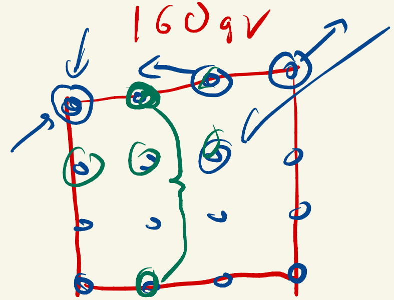
State Space Representation

Dynamics.

Transf.	t
Repres.	$t + dt$
Coords	

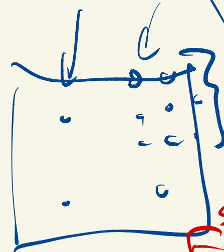
Particle system.



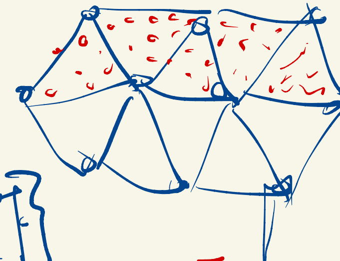
$m_i = 10$ gr.

$m\ddot{x} = \sum F$

$\ddot{x} = \frac{\sum F}{m}$

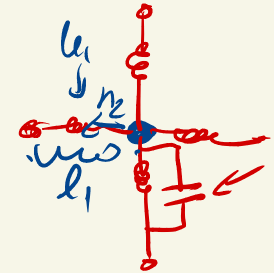


$F_1 = |F_1| \hat{n}_2$



$|F_1| = k_1 \cdot (l_1 - l_0)$

$\frac{x_2 - x_1}{|x_2 - x_1|} = n_2$



$\bar{x}$

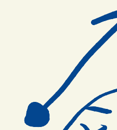
$F_i^s = k_i(l - l_0)$

$\dot{x}$

$F_i^d = k_d(|v_i| - |v_0|)$

$m \cdot g$

m



$\bar{x}_t$

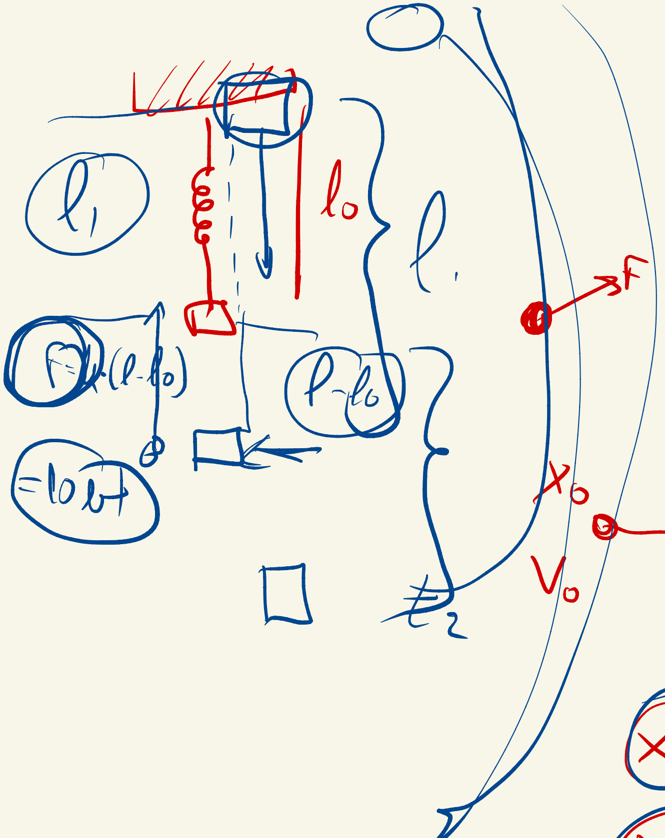
$\dot{x}_t$

$$\dot{x}_{t+dt} = \dot{x}_t + \cancel{\ddot{x}} \cdot dt \rightarrow \frac{\sum F}{m} \cdot dt$$

