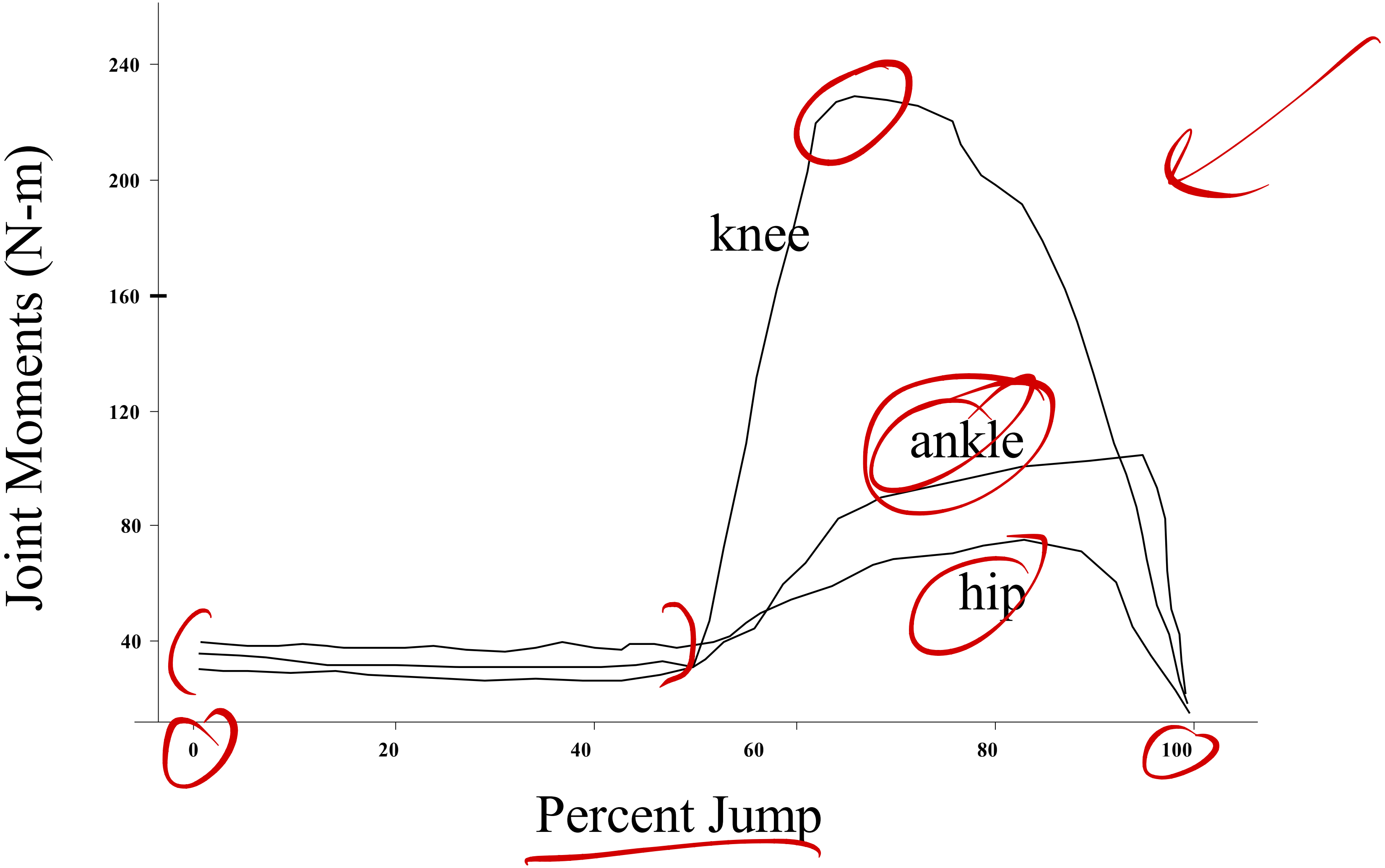


Multibody Dynamics

- Planar 3 degrees of freedom
- Position (orientation) in global coordinate system
- Segment length = l_i
- Distance to mass center = r_i
- Moments of inertia about mass center
- Foot has no mass and remains on ground

