Neuromusculoskeletal Modeling

Why are muscle forces important?

- Moving (walking, running, waving, …)
- Talking
- Breathing
- Seeing
- Hearing
- Digesting (smooth muscle)
- Pumping (cardiac muscle)

Muscle and Tendon Properties Influence Performance

Muscle strength and the rate at which muscles contract are major determinants of running speed

Kangaroos can run more efficiently at fast speeds than at slow speeds, partly because of the compliance of their tendons

Facts

- Strong quadriceps can lift a small car off the ground

$$F_o^M \approx 10{,}000\,N \;\approx\; 1{,}000\;kg$$

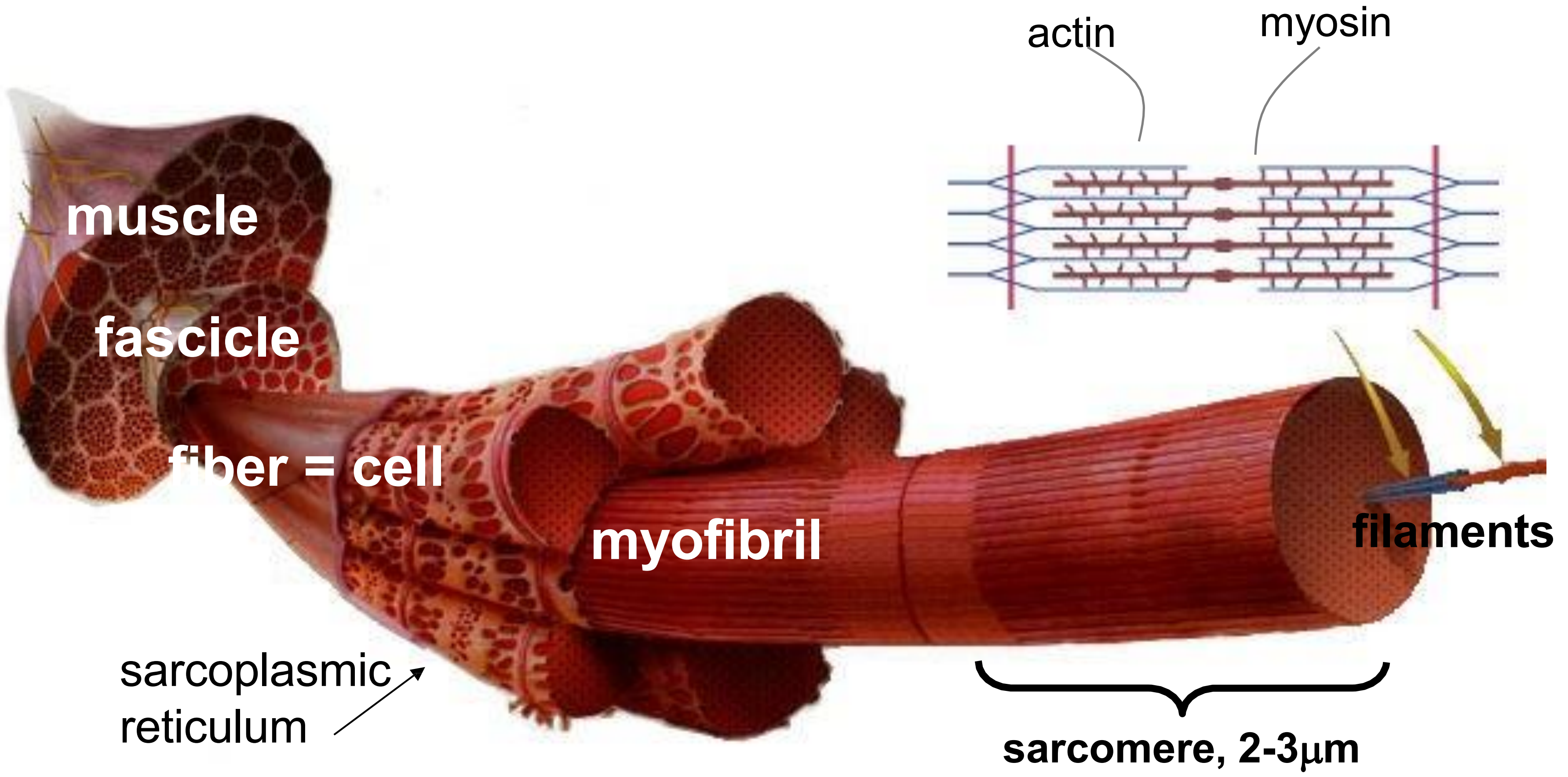
$$VW\;\; Bug \approx 1{,}180\;kg.$$

- Joint contact forces:
  - 3 * Body Weight during walking
  - 5 * Body Weight during running

Muscles Actuate Movement by Developing Tension

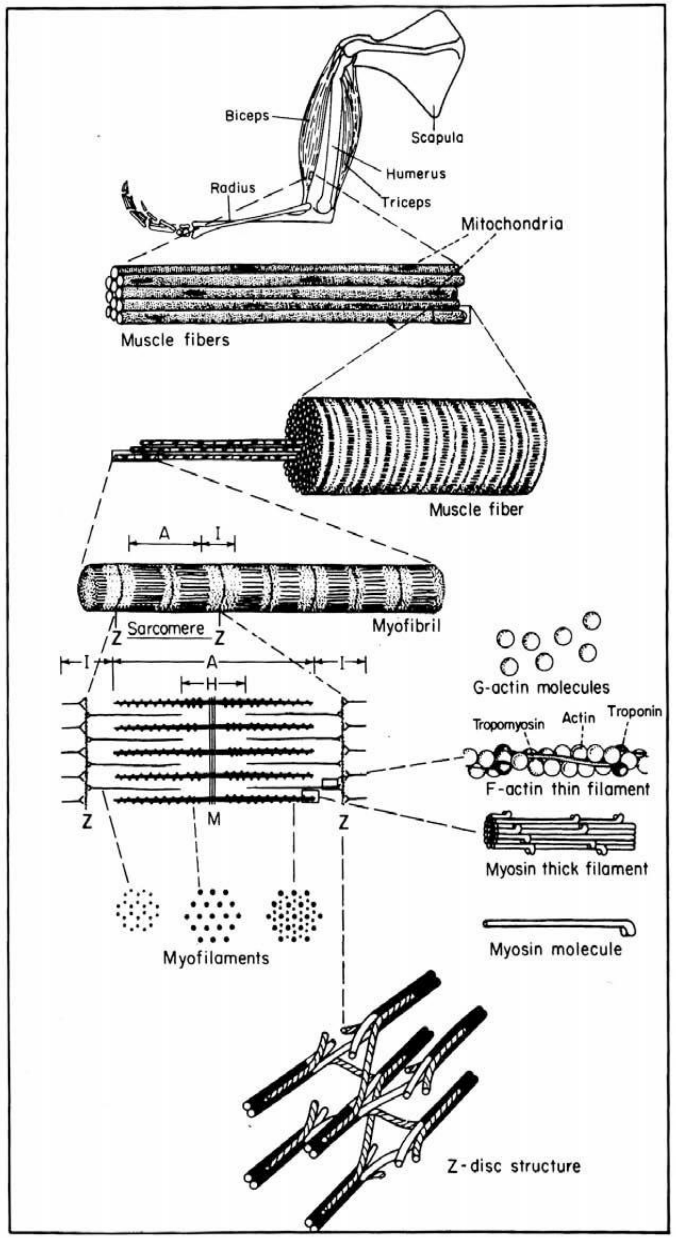
- Muscles pull, not push
- Muscles are grouped into antagonistic pairs
- Tendon connects muscle to bone

Hierarchical Muscle Structure

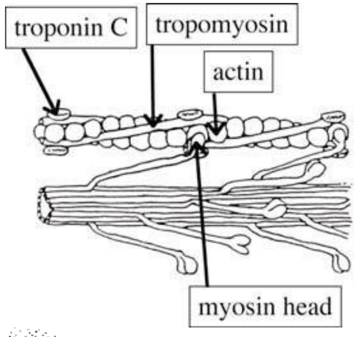


Muscle Force Results From Interaction Between Contractile Proteins

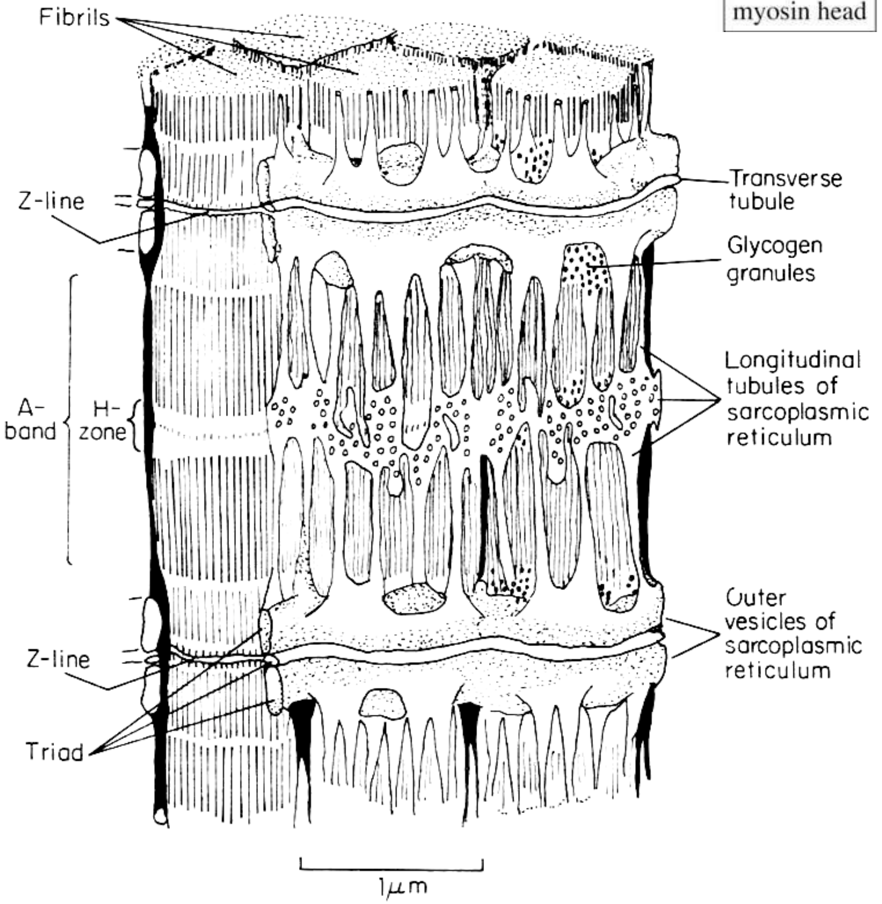
- Sarcomere: the smallest anatomical unit that contracts like a muscle
- Sliding filament model proposes that muscle force arises from cyclic binding between thick and thin filaments of the sarcomere
- Thin filaments contain actin, troponin C, and tropomyosin
- Thick filaments contain myosin
- In the absence of calcium, tropomyosin prevents myosin from attaching to actin



Calcium is Needed for Muscle Contraction



- At the onset of an action potential, the sarcoplasmic reticulum (SR; a membrane that surrounds the myofibrils) releases calcium
- Calcium binds to troponin, causing a conformal change in tropomyosin which reveals myosin binding sites on the actin
- Simultaneously, adenosine triphosphate (ATP) is hydrolyzed by ATPase in the myosin head, providing the energy for cross-bridge attachment
- The SR re-sequesters calcium at the end of the action potential, thereby inducing muscle relaxation

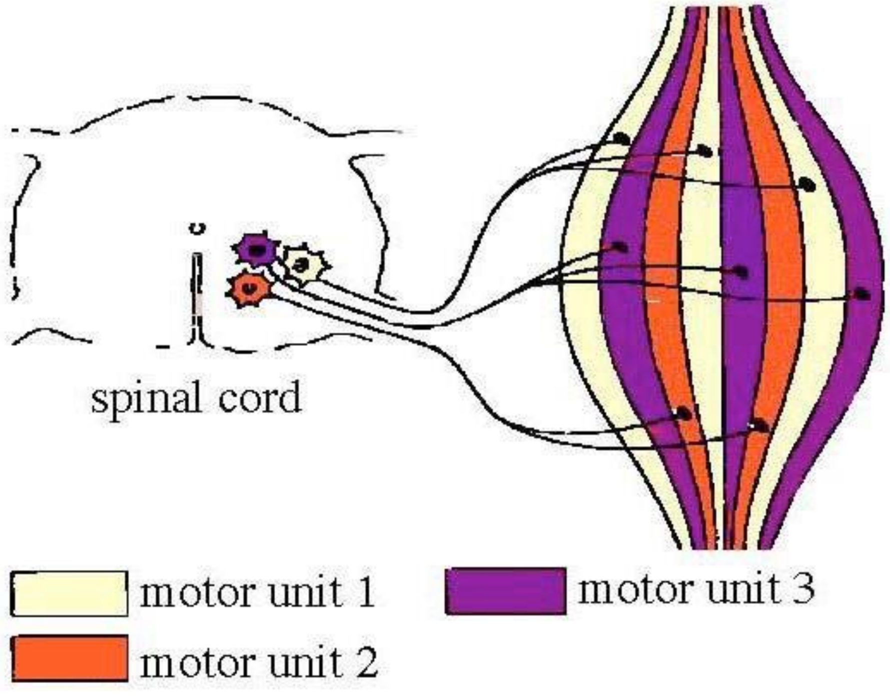


Factors Affecting Muscle Force Development

- Muscle fiber type
- Number of activated motor neurons, frequency of discharge
- Muscle length
- Velocity of shortening/ lengthening
- Muscle geometry (physiological cross-sectional area (PCSA), angle of pennation)

