

Mid-term  
exams will be  
on 23/11/2021,  
18:00

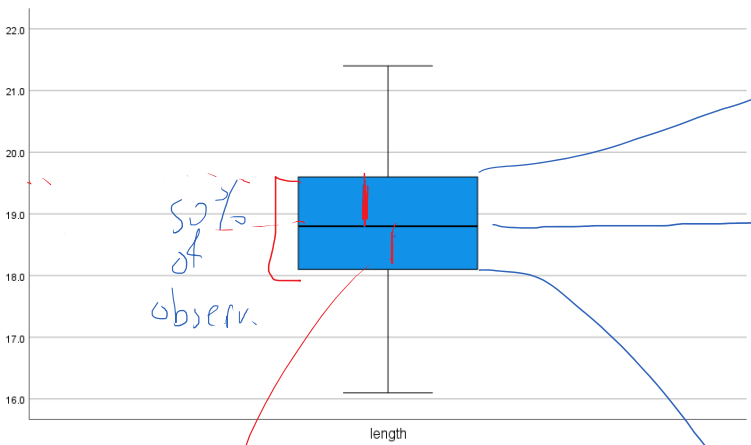
$$CV = \frac{s}{|\bar{x}|} = \frac{1.1205}{10.302}$$

$$= 0.0595 < 0.1$$

rule

if  $CV > 0.1$  (10%) the sample comes from an heterogeneous population

if  $CV < 0.1$  (10%) the sample comes from an homogen. population



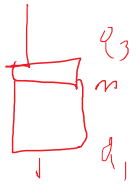
$Q_3$   $\left\{ \begin{array}{l} 25\% \\ 75\% \end{array} \right.$   
median  $m$   $\left\{ \begin{array}{l} 50\% \geq 0 \\ 50\% \leq \end{array} \right.$

$Q_1$   $\left\{ \begin{array}{l} 75\% \\ 25\% \end{array} \right.$

$m - Q_1 \approx Q_3 - m$   
Symmetric

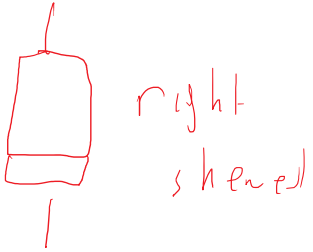
$Q_1 \approx 19.1$

$Q_3$   
 $Q_3$



$-m < m - Q_1$   
 left skewed

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$-m > m - Q_1$

### Estimated Distribution Parameters

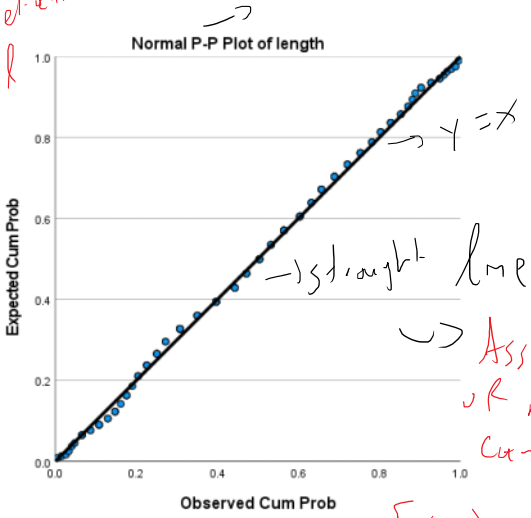
		length
Normal Distribution	Location	18.802
	Scale	1.1205

The cases are unweighted.

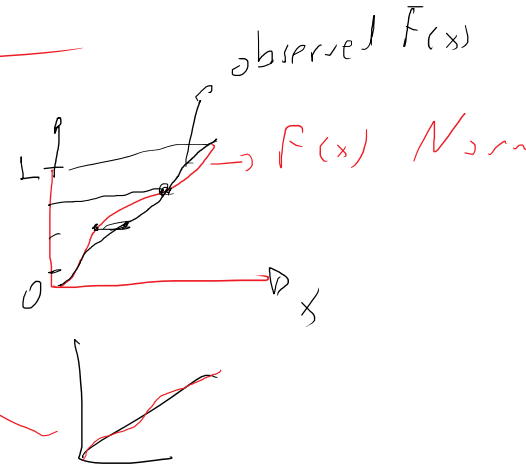
Assume  $X \sim N(\mu, \sigma)$

$\hat{\mu}$  estimator of  $\mu$   
 $\hat{\sigma}$  estimator of  $\sigma$   
 $S$