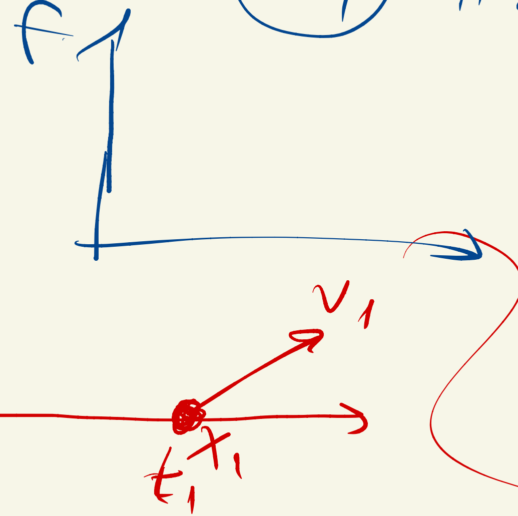
$$x_{t+dt} = x_t + \dot{x}\, dt$$

$$m\ddot{x} + d \cdot \dot{x} + k \cdot x = F$$

$$m \cdot \ddot{x} = \sum F$$



$dt = 0{,}1$ sec.

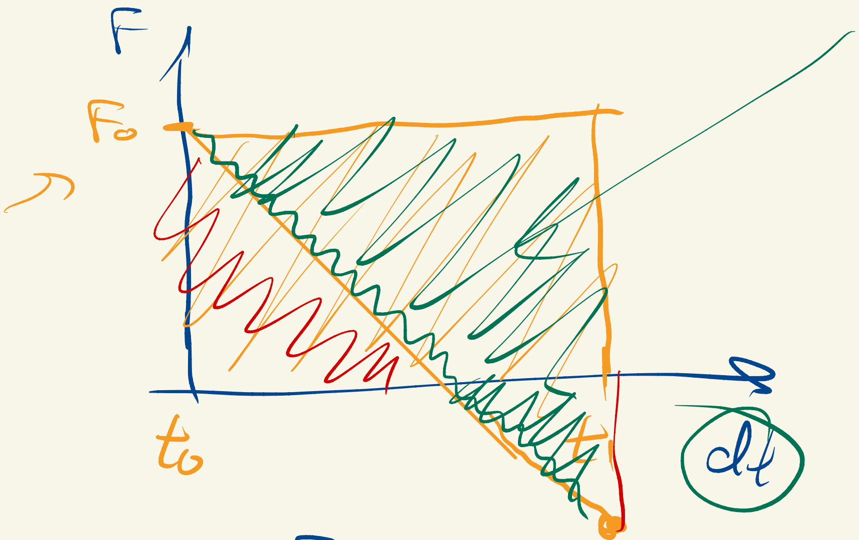


Forward Euler

$$x_1 = x_0 + v_0 \cdot dt$$

$$v_1 = v_0 + a_0 \cdot dt$$

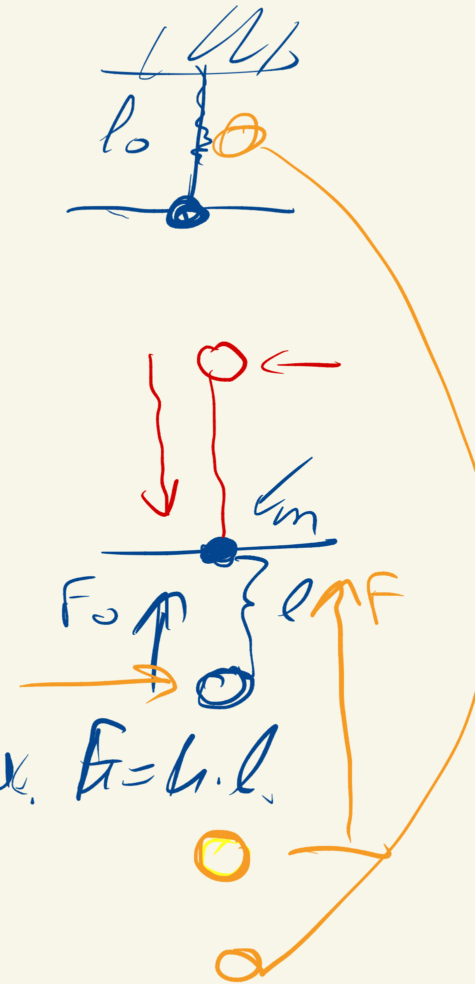
$$a_0 = \frac{F_0}{m}$$



$$v_1 = v_0 + \frac{F}{m} dt$$

$$x = x_0 + v_{1/0} \cdot dt.$$

t_1



$t_1 = t_0 + dt.$ $F = k \cdot l.$

Definitions

X – State vector

- Information of the current condition of the internal variables
- N is the "dimension" of the state model (number of internal state variables)

$\dot{X}$ – "Next state" vector

- Derivative of the state vector
- Provides knowledge of where the states are going
  - Direction (+ or -)
  - How fast (magnitude)
- A function fo the input and the present state of the internal variables

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}$$

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_N(t) \end{bmatrix}$$

State-Space Equations

General form of the state-space model

Two equations – $\dot{x}(t)$
$y(t)$

$$\dot{x}(t) = f\left(x(t), v(t), t\right)$$
$$y(t) = g\left(x(t), v(t), t\right)$$

Linear State-Space Equations

$$\dot{x}(t) = Ax(t) + Bv(t)$$