Muscle-Nerve Interaction

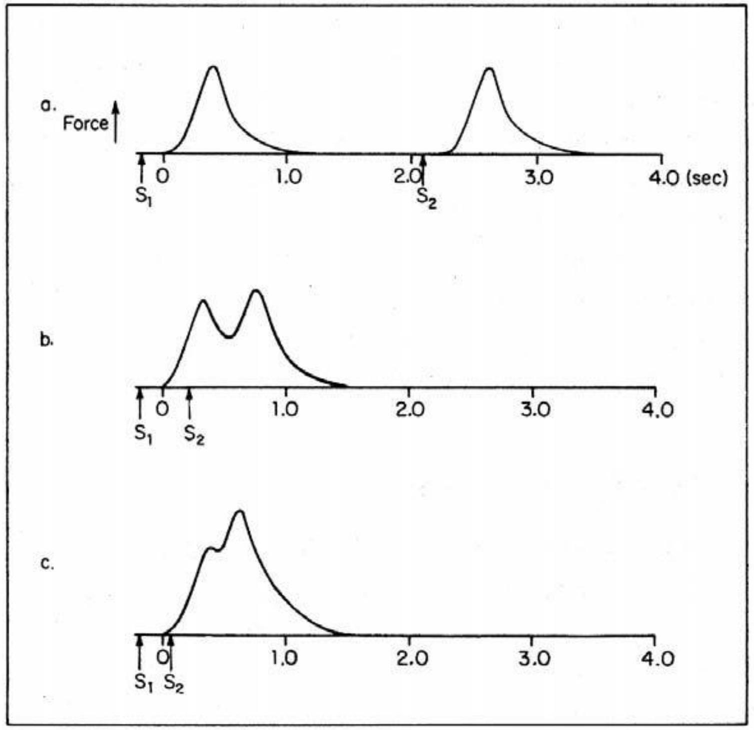
- A motor nerve enters muscle and splits into numerous axons; each axon contacts 10- 2000 muscle fibers
- Each muscle fiber is innervated by only one motor nerve axon, and contracts in response to an action potential in that axon
- Motor unit: a single motor nerve axon and all the muscle fibers it contacts

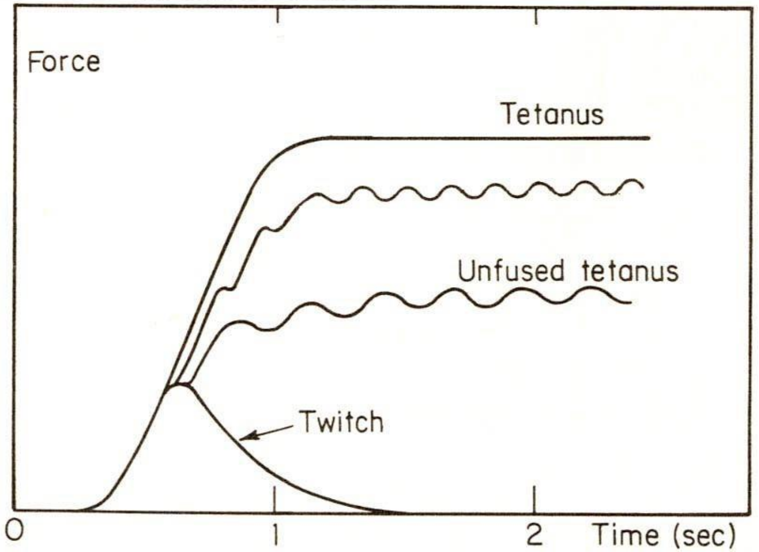


muscle	# muscle fibers	# motor units	av. fibers per motor u.
platysma	27,100	1,100	25
Brachioradialis	130,000	330	410
Tibialis anterior	250,000	450	600
gastrocnemius	1,120,000	580	2,000

Stimulation Frequency Affects Muscle Force: Twitch and Tetanus

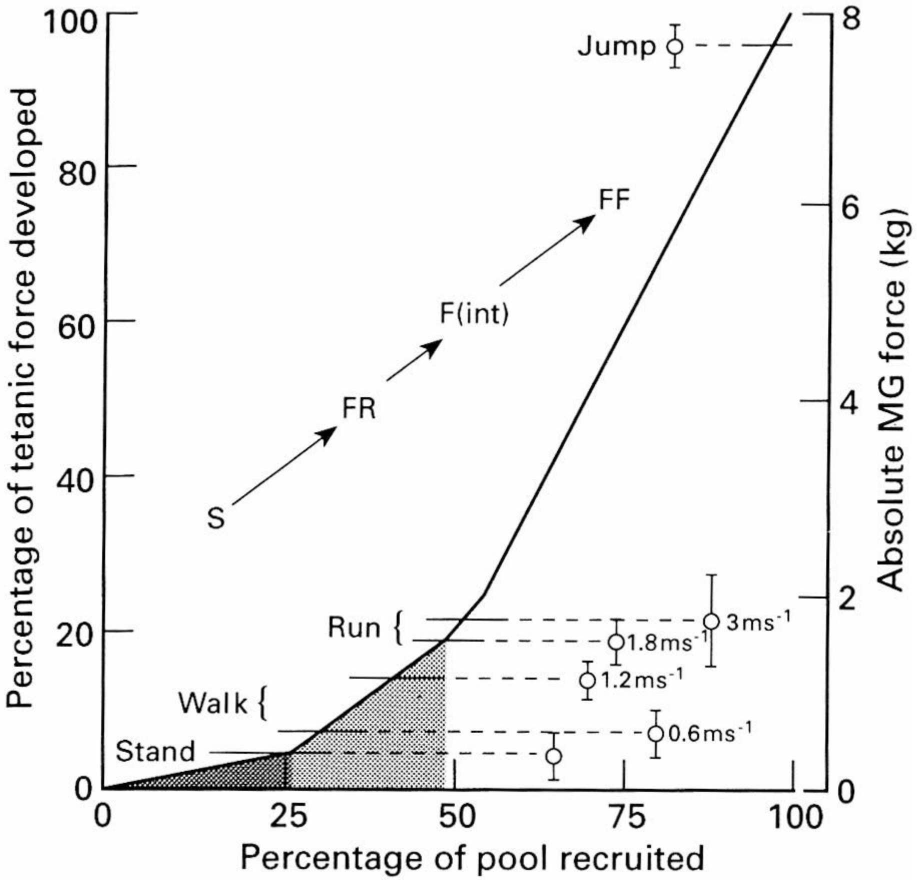
- Muscle force can be modulated by varying: (1) the number of recruited motor neurons, and (2) the frequency of discharge (i.e., stimulation rate) in motor neurons
- A single action potential (S1) produces a twitch contraction, a quick rise and slow fall in force
- A tetanus occurs when a new action potential (S2) arrives before the previous twitch has dissipated, and there is force summation
- At stimulation frequencies >30/s, there are no twitch transients (fused tetanus)



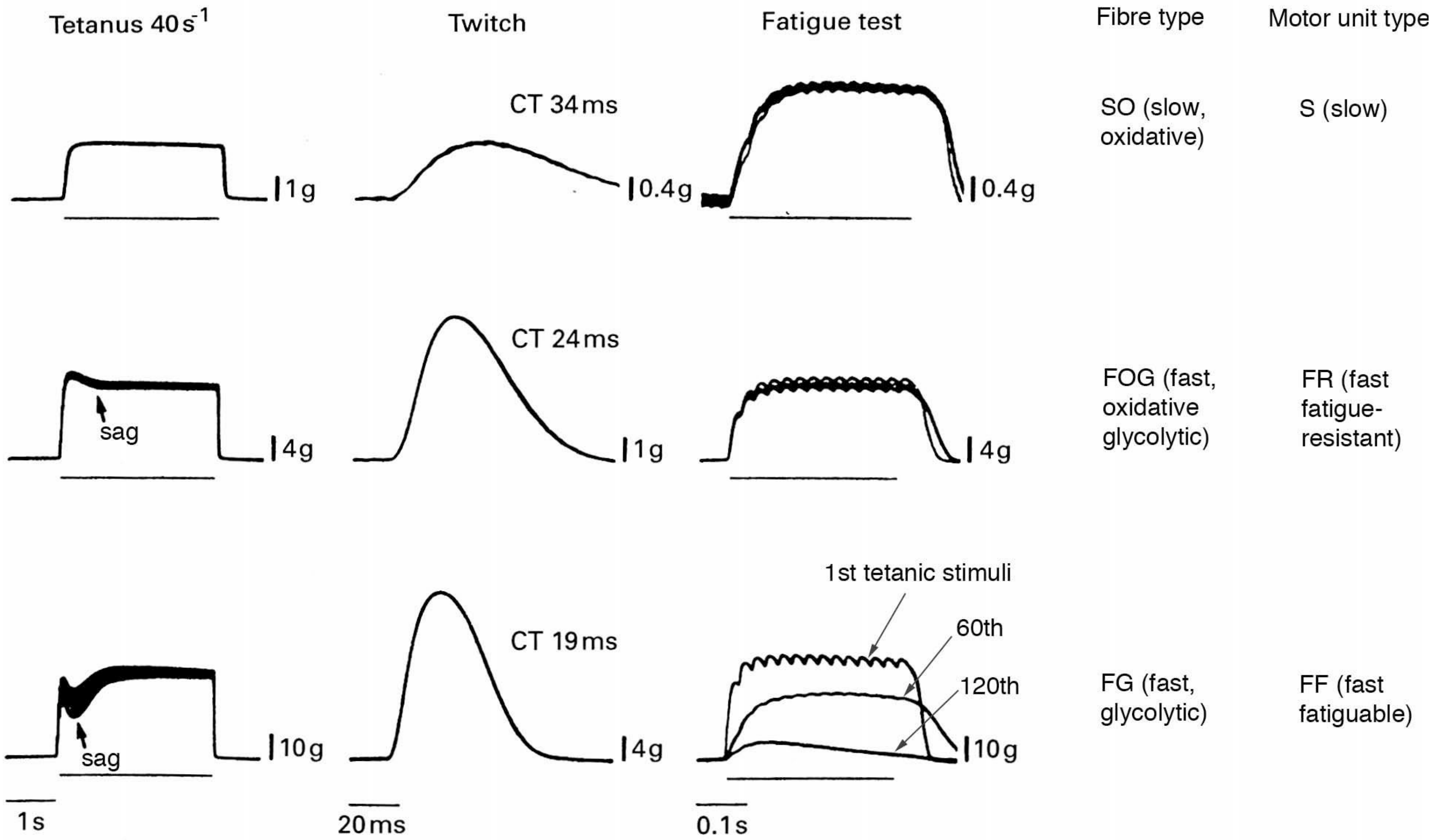


Size Principle

- When a stimulus is applied to the ventral aspect of the spinal cord, the smallest and most excitable motor units are activated first. These tend to be slow (S) motor units which innervate slow oxidative (SO) muscle fibers. Larger FR and FF motor units that innervate FOG and FG fibers are recruited only at high levels of force
- Sequence is reversed when force level falls, with largest motor units dropping out first

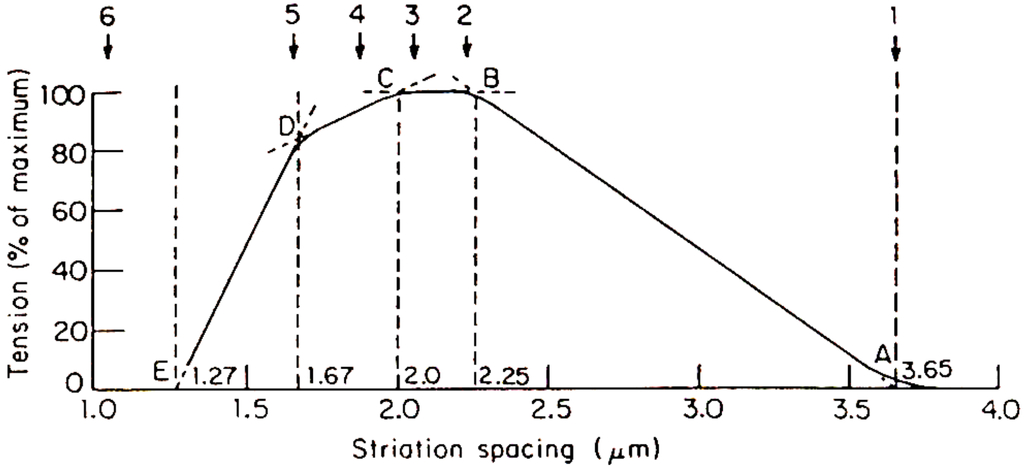


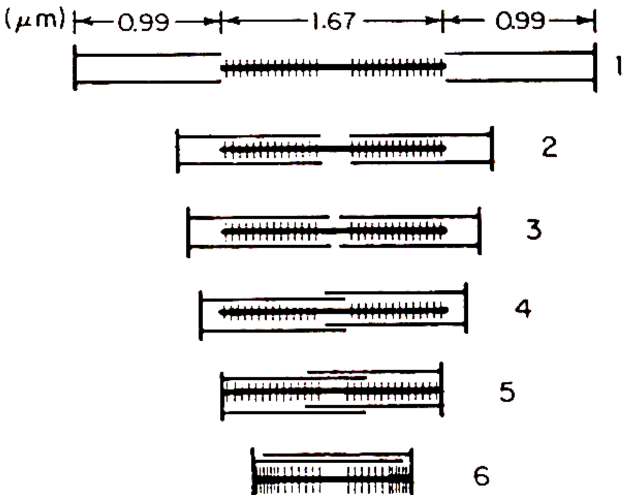
Three Types of Muscle Fibers and Motor Units



Active Force Development in the Sarcomere Depends on Actin-Myosin Overlap

- (A): no overlap between actin and myosin, zero developed tension
- Between (A) and (B): tension increases linearly as overlap increases
- Between (B) and (C): maximum overlap & maximum tension
- Left of (C): interference between actin filaments reduces ability of crossbridges to develop tension
- Left of (D): myosin filaments collide with Z-lines and fold, and force declines rapidly