theoretical Normal  $F(x)$



P-P plot

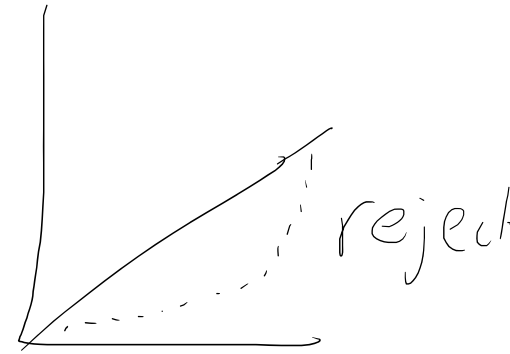
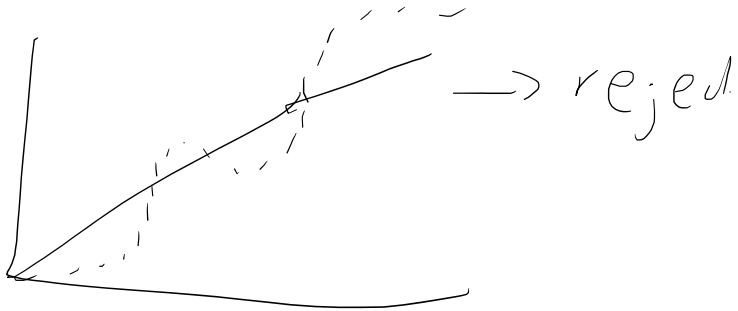
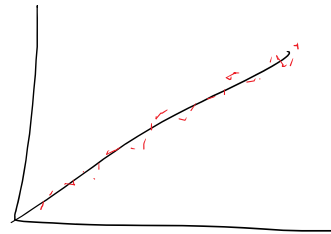
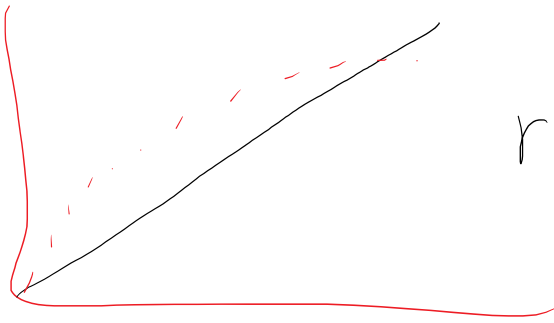


Assumptions of normal distrib can not be rejected  
 $F(x) = P(X \leq x)$

n-1

# Normal $\rho$ - $\rho$ plot.

The normal is rejected



is not  
rejected

→

→



**Statistics**

		tire	driver
N	Valid	75	75
	Missing	0	0
Mean		1.9733	.5333
Median		2.0000	1.0000
Mode		2.00	1.00

**Frequency Table**

tire → 3 types

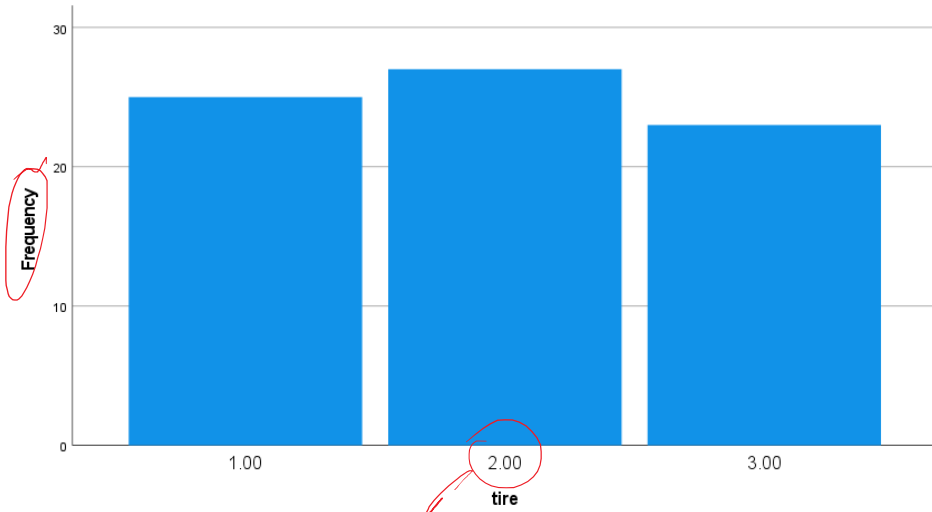
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1.00	25	33.3	33.3	33.3
	2.00	27	36.0	36.0	69.3
	3.00	23	30.7	30.7	100.0
Total		75	100.0	100.0	

$\hat{p}_1 = 0.333 = 33.3\%$   
 $\hat{p}_2 = 0.36 = 36\%$   
 $\hat{p}_3 = 0.307 = 30.7\%$

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	.00	35	46.7	46.7	46.7
	1.00	40	53.3	53.3	100.0
	Total	75	100.0	100.0	