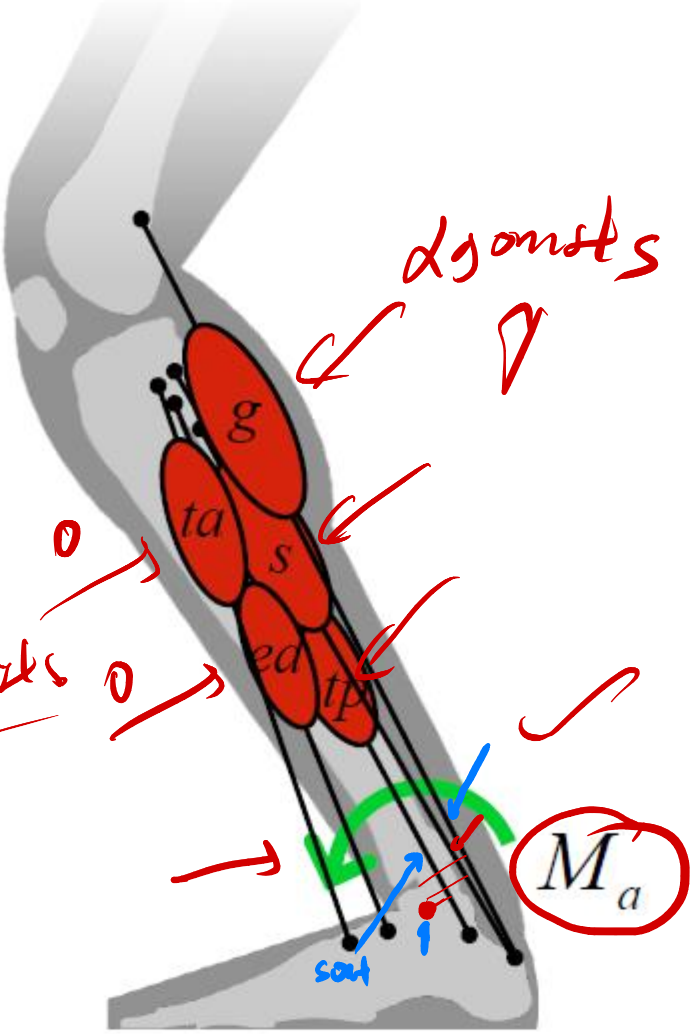


Net Joint Moment



Static Optimization

- Kinematics
  - coordinates and their velocities and accelerations
- Kinetics torques
  - muscle forces
- Muscle physiology
  - activation-contraction dynamics and force-length-velocity relations
- Dynamics
  - equations of motion
- Musculoskeletal geometry
  - muscle moment arm
- Optimization
  - the “distribution” problem