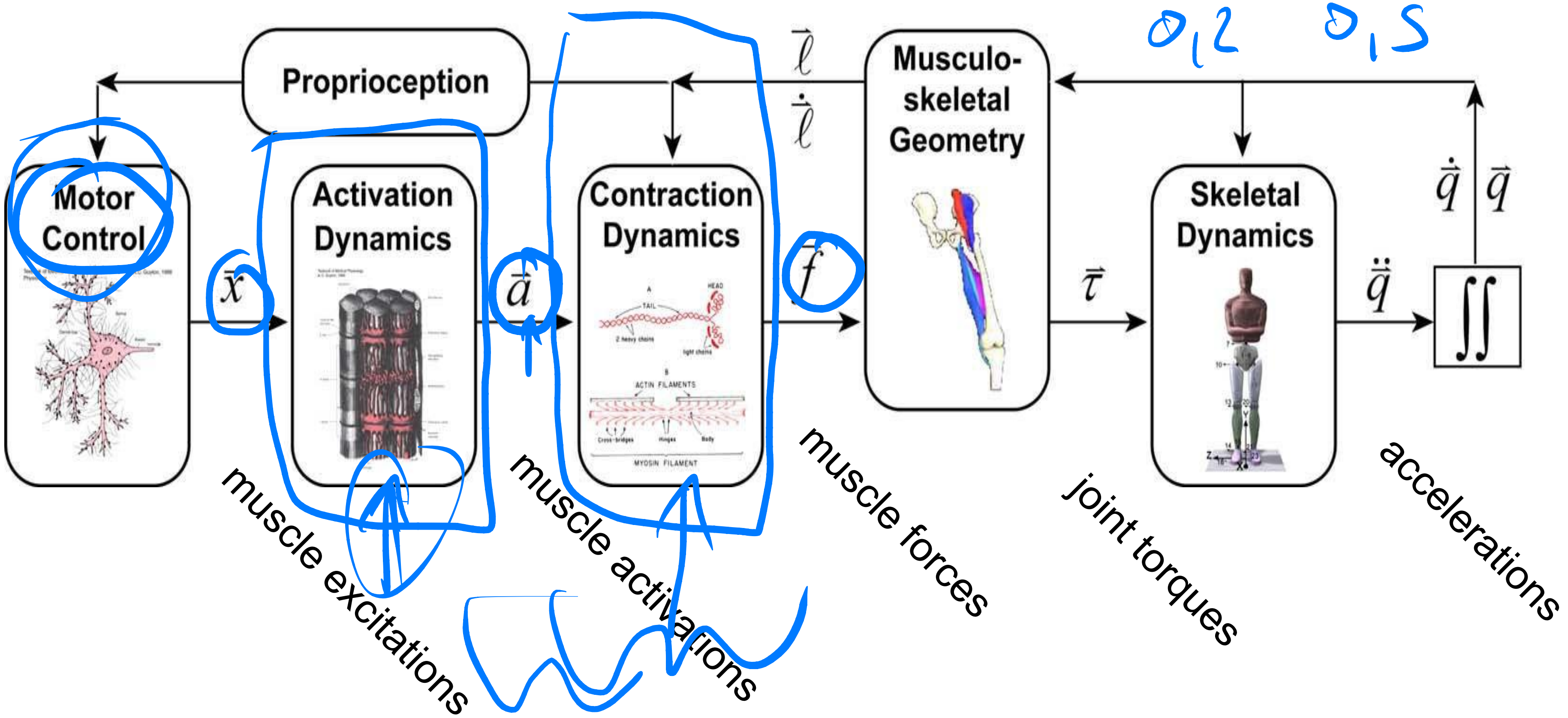




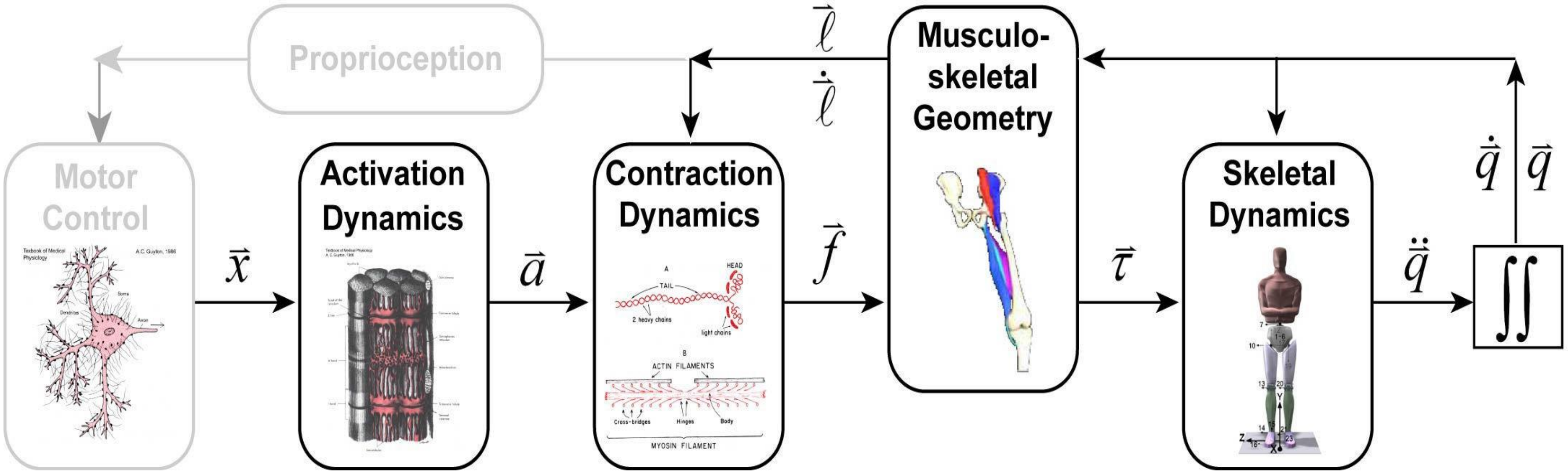
How Movement is Generated/Simulated?

