

• **Addition Rule and Generalized Addition Rule:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P\left(\bigcup_{i=1}^n E_i\right) = \sum_{i=1}^n P(E_i) - \sum_{1 \leq i < j \leq n} P(E_i \cap E_j) + \sum_{1 \leq i < j < k \leq n} P(E_i \cap E_j \cap E_k) + \dots + (-1)^{n-1} P\left(\bigcap_{i=1}^n E_i\right)$$

Alternatively:

$$P\left(\bigcup_{i=1}^n E_i\right) = 1 - P\left(\bigcap_{i=1}^n E_i^c\right)$$

Product Rule:

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

$$P(E_1 \cap E_2 \cap \dots \cap E_k) = P(E_1)P(E_2|E_1) \dots P(E_k|E_1 \cap E_2 \cap \dots \cap E_{k-1})$$

Total Probability and Bayes Theorem:

$$P(A) = \sum_{i=1}^n P(A|H_i)P(H_i) \quad \text{and} \quad P(H_i|A) = \frac{P(A|H_i)P(H_i)}{\sum_{i=1}^n P(A|H_i)P(H_i)}$$

• **Random Variables:**

$$\text{Expected Value: } E(X) = \begin{cases} \sum_x x f(x) & \text{if } X \text{ discrete} \\ \int_x x f(x) dx & \text{if } X \text{ continuous} \end{cases}$$

$$E(g(X)) = \begin{cases} \sum_x g(x) f(x) & \text{if } X \text{ discrete} \\ \int_x g(x) f(x) dx & \text{if } X \text{ continuous} \end{cases}$$

$$E(aX + b) = aE(X) + b \quad \text{and} \quad E(X + Y) = E(X) + E(Y)$$

$$\text{Median: } F(\delta) = P(X \leq \delta) \geq \frac{1}{2} \geq P(X < \delta) = F(\delta^-)$$

$$p\text{-Percentile: } F(x_p^-) = P(X < x_p) \leq p \leq P(X \leq x_p) = F(x_p)$$

$$\text{Variance: } Var(X) = \sigma^2 = E((X - \mu)^2) = \begin{cases} \sum_x (x - \mu)^2 \cdot f(x) & \text{if } X \text{ discrete} \\ \int_x (x - \mu)^2 \cdot f(x) dx & \text{if } X \text{ continuous} \end{cases}$$

$$\text{or } Var(X) = E(X^2) - [E(X)]^2 \quad \text{and} \quad Var(\alpha X + \beta) = \alpha^2 Var(X)$$

$$\text{Covariance: } Cov(X, Y) = E((X - E(X))(Y - E(Y))) = E(XY) - E(X)E(Y)$$

$$Var(\alpha X \pm \beta Y) = \alpha^2 Var(X) + \beta^2 Var(Y) \pm 2\alpha\beta Cov(X, Y)$$

$$\text{Correlation Coefficient: } \rho = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

• **Sensitivity, Specificity, Prediction Values and Likelihood ratios**

$$Se = TP/(TP + FN) = TP/nD \quad \text{and} \quad Sp = TN/(TN + FP) = TN/nD^c$$

$$PPV = TP/(TP + FP) = TP/nT^+ \quad \text{and} \quad NPV = TN/(TN + FN) = TN/nT^-$$

$$LRP = \frac{Se}{1-Sp} \quad \text{and} \quad LRN = \frac{1-Se}{Sp}$$

PV with prevalence:

$$PPV = \frac{Se \times Pre}{Se \times Pre + (1-Sp) \times (1-Pre)} \quad \text{and} \quad NPV = \frac{Sp \times (1-Pre)}{Sp \times (1-Pre) + (1-Se) \times Pre}$$

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- Binomial $\mathcal{B}(n, p)$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x \in \{0, 1, \dots, n\}, \quad E(X) = np, \quad \text{Var}(X) = np(1-p)$$

- Poisson $\mathcal{P}(\lambda)$

$$f(x) = e^{-\lambda} \frac{\lambda^x}{x!}, \quad x \in \{0, 1, \dots\}, \quad E(X) = \lambda, \quad \text{Var}(X) = \lambda$$

- Exponential $\mathcal{E}(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = \frac{1}{\lambda}, \quad \text{Var}(X) = \frac{1}{\lambda^2}$$

- Normal $\mathcal{N}(\mu, \sigma)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \text{ for } -\infty < x < \infty, \quad E[X] = \mu, \quad \text{Var}(X) = \sigma^2$$