agonists

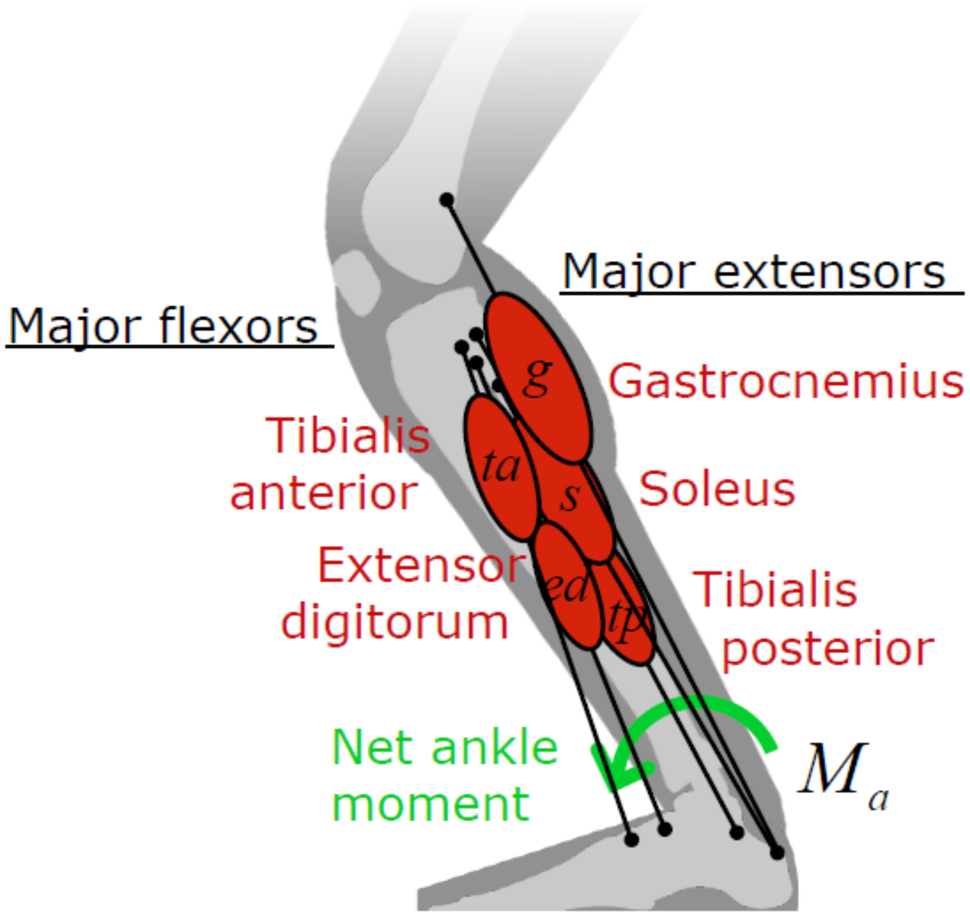
antagonists

0

0

Static Optimization

- Determines the “best” set of muscle forces that
  - Produce net joint moments at a discrete time
  - Do not violate muscle force limits
  - Optimize a performance criterion
- Performance criterion attempts to capture the goal of the neural control system
  - Minimize muscle force?
  - Minimize muscle stress?



The Muscle Force Distribution Problem

M_j = $\sum$ muscle moments + $\sum$ moments due to other structures

number of flexors

number of extensors

$$M_j = \underbrace{\sum_{f=1}^{nf} F_f r_f}_{\text{flexion moment}} - \underbrace{\sum_{e=1}^{ne} F_e r_e}_{\text{extension moment}}$$