$F = \alpha F_0$

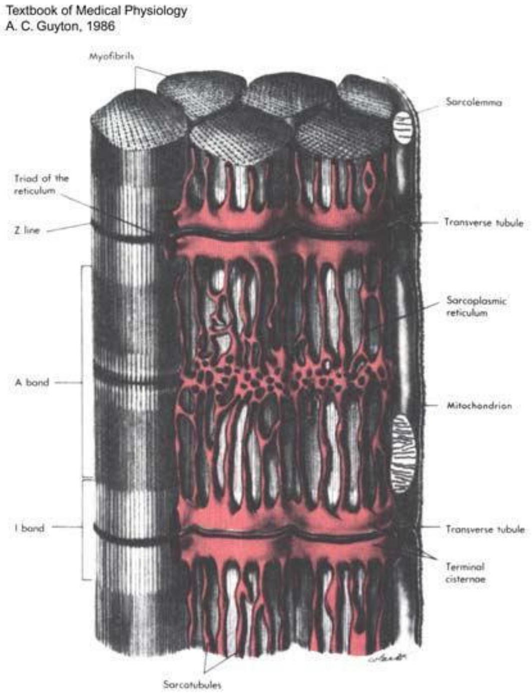
0,2 0,5

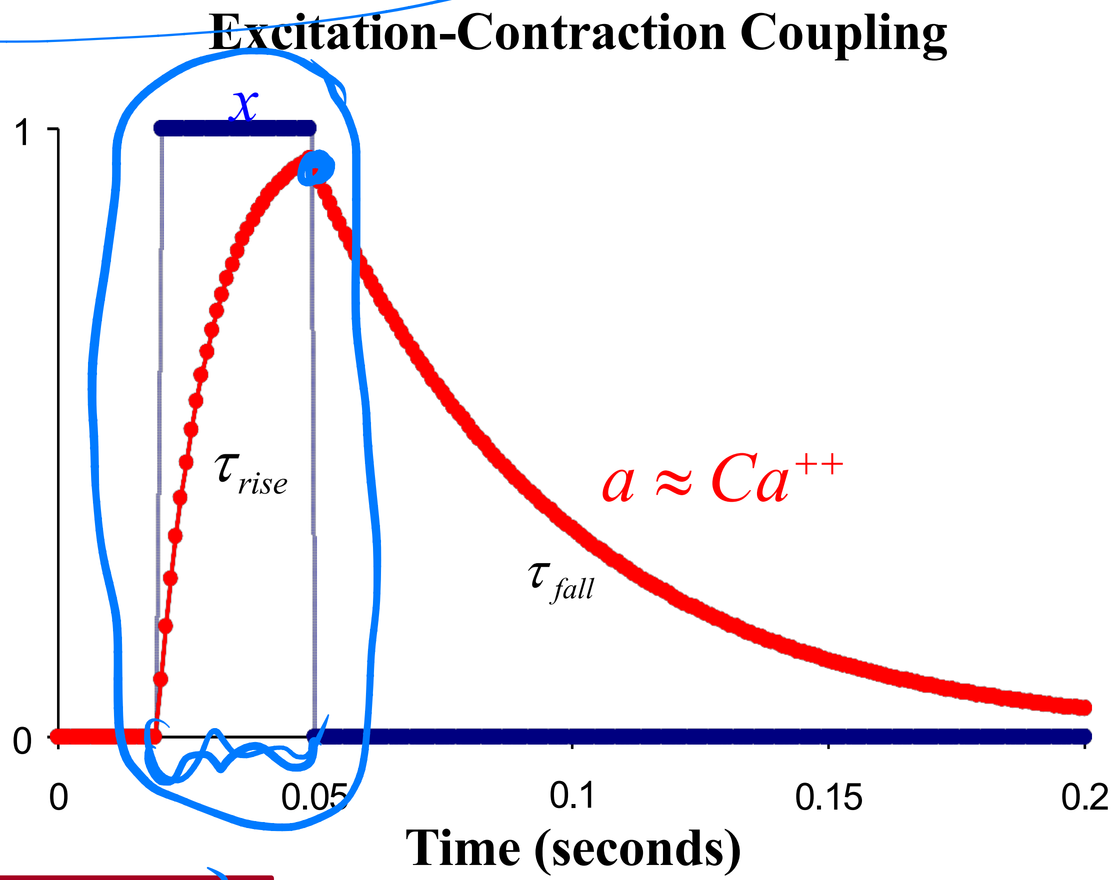


Muscle Activation Dynamics



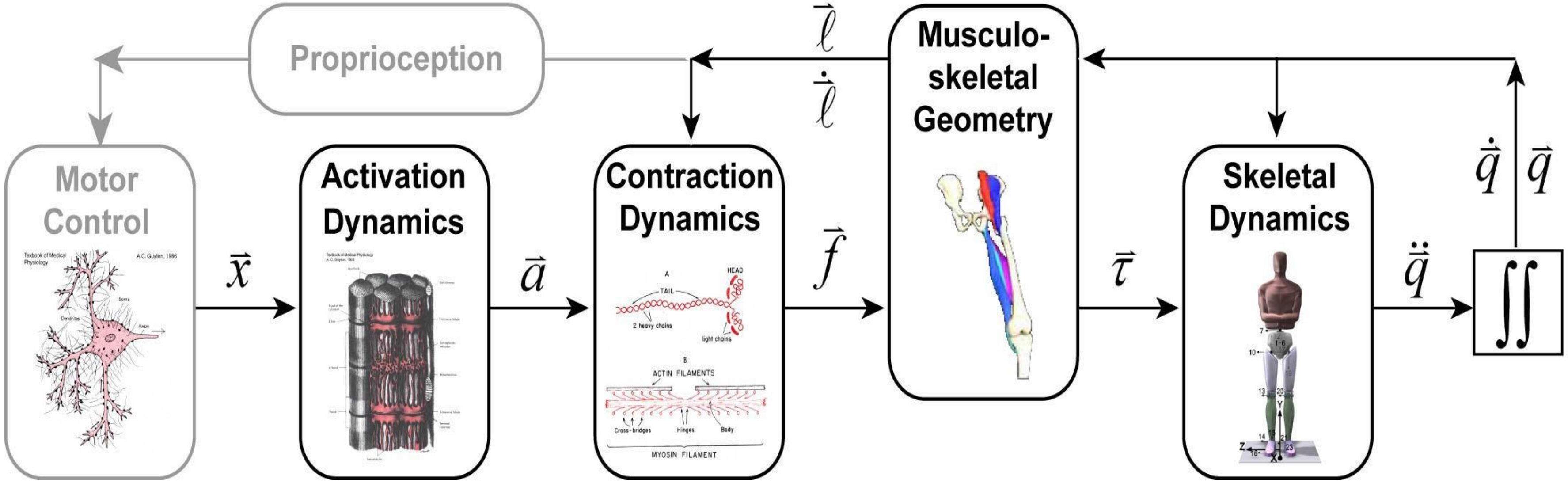
Muscle Activation Dynamics





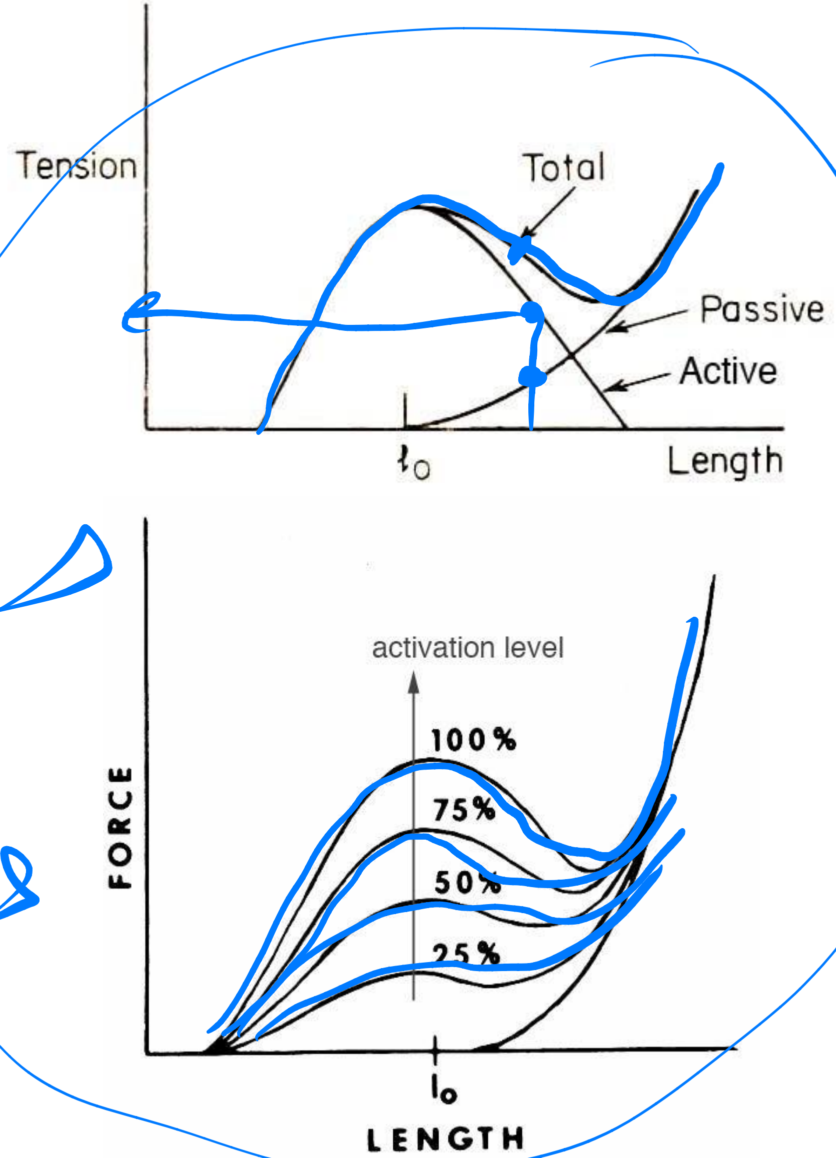
$$\boxed{\frac{d\,a}{dt} = \frac{x^2 - xa}{\tau_{rise}} + \frac{x - a}{\tau_{fall}}}$$

Muscle Contraction Dynamics



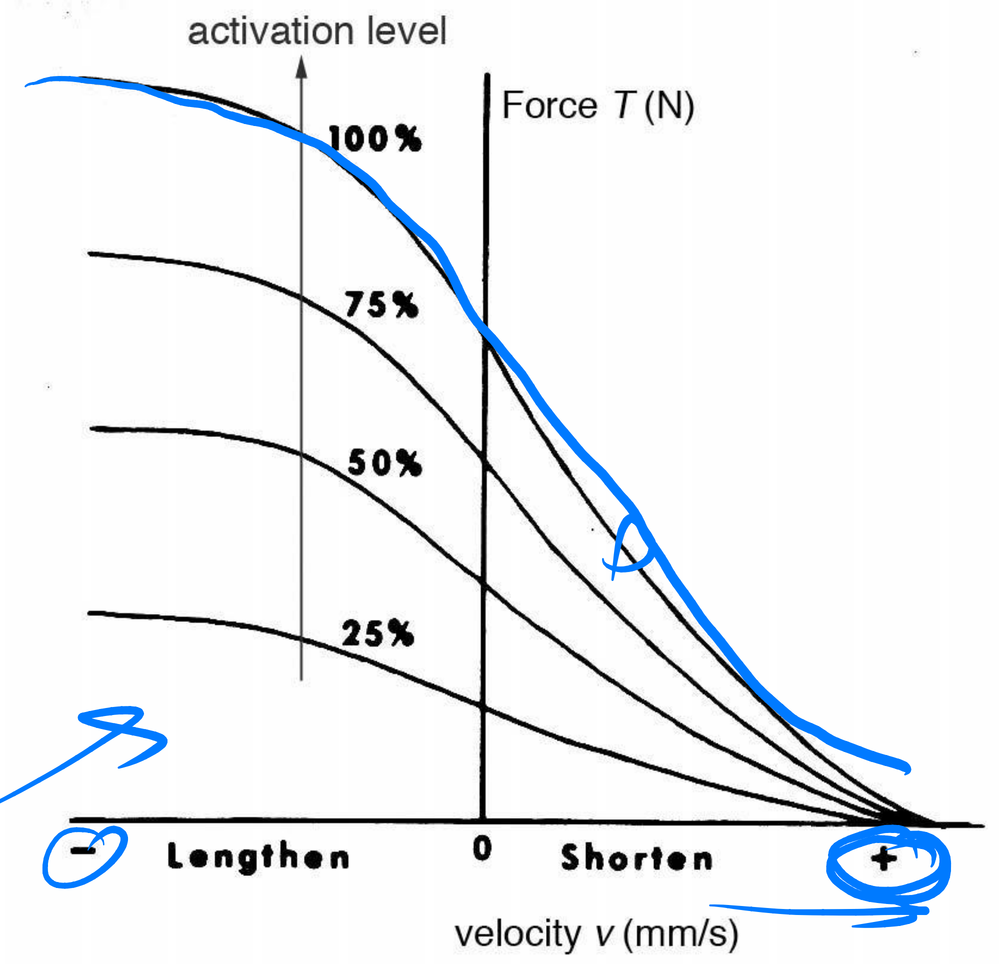
Muscle Length Affects Force Development

- The tension developed in a whole muscle is the sum of active force due to muscle contraction and passive force due the passive stiffness of tendon and muscle
- The passive force is negligible for lengths less that the normal resting length (l_0)
- The active force follows the tension-length behavior of the sarcomere, and scales with muscle activation



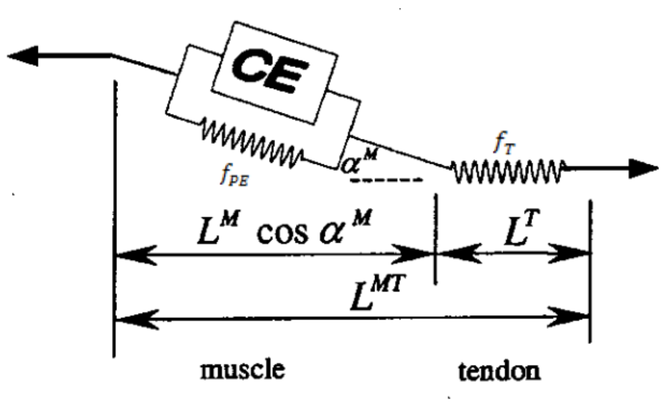
Muscle Velocity Affects Force Development

- Force (T) is greater during lengthening than shortening contractions
- The greater the shortening velocity (v), the smaller the force (explains why we cannot lift heavy objects quickly)
- In the shortening regime, mechanical power output is maximum when T and v are around one-third their maximum values



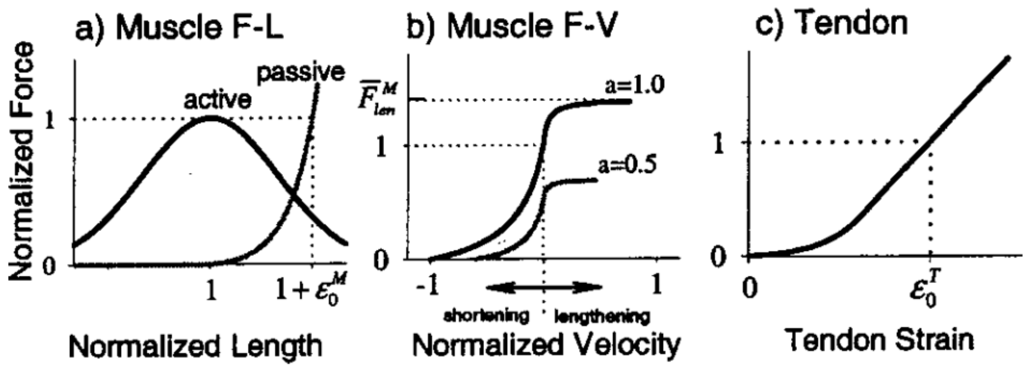
Muscle Contraction Dynamics

- Contractile element (active force)
- Parallel element (fibers passive forces)
- Series tendon (transmits muscle force)



$$F^{CE} = a(t) \cdot F_o^M \cdot \tilde{F}_L^M(L^M) \cdot \tilde{F}_V^M(\dot{L}^M)$$

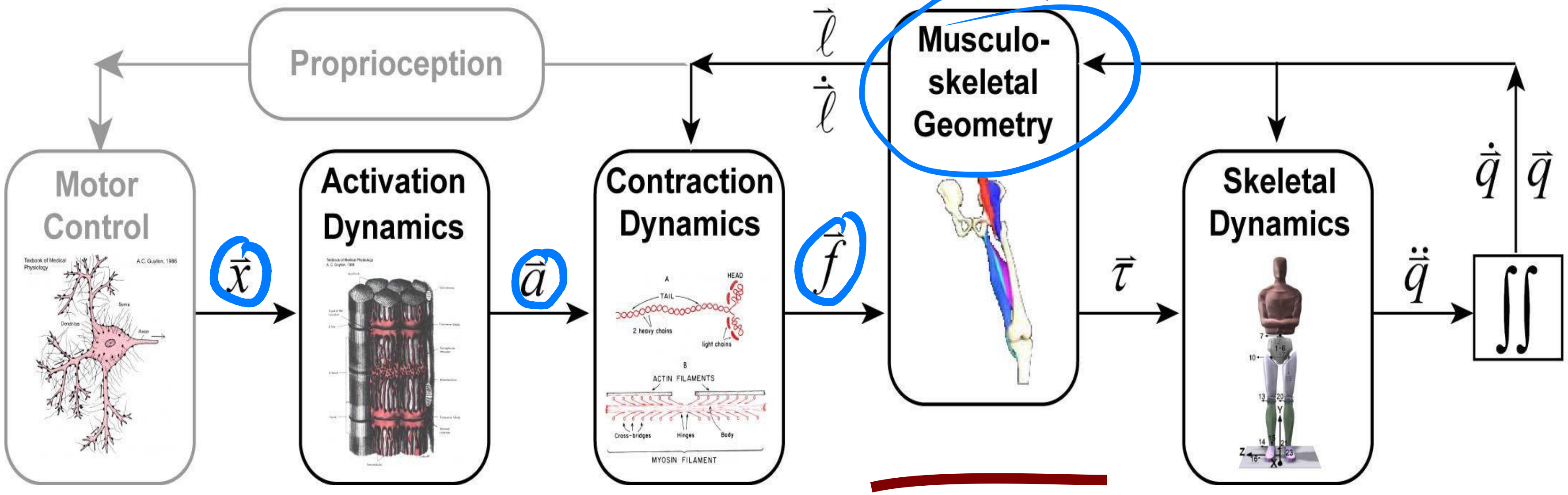
$$F^T = (F^{CE} + F^{PE}) \cdot \cos\alpha$$