$$y(t) = Cx(t) + Dv(t)$$

$x(t),\ \dot{x}(t) \to N \times 1$ vectors

$v(t) \to R \times 1$ vector

$y(t) \to M \times 1$ vector

$A \to N \times N$ system matrix

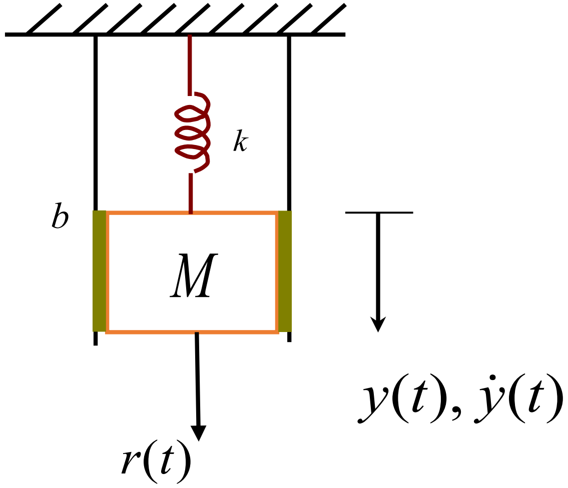
$B \to N \times R$ input matrix

$C \to M \times N$ output matrix

$D \to M \times R$ matrix representing direct coupling from system inputs to system outputs

If A, B, C, D are constant over time, then the system is also time invariant
→ Linear Time Invariant (LTI) system

Example



By Newton's Law

$\dot{x}_2 \quad x_2 \quad x_1$

$$F = M\ddot{y} \qquad \Rightarrow M\ddot{y} + b\dot{y} + ky = r$$

$$r - ky - b\dot{y} = M\ddot{y} \qquad \text{let} \quad x_1 = y,\ x_2 = \dot{y}$$

$$\Rightarrow \begin{cases} \dot{x}_1 = \dot{y} = x_2 \\ \dot{x}_2 = \ddot{y} = -\dfrac{b}{M}\dot{y} - \dfrac{k}{M}y + \dfrac{1}{M}r \end{cases}$$

$$= -\frac{b}{M}x_2 - \frac{k}{M}x_1 + \frac{1}{M}u \qquad (u = r)$$

$$\Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{k}{M}x_1 - \dfrac{b}{M}x_2 + \dfrac{1}{M}u \end{cases}$$

$y = x_1$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\dfrac{k}{M} & -\dfrac{b}{M} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{M} \end{bmatrix}}_{B} \cdot u \qquad y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u$$