moment arm

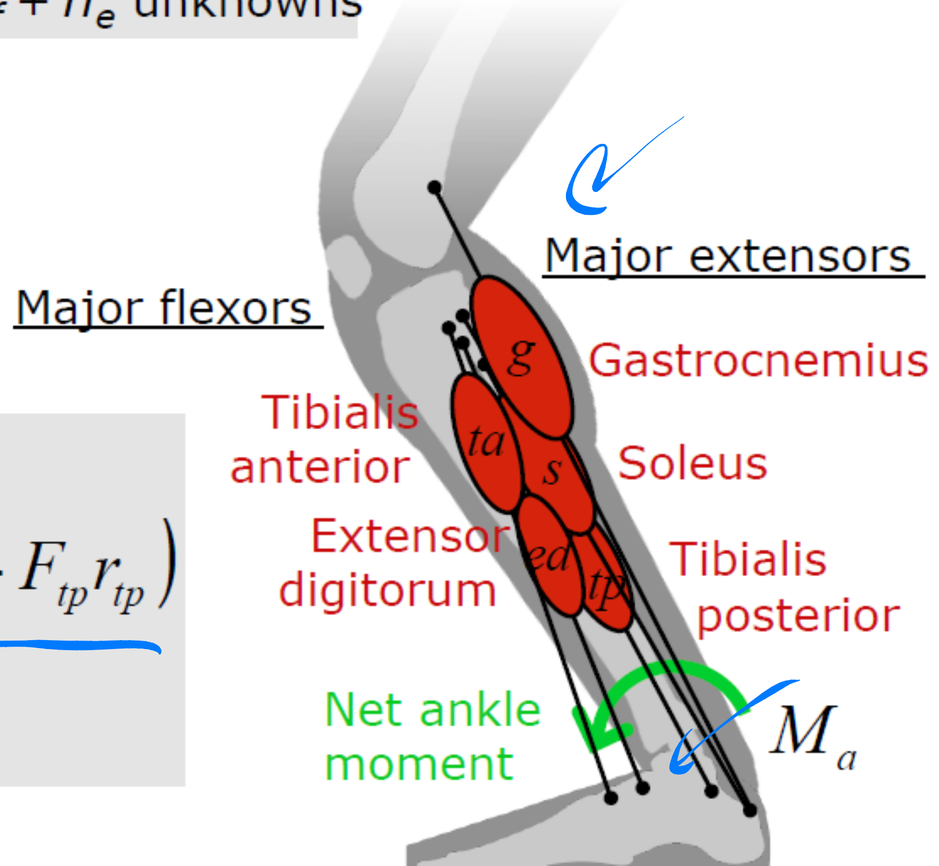
1 equation with $n_f + n_e$ unknowns

Ankle example

$\alpha \cdot F_p$

$$M_a = (F_{ta} r_{ta} + F_{ed} r_{ed}) - (F_g r_g + F_s r_s + F_{tp} r_{tp})$$

How can we solve this?



Static Optimization Formulation

minimize $f(F_m)$ Function of muscle forces

subject to $M_a(t) = [F_{ta}(t)r_{ta}(t) + F_{ed}(t)r_{ed}(t)] - [F_g(t)r_g(t) + F_s(t)r_s(t) + F_{tp}(t)r_{tp}(t)]$