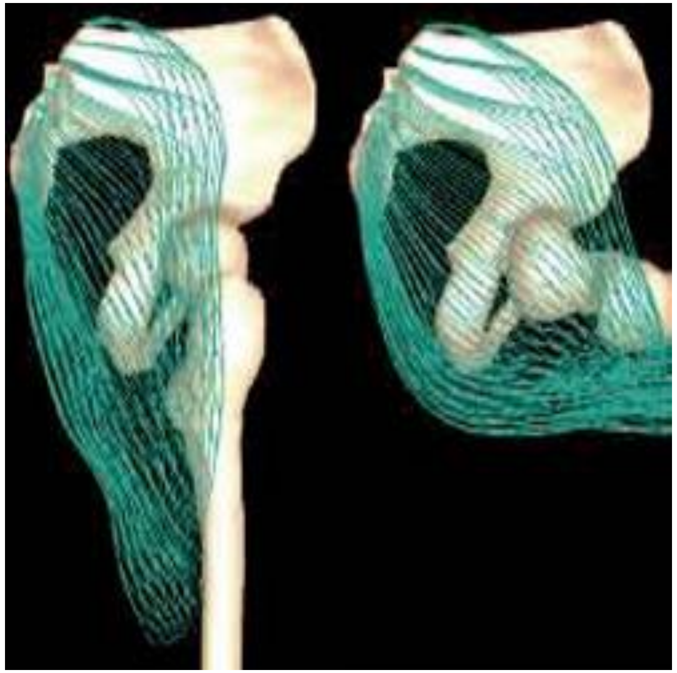
Musculoskeletal Geometry

$F \quad \alpha$

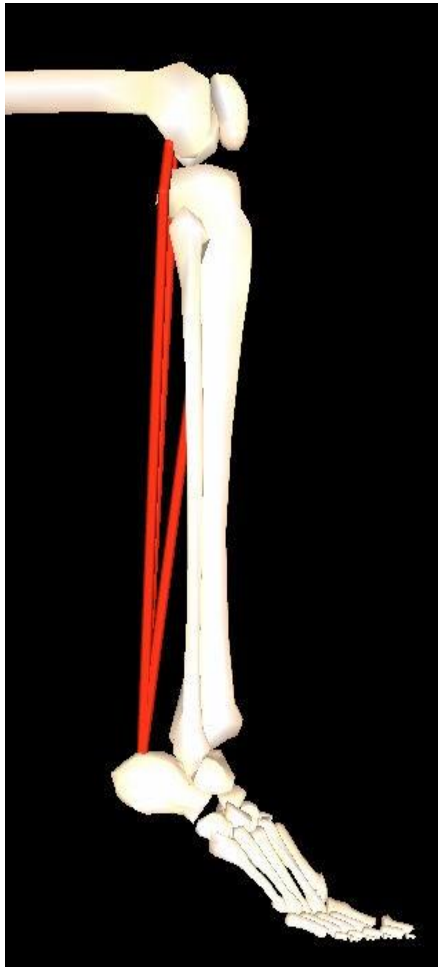
$\tau = F \cdot \alpha$



Musculoskeletal Geometry

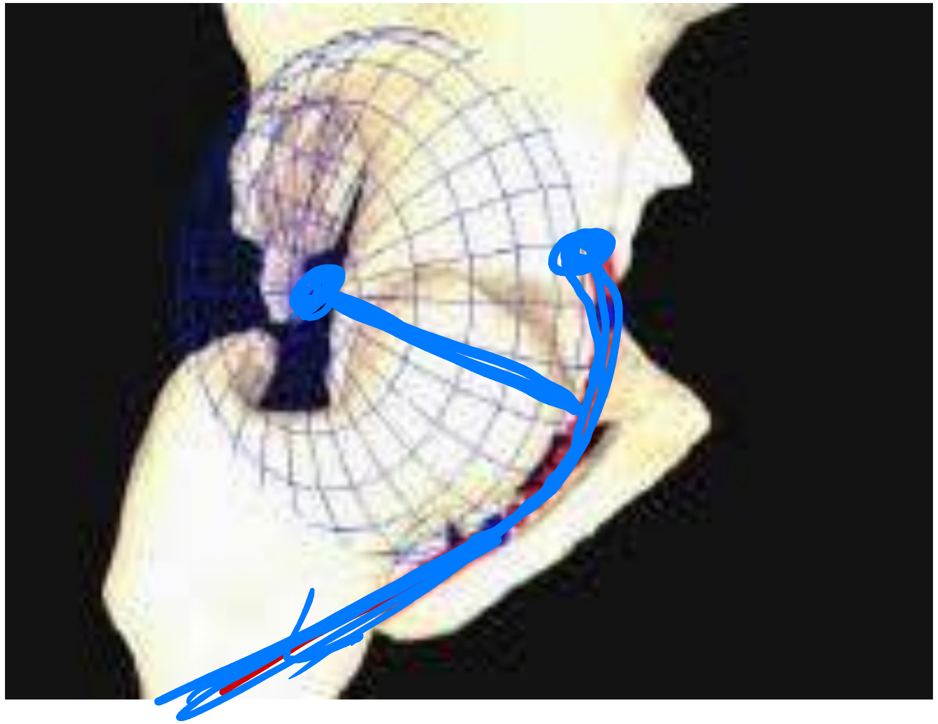


Muscle Volume



Straight Lines

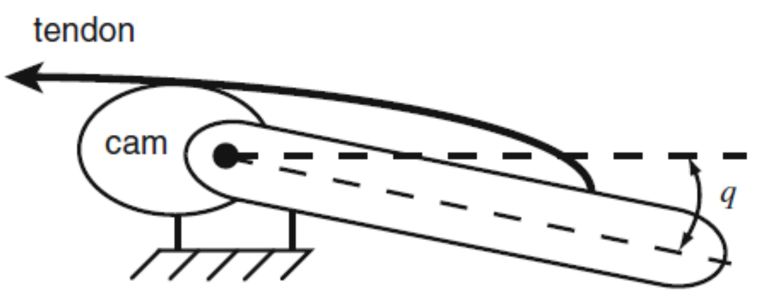
Wrapping

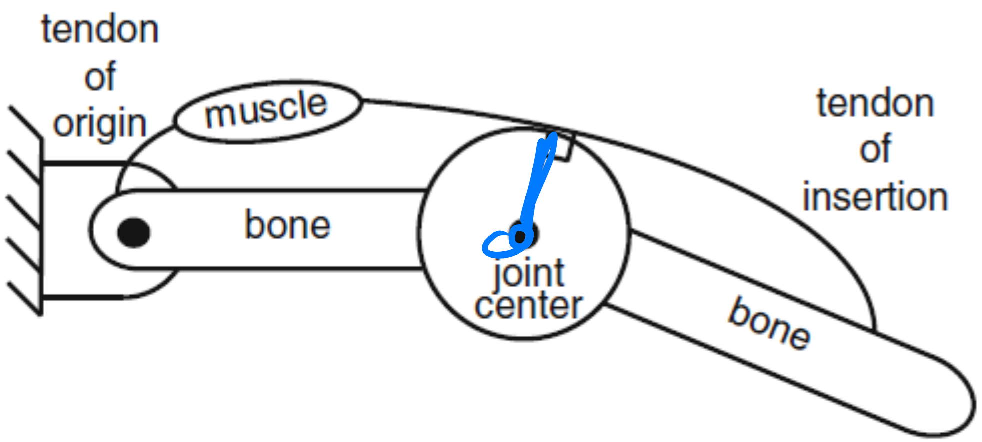


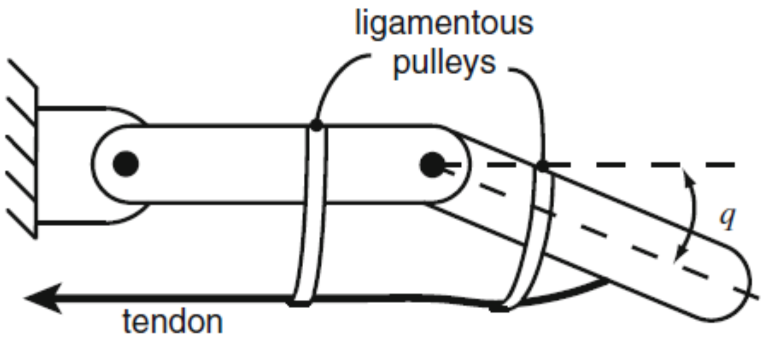
What is Moment Arm?

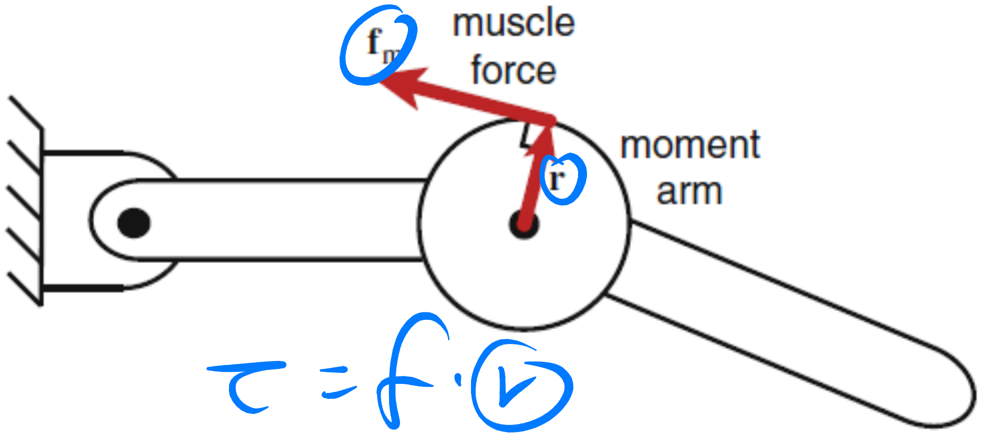
$$\tau = r\ x\ f_m$$

- Relates the muscle force to the torque developed at a joint





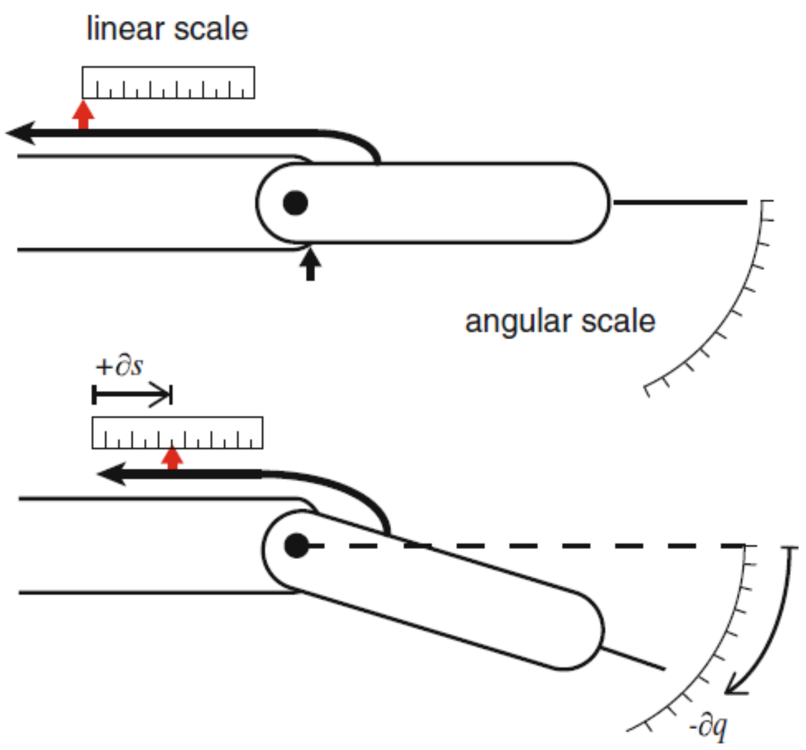


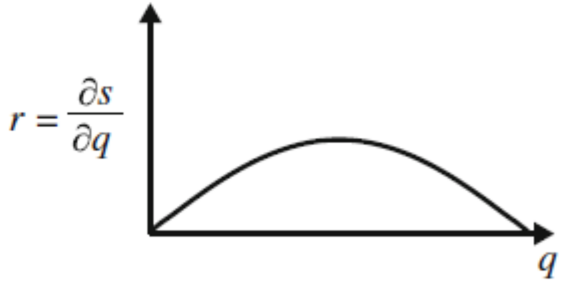


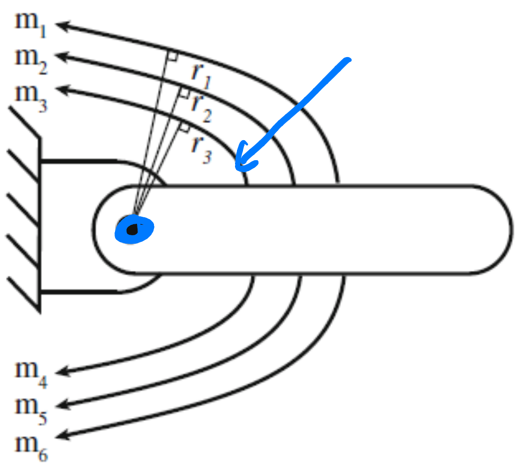
How to Measure Moment Arm?

- Measure muscle length for changes in the joint angle

$$r = \frac{ds}{dq}$$

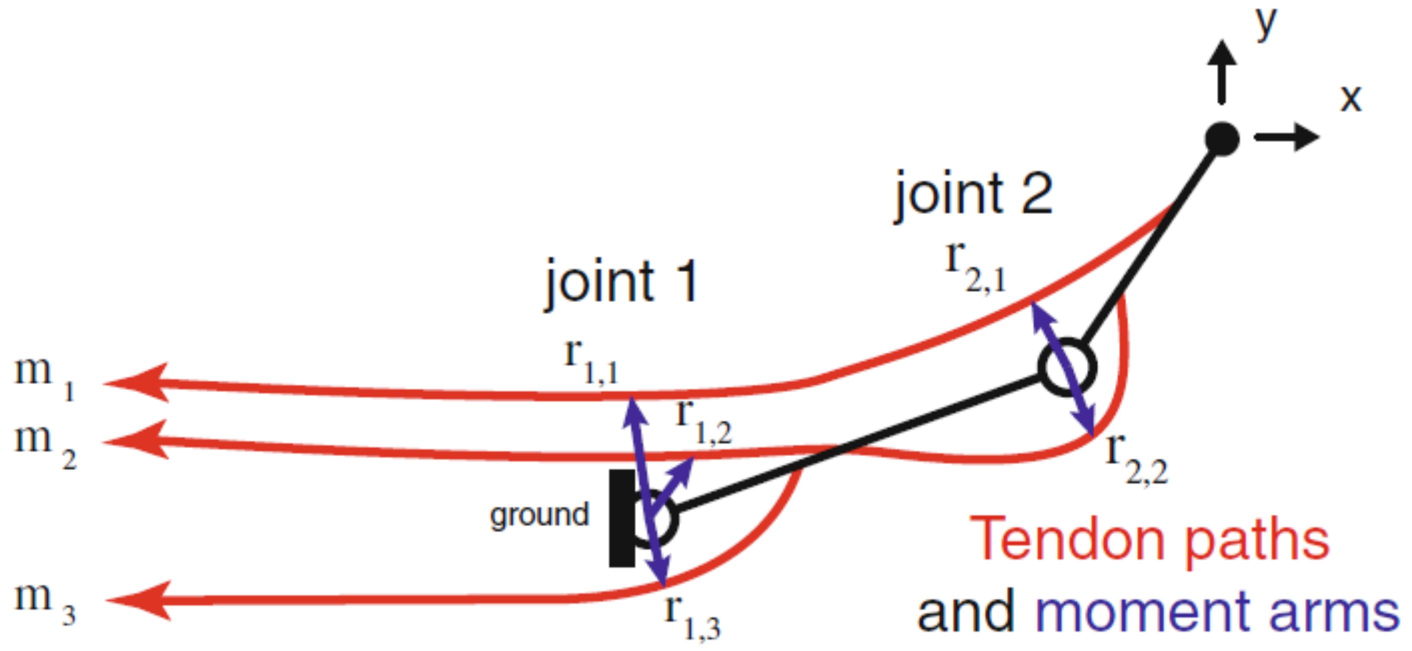




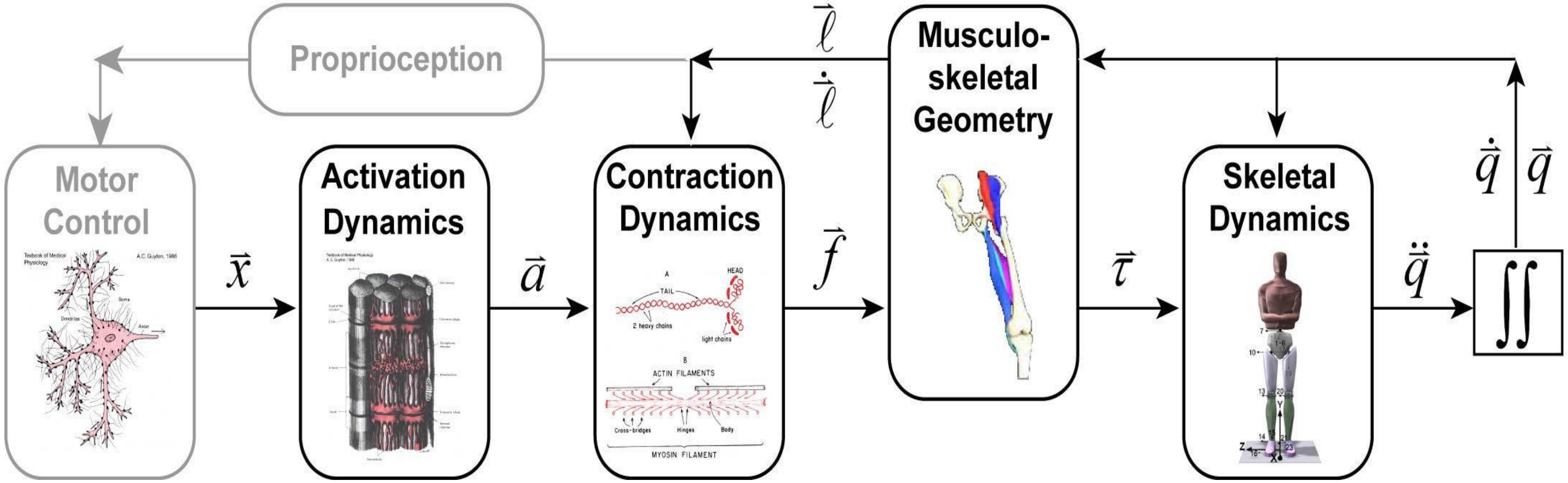


Muscles Can Spann Multiple Joints

- A muscle can contribute to the acceleration of multiple joints (How?)
- The moment arm of a muscle can depend on multiple joint angles (Why?)

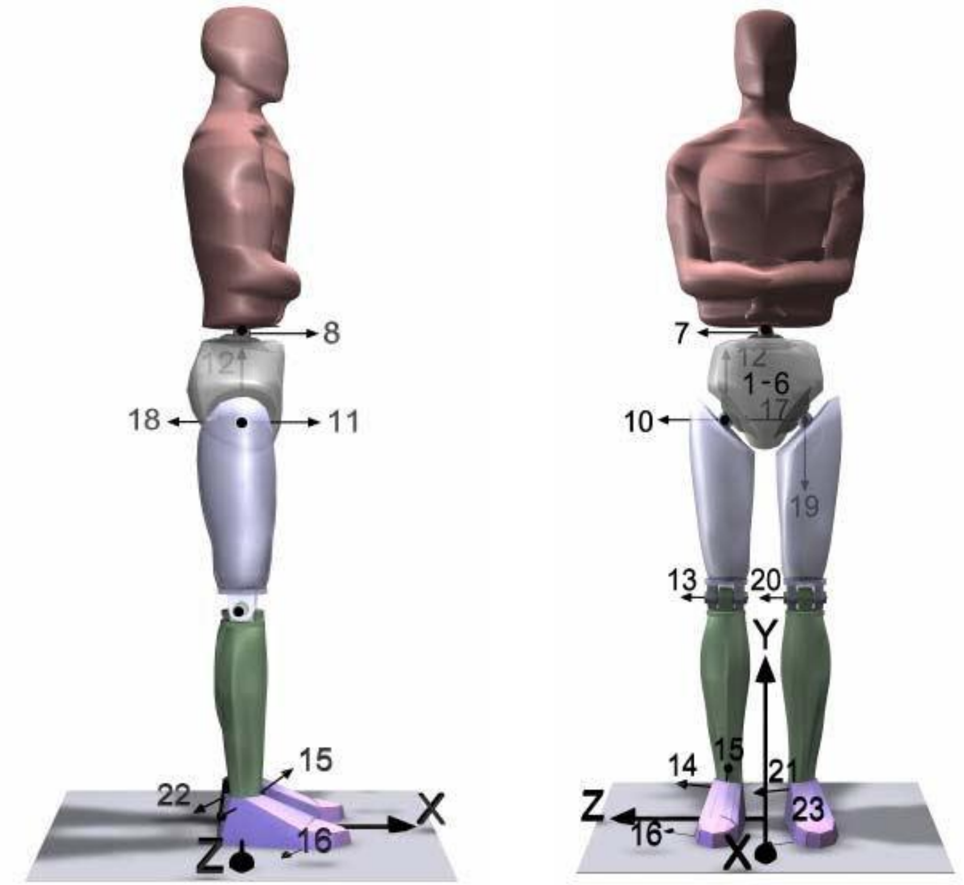


Skeletal Dynamics



Musculoskeletal Dynamics

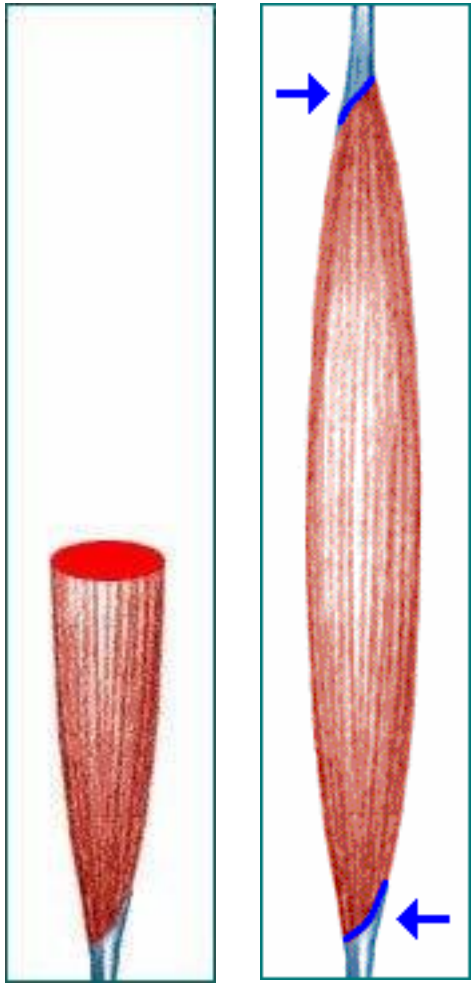
- 10 Rigid Bodies
  - mass
  - center of mass
  - inertia tensor
- 10 Joints
- 23 Degrees of Freedom



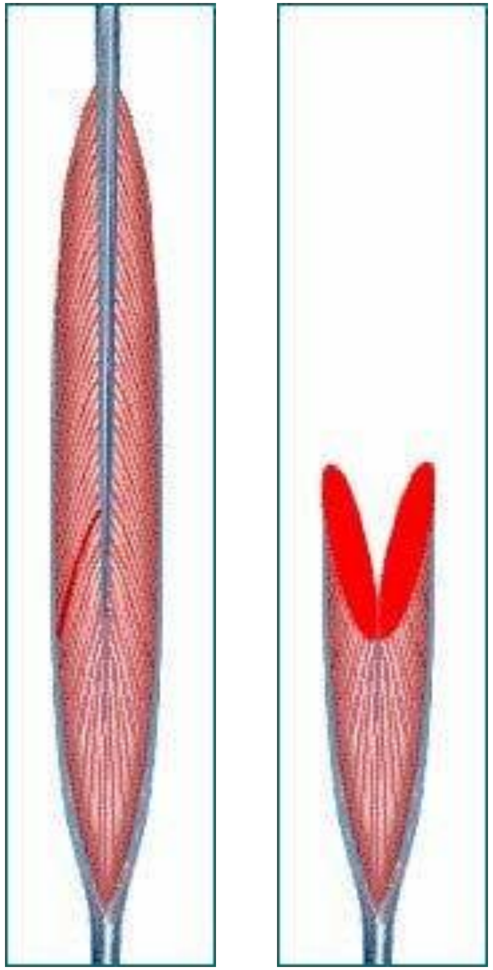
$$M(q)\ddot{q} = \tau + V(q,\dot{q}) + G(q) + F(q,\dot{q}) \rightarrow$$
$$M(q)\ddot{q} = R(q)F_M + V(q,\dot{q}) + G(q) + F(q,\dot{q})$$

M: $mass\ matrix$, V: $Coriolis\ and\ centrifugal$ torques,
G: gravity torques, F: other forces (ligaments), τ: active forces (muscles)
R: moment arm matrix, FM a vector of muscle forces