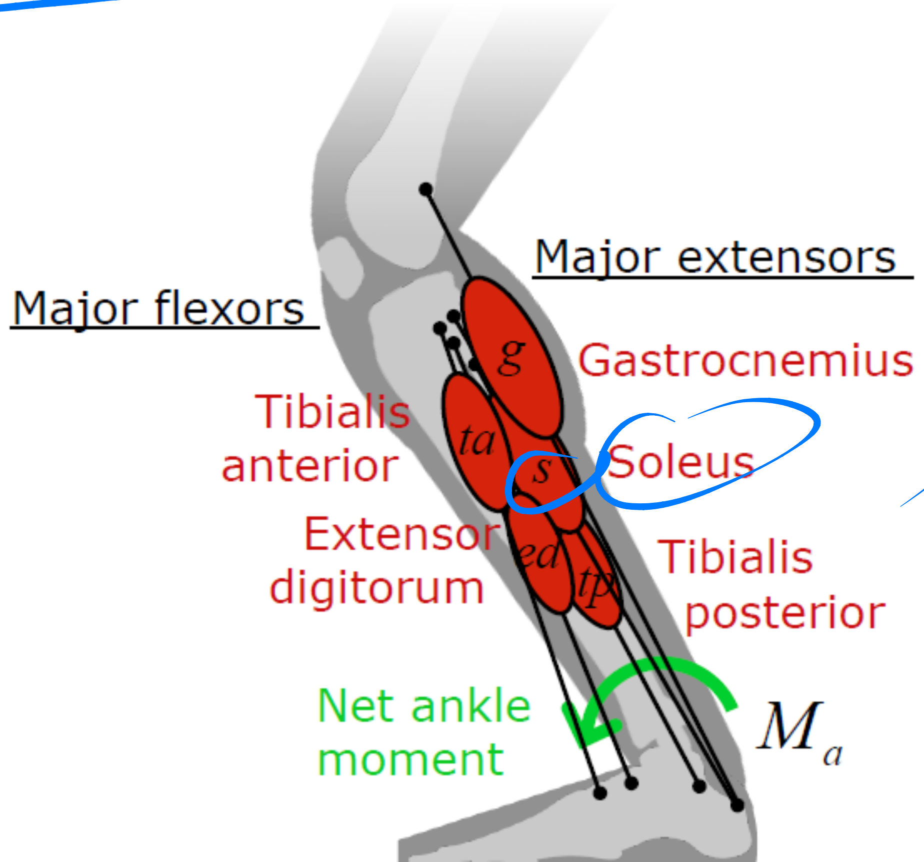
$F_{ta}(t) \leq 900\text{N}$

$F_{ed}(t) \leq 800\text{N}$

$F_g(t) \leq 1500\text{N}$

$F_s(t) \leq 2500\text{N}$

$F_{tp}(t) \leq 1500\text{N}$



Example Performance Criteria

Difficult to define and validate a good criterion

$$f(F_m)=\sum_{m=1}^{nm} F_m$$

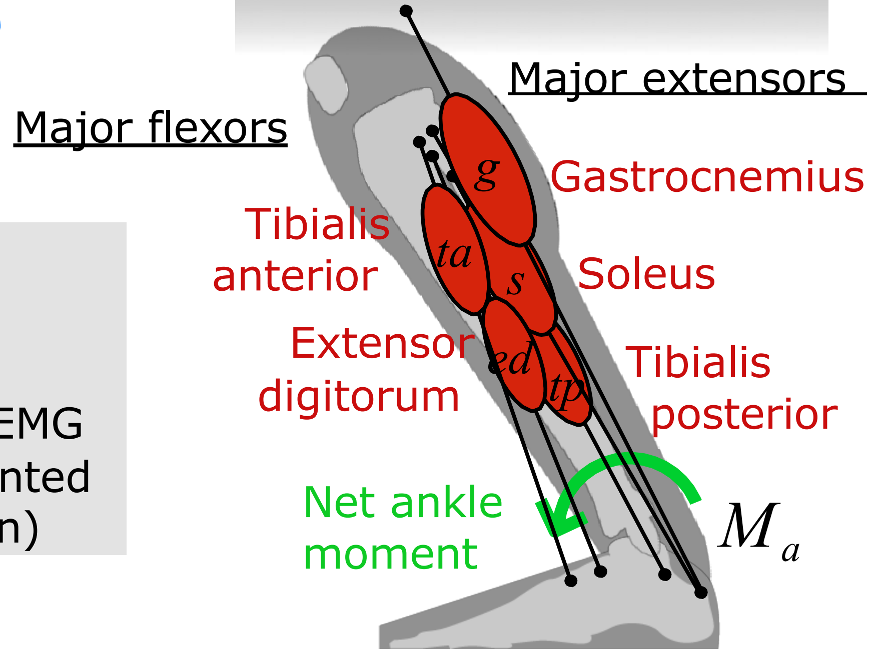
Muscle force

$$f(F_m)=\sum_{m=1}^{nm}\left(\frac{F_m}{PCSA_m}\right)^3$$

(Muscle stress)^3 ~ Metabolic energy

$$f(F_m)=\sum_{m=1}^{nm}\left(k\frac{F_m}{PCSA_m}\right)^2 \approx \sum_{m=1}^{nm}(a_m)^2$$

(Muscle activation)^2



Possible validations

- Use output to drive a forward dynamic simulation
- Compare qualitatively to experimental EMG
- Compare to measured forces (instrumented hip implant, buckle transducer in tendon)

Formulation of the Optimization Problem

- Ideal force generators
- Constrained by force-length-velocity properties
- Minimizing the objective function

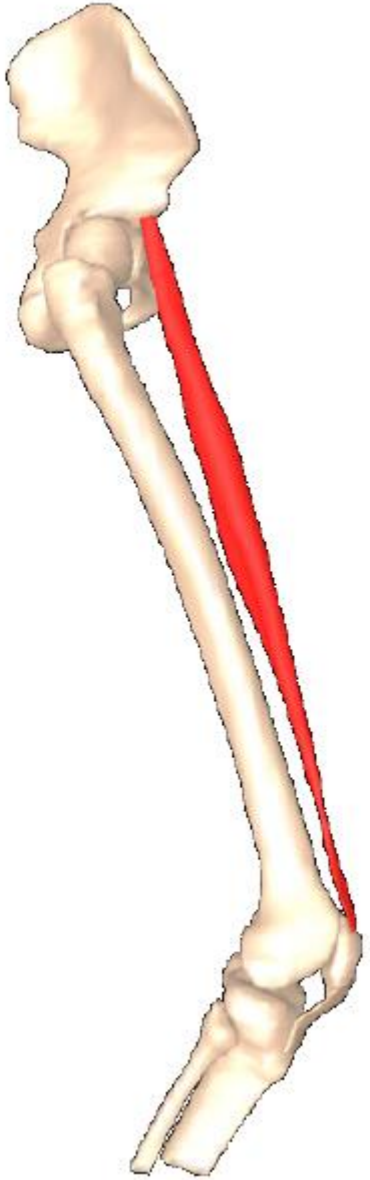
$$\sum_{m=1}^{n} (a_m F_m^0) r_{m,j} = \tau_j$$

$$\sum_{m=1}^{n} [a_m f(F_m^0, l_m, v_m)] r_{m,j} = \tau_j$$

$$J = \sum_{m=1}^{n} (a_m)^p$$

Static Optimization Exercise

- Given that the rectus femoris muscle has a peak isometric force of 1169 N and it is at its optimal fiber length and zero velocity, what is the force generated for an activation of 0.86?