Muscle Architecture



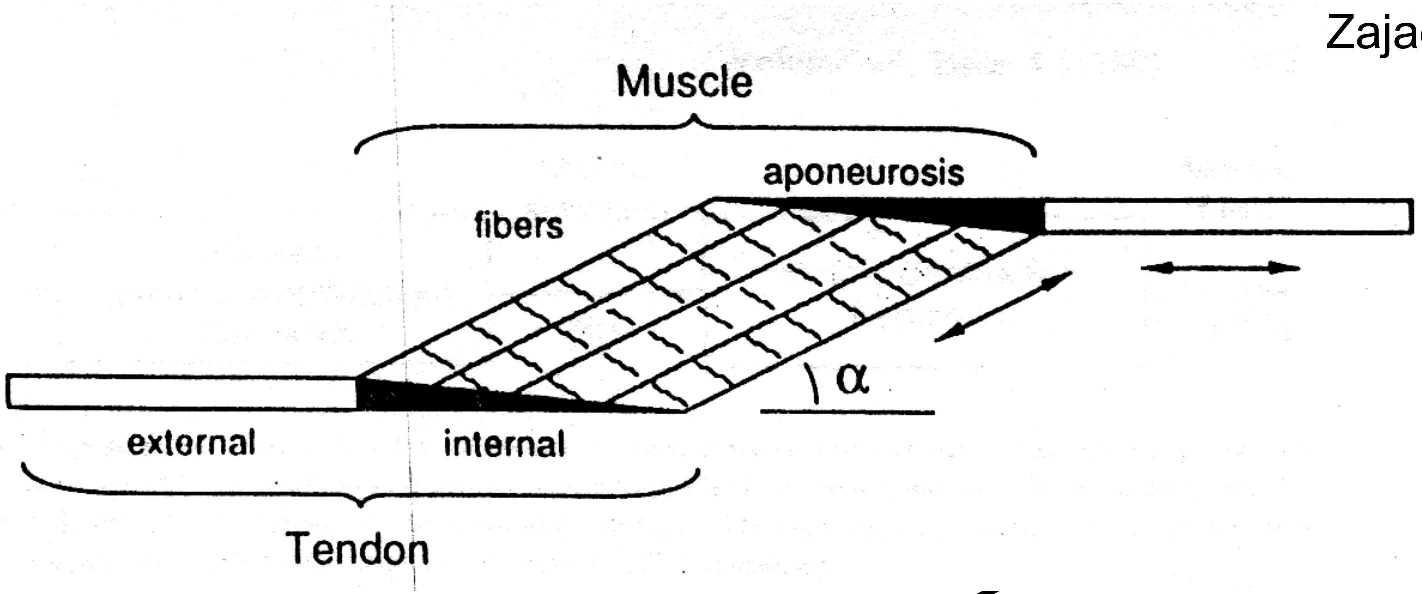
Parallel Fibered

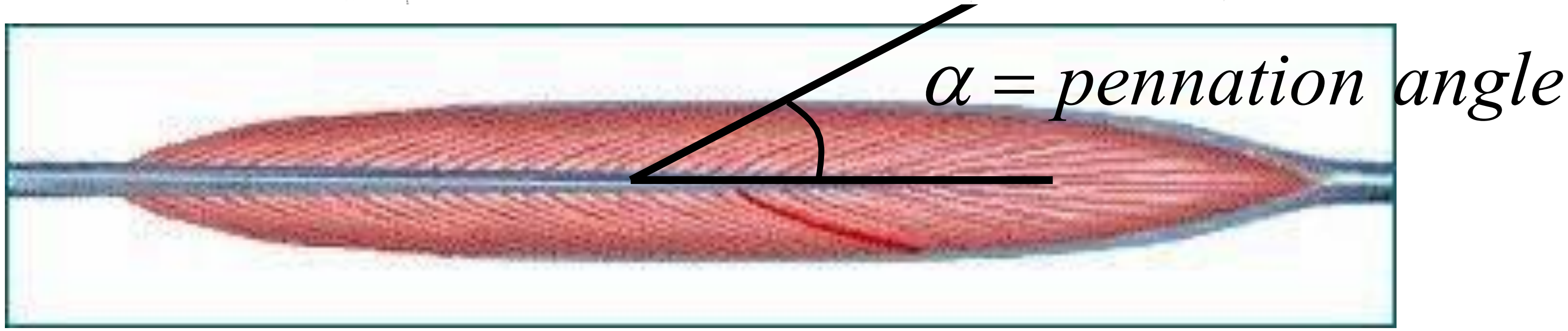


Pennate Muscle

Muscle, Tendon and Pennation Angle

Zajac, 1989

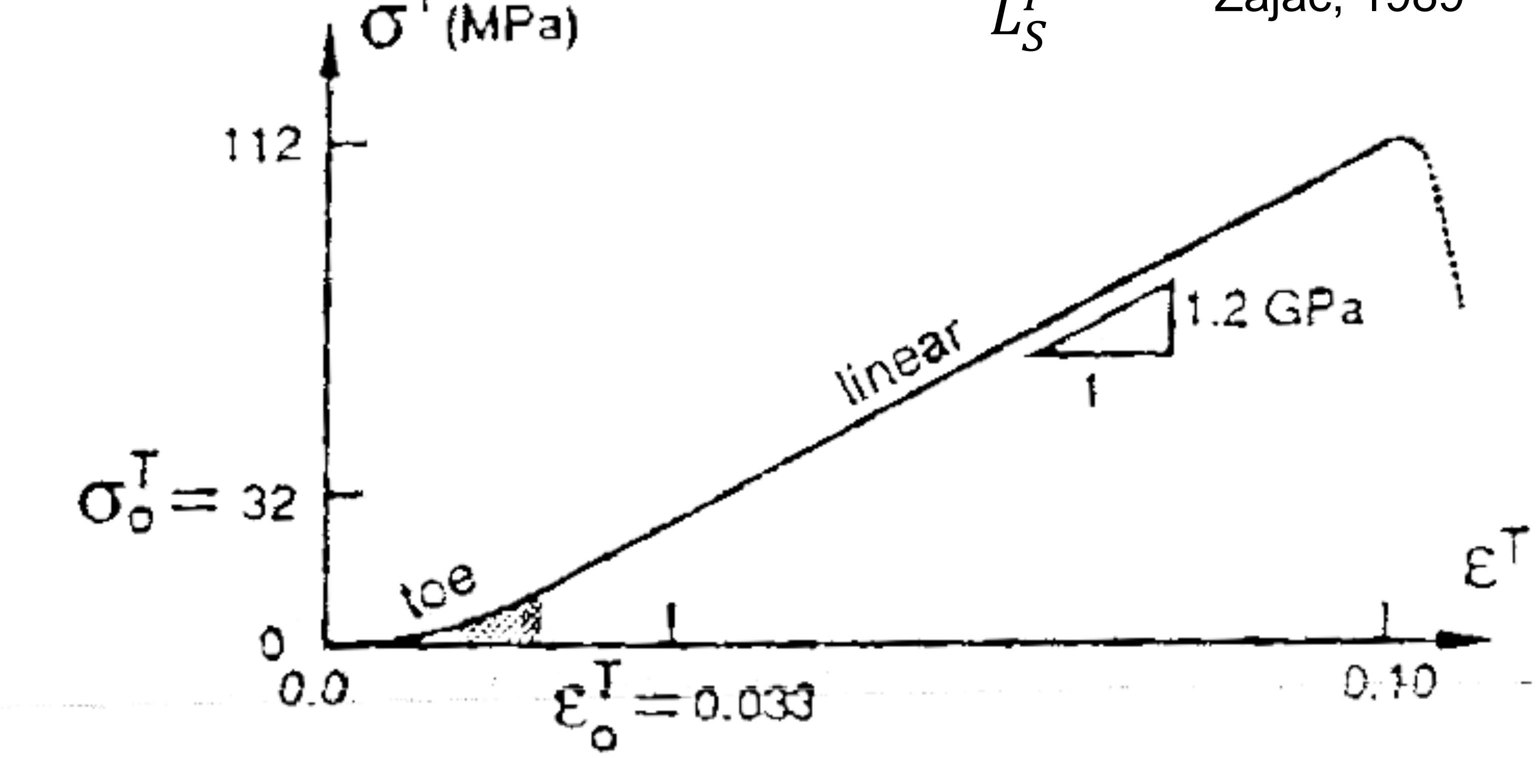




Tendon Stress-Strain Properties

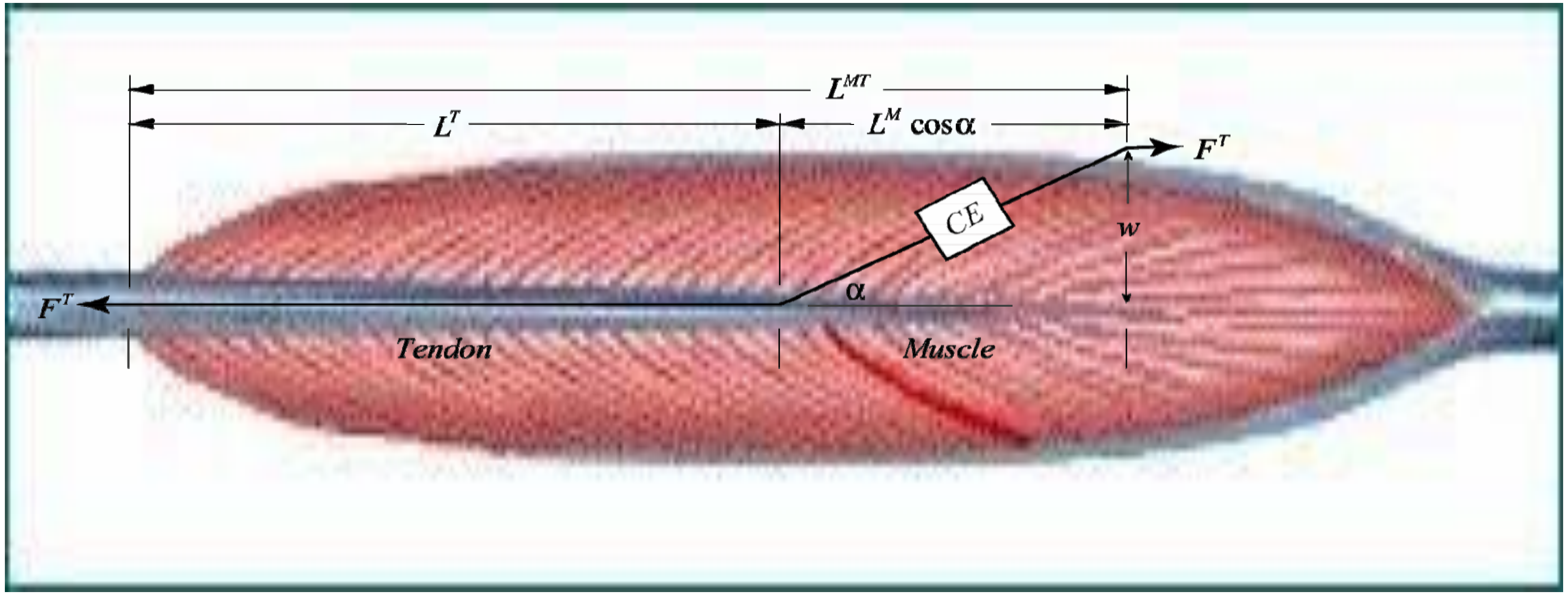
$$\varepsilon^{\mathrm{T}} = \frac{\mathrm{L}^{T} - L_S^T}{L_S^T}$$

Zajac, 1989

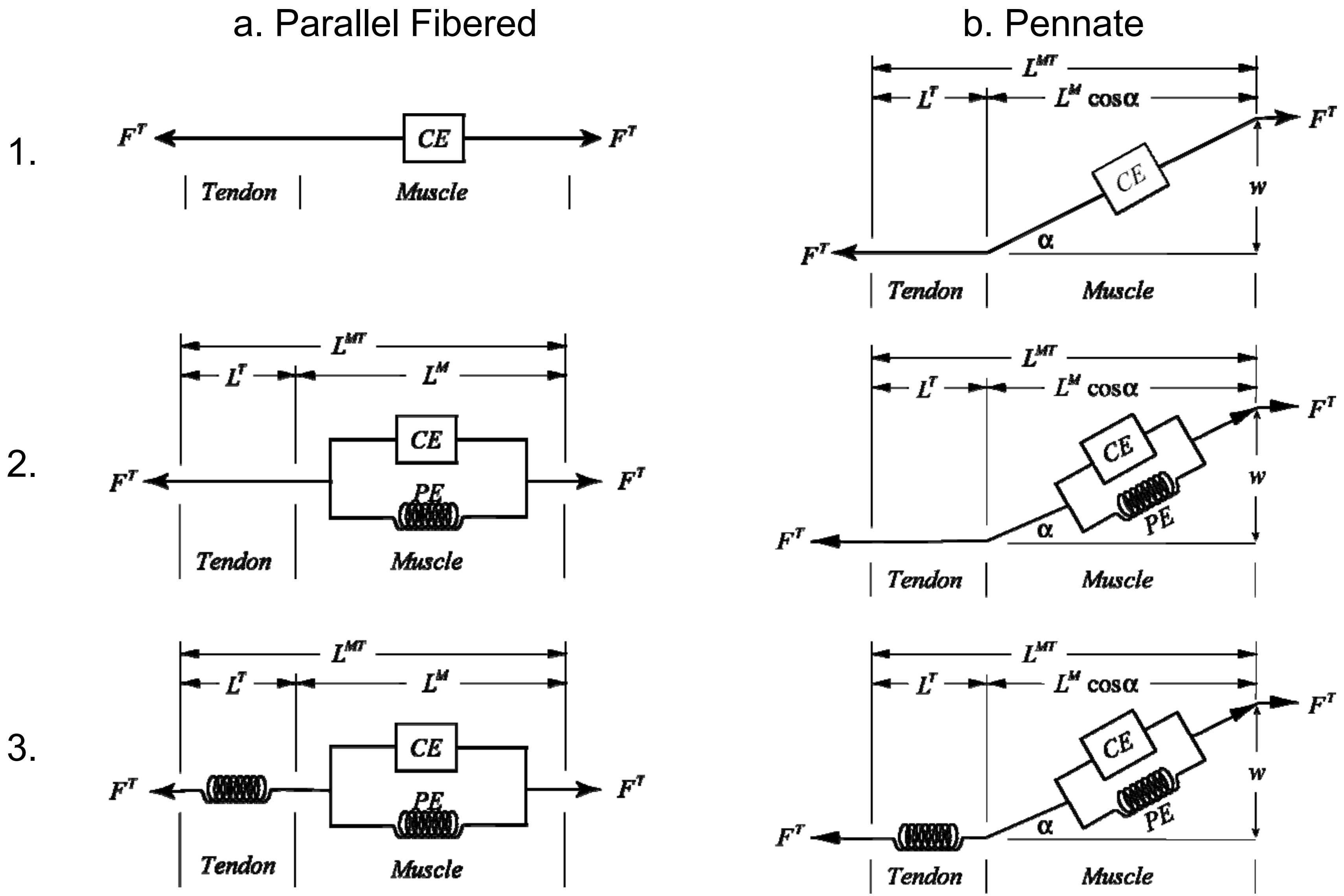


Lumped-Parameter Model

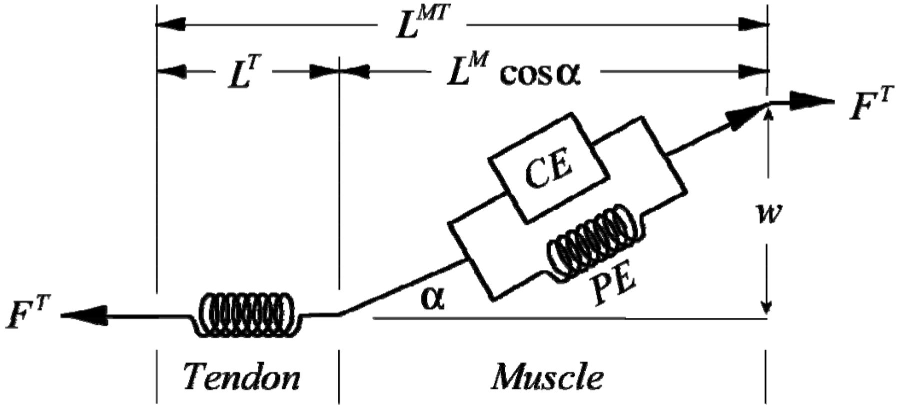
- The distributed properties of all muscle fibers are lumped into a single ideal fiber characterized by parameters appropriate for the whole muscle.
  - All fibers are the same length, at the same pennation angle, etc
  - The strength of the muscle is the summed strength of the individual fibers



Muscle Models



Definitions



L = Length
F = Force

T = Tendon
M = Muscle
α = Pennation angle

L^T = Length of tendon
L^M = Length of muscle
L^{MT} = Length of actuator

CE = Contractile Element.
Models the active force generating properties of muscle.

PE = Parallel Elastic Element
Models the passive force generating properties of muscle.