A. 164 N
B. 952 N
C. 1005 N
D. 1058 N

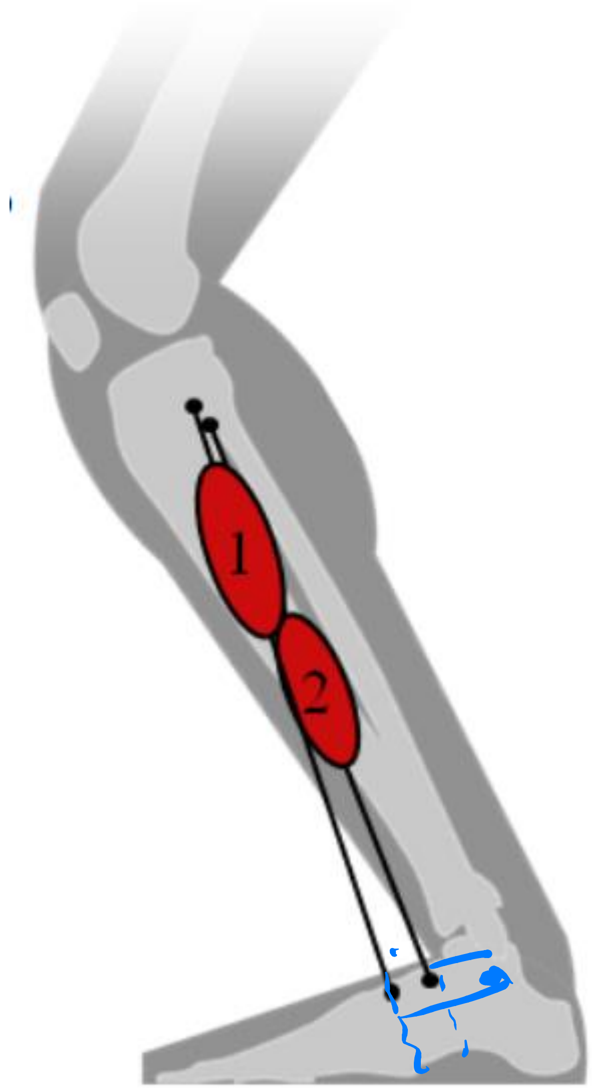
Static Optimization Exercise

- For the model shown on the right, which muscle has the largest moment arm about the **ankle** joint?