Five Parameters

F_o^M Optimal muscle force.
Maximum isometric strength of muscle.

$$F_o^M = \sigma \frac{Volume}{L_o^M}$$

L_o^M Optimal muscle fiber length
Length at which F_o^M is generated

α_o Optimal pennation angle
Pennation angle when the fibers are at L_o^M

V_{max}^M Maximum shortening velocity of muscle
normalized by fiber length

$$\underset{slow\ twitch}{2.0 \cdot L_o^M} < V_{max}^M < \underset{fast\ twitch}{10 \cdot L_o^M}$$

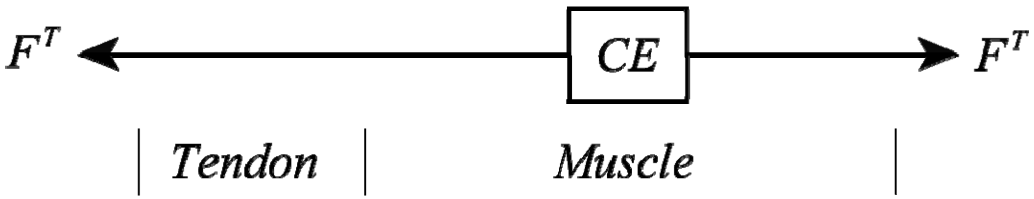
L_s^T Slack length of tendon.
Length at which tendon starts
to develop force

$$L_s^T = L_{external}^T + L_{internal}^T$$

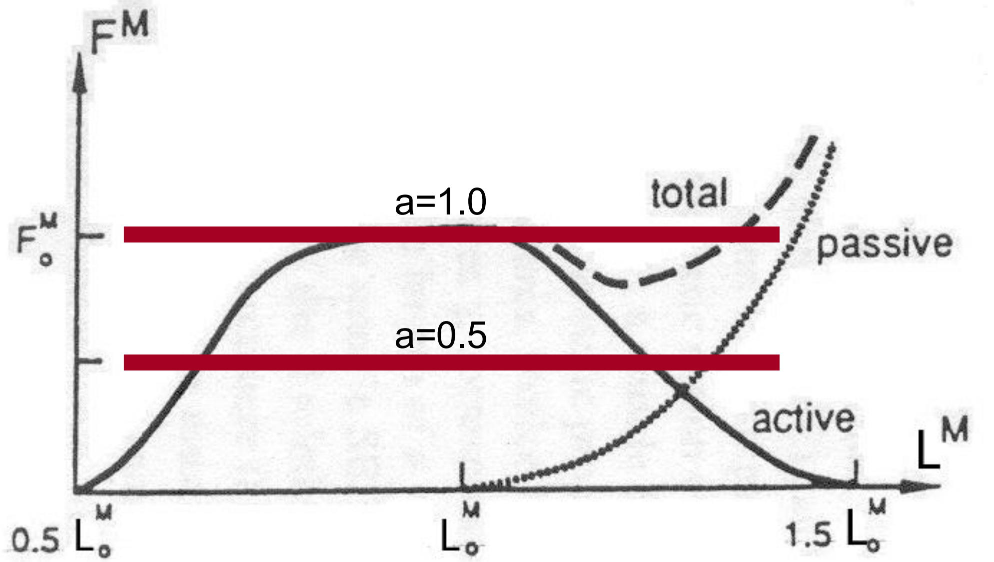
Simplest

- Assumptions
  - Tendon is inelastic
  - No dependence on length or velocity
  - Parallel fibered
- Parameters
  F_o^M
- Time-varying inputs
  a

$$0\ (off) \leq\ a\ \leq\ 1.0\ (fully\ on)$$

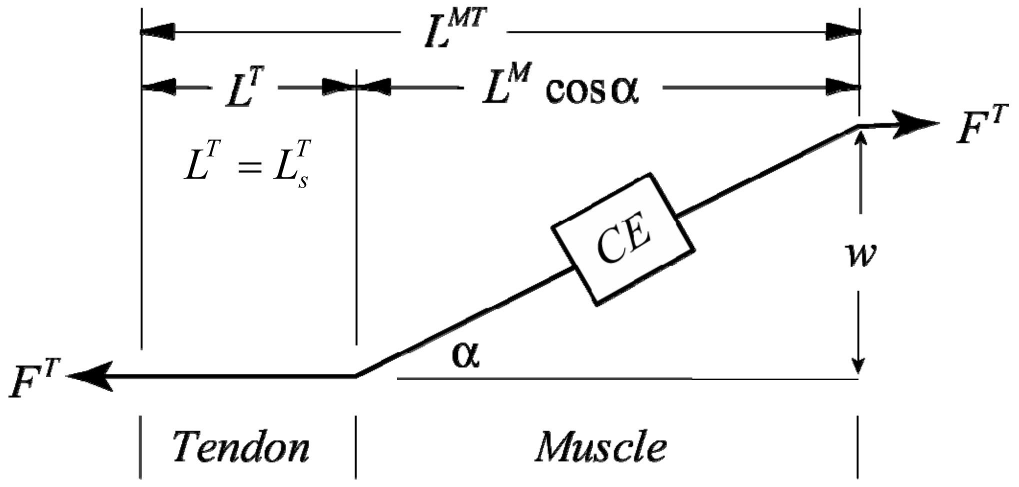


$$F^T = F^{CE} = a(t) \cdot F_o^M$$



Simplest with Pannate Fibers

- Assumptions
  - Tendon is inelastic
  - No dependence on length or velocity
  - Pennate
- Parameters
  $F_o^M \quad L_o^M \quad \alpha_o \quad L_s^T$
- Time-varying inputs
  $a \quad L^{MT}$



$$F^T = F^{CE} \cdot \cos\alpha = a(t) \cdot F_o^M \cdot \cos\alpha$$

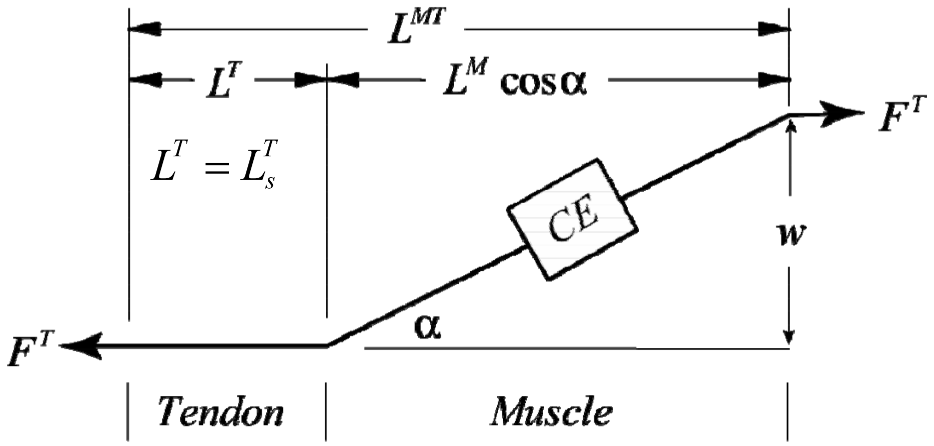
But, α changes with the length of the muscle!

Simplest with Pannate Fibers

$$L^{MT} = L_s^T + L^M \cos\alpha$$

$$w = L_o^M \sin\alpha_o$$

Width is assumed to be constant.



Some algebra and trig…

$$\cos\alpha = \sqrt{\frac{\left(\frac{L^{MT} - L_s^T}{w}\right)^2}{1 + \left(\frac{L^{MT} - L_s^T}{w}\right)^2}}$$

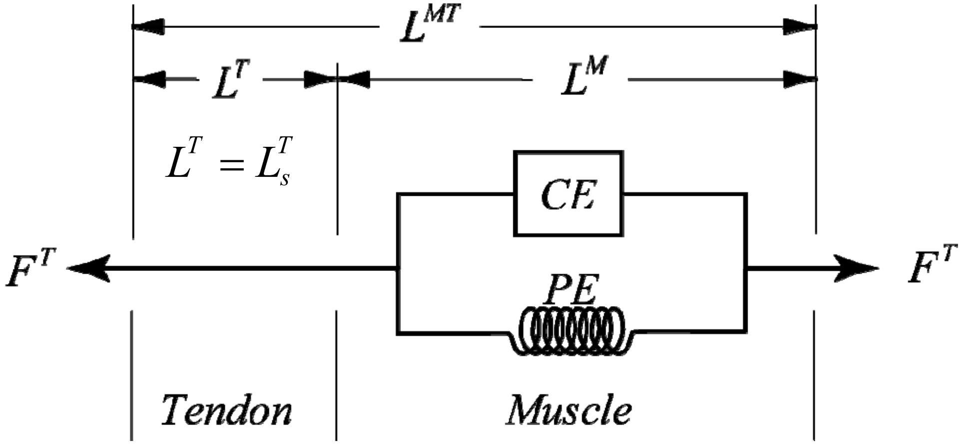
$$F^T = F^{CE} \cdot \cos\alpha = a(t) \cdot F_o^M \cdot \cos\alpha$$

Force-Length-Velocity Properties and Inelastic Tendon

- Assumptions
  - Tendon is inelastic
  - Dependence on length or velocity
  - Parallel fibered
- Parameters

  $F_o^M \; L_o^M \; L_S^T \; V_{max}$
- Time-varying inputs

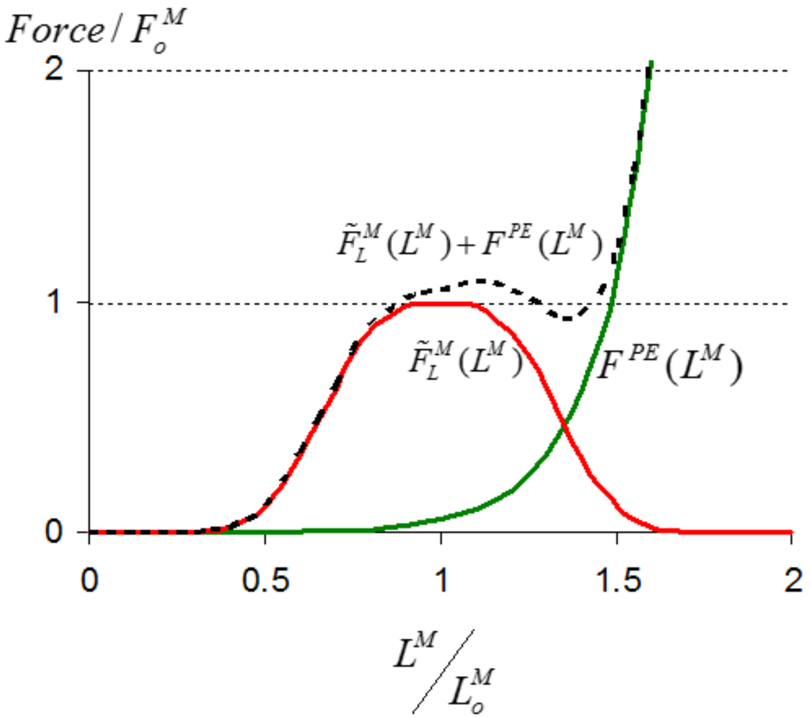
  $a \;\; L^{MT} \; \dot{L}^{MT}$

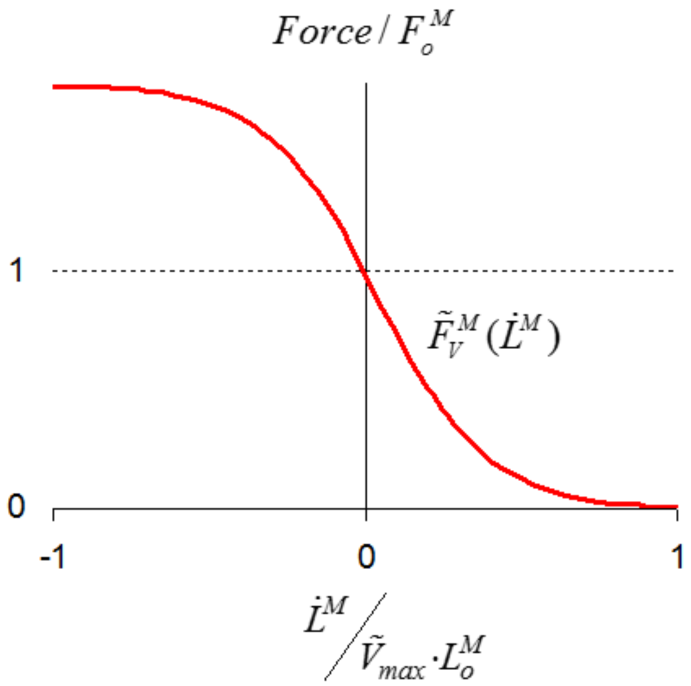


$$F^T = F^{CE} + F^{PE}$$

$$F^{CE} = a(t) \cdot F_o^M \cdot \tilde{F}_L^M(L^M) \cdot \tilde{F}_V^M(\dot{L}^M)$$

Length, Velocity and Passive Forces





$$F^{PE} = F_o^M \cdot 3 \cdot 10^4 \cdot \exp[6 \cdot (\frac{L^M}{L_o^M} - 3.2)]$$

$$\tilde{F}_L^M(L^M) = \exp[\ 17.33 \cdot \left| \frac{L^M}{L_o^M} - 1.0 \right|^3\ ]$$

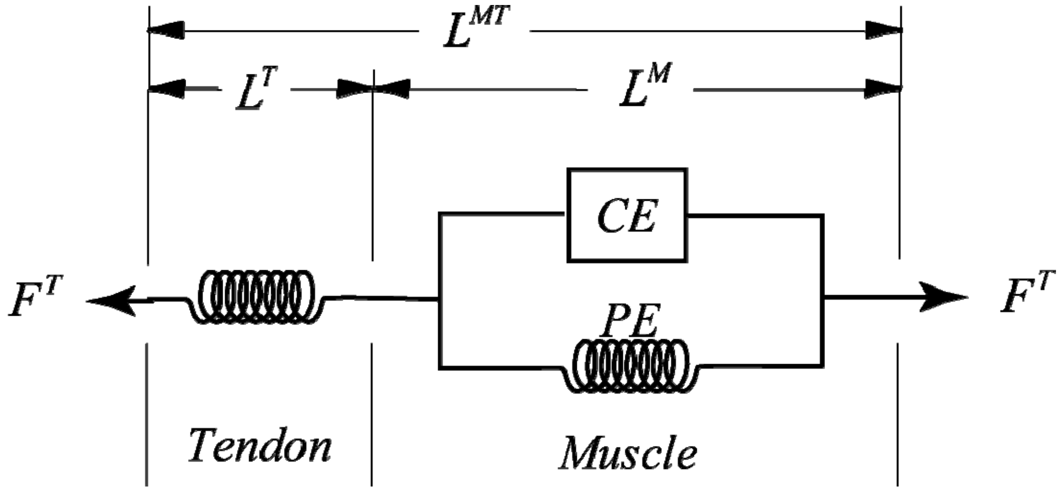
$$\tilde{F}_V^M(\dot{L}^M) = 1.8 - \frac{1.8}{1.0 + \exp[\ \frac{0.04 - \frac{\dot{L}^M}{\tilde{V}_{max} \cdot L_o^M}}{0.18}\ ]}$$

Force-Length-Veloctiy Properties and Elastic Tendon

- Assumptions
  - Tendon is elastic
  - Dependence on length or velocity
  - Parallel fibered
- Parameters

  $F_o^M \; L_o^M \; L_S^T \; V_{max}$
- Time-varying inputs

  $F_o^M \; L_o^M \; L_S^T \; V_{max}$



$$L^{MT} = L^T + L^M$$
$$\dot{L}^{MT} = \dot{L}^T + \dot{L}^M$$

A closed-form expression for F^T is generally not possible.