A. 1

B. 2

C. Neither (are identical)



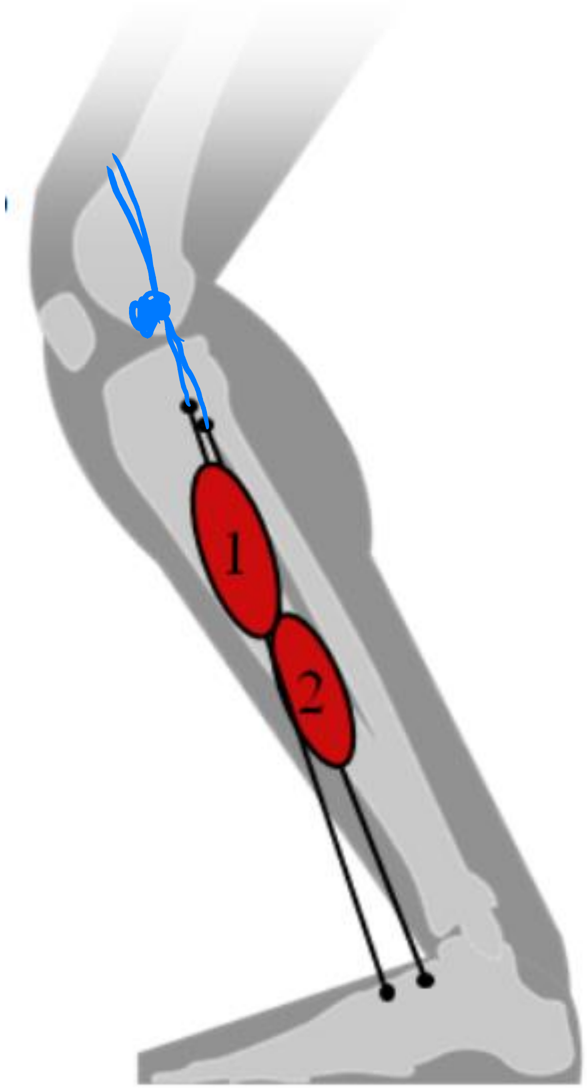
Static Optimization Exercise

- For the model shown on the right, which muscle has the largest moment arm about the **knee** joint?

A. 1

B. 2

C. Neither (are identical)



$F_1 = 200 \times \alpha_0$

$F_1 = 900 \times \alpha_0$

$F_1^0 = 200$

$F_2^0 = 300$

$T_1 = 200\alpha \cdot 3{,}6 = 720\alpha$

$T_2 = 300\alpha \cdot 3 = 900\alpha$

Static Optimization Exercise

For the model shown on the right, muscle 1 and 2 have the following properties

Muscle	**Peak Isometric Force (N)**		**Moment Arm (cm)**
1	905	500	3.6
2	512	900	3.0

To solve the "distribution" problem minimizing the sum of squared activations, which muscle would be activated more for a given dorsiflexion moment?

A. 1

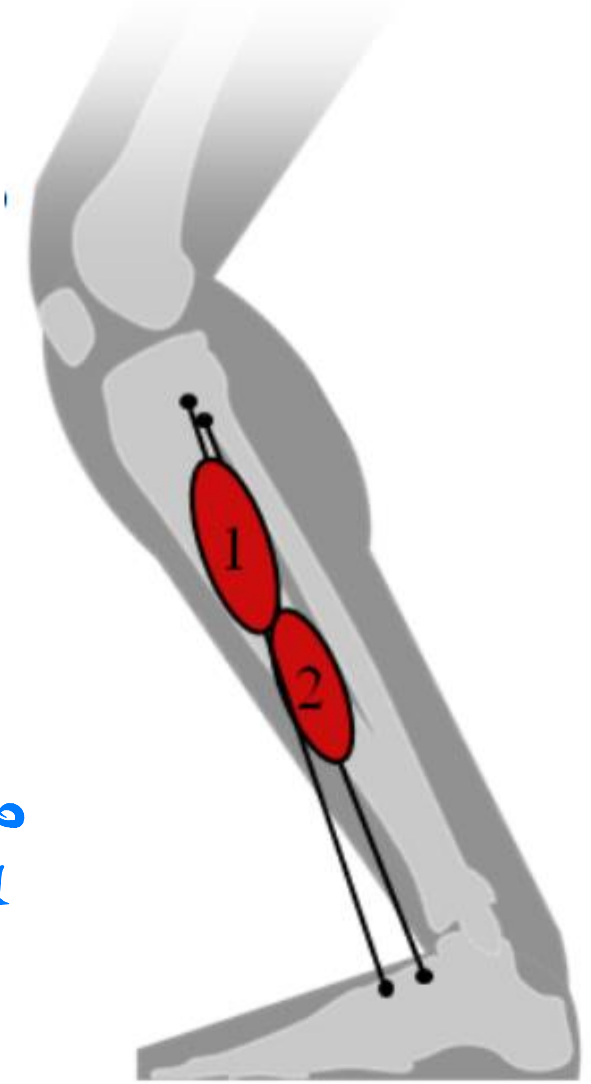
B. 2

C. Neither (are identical)

$\alpha_1 = \alpha_2$

$F_1 = \alpha_0 \cdot P_1^0$

$F_2 = \alpha_0 \cdot P_2^0$



Tips and Tricks

- **Inputs:** Can use kinematics from IK or RRA. If using IK, need to filter kinematics
- **Residuals:** Add residual actuators to pelvis
- **Reserves:** Add reserve torque actuators to trouble-shoot a weak model
- **Minimizing residuals & reserves:** Increase maximum control value (default = 1) and lower the maximum force -> penalizes activity
- **Command Line:** analyze –S setup_file.xml