$\hat{p}_0 = 46.7\%$   
 $\hat{p}_1 = 53.3\%$

tire



**Statistics**

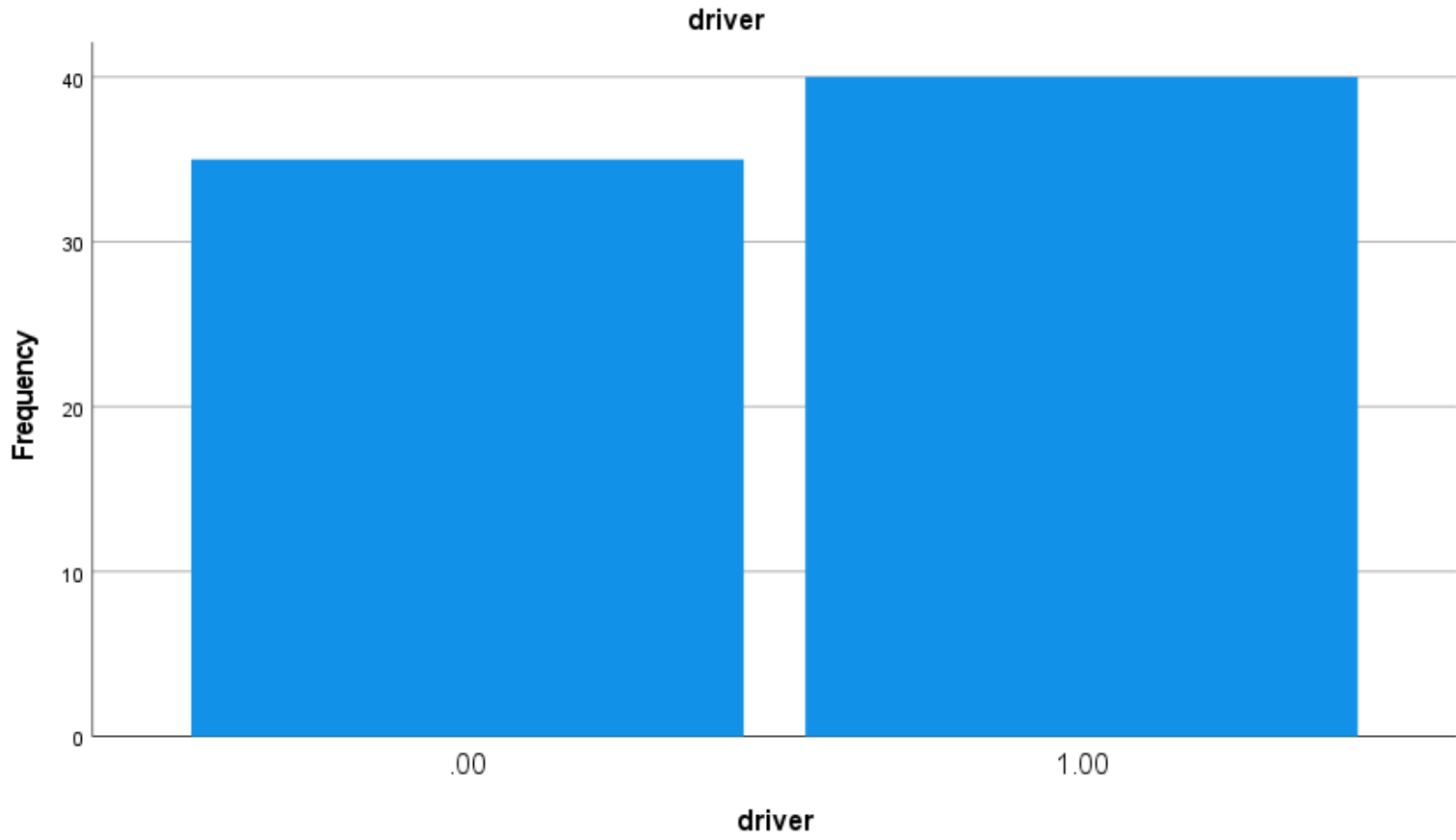
		tire
N	Valid	75
	Missing	0
Mean		1.9733
Median		2.0000
Mode		2.00

≤ 50%  
7, 10, 15

mode

$\bar{x} \approx \text{median} \approx \text{mode}$   
Symmetric  
dist.

driver
75
0
.5333
1.0000
1.00



*no symmetry*

**tire \* driver Crosstabulation**

		driver		Total	
		.00	1.00		
tire	1.00	Count	12	13	25
		% within tire	48.0%	52.0%	100.0%
		% within driver	34.3%	32.5%	33.3%
		% of Total	16.0%	17.3%	33.3%
2.00		Count	11	16	27
		% within tire	40.7%	59.3%	100.0%
		% within driver	31.4%	40.0%	36.0%
		% of Total	14.7%	21.3%	36.0%
3.00		Count	12	11	23
		% within tire	52.2%	47.8%	100.0%
		% within driver	34.3%	27.5%	30.7%
		% of Total	16.0%	14.7%	30.7%
Total		Count	35	40	75
		% within tire	46.7%	53.3%	100.0%
		% within driver	100.0%	100.0%	100.0%
		% of Total	46.7%	53.3%	100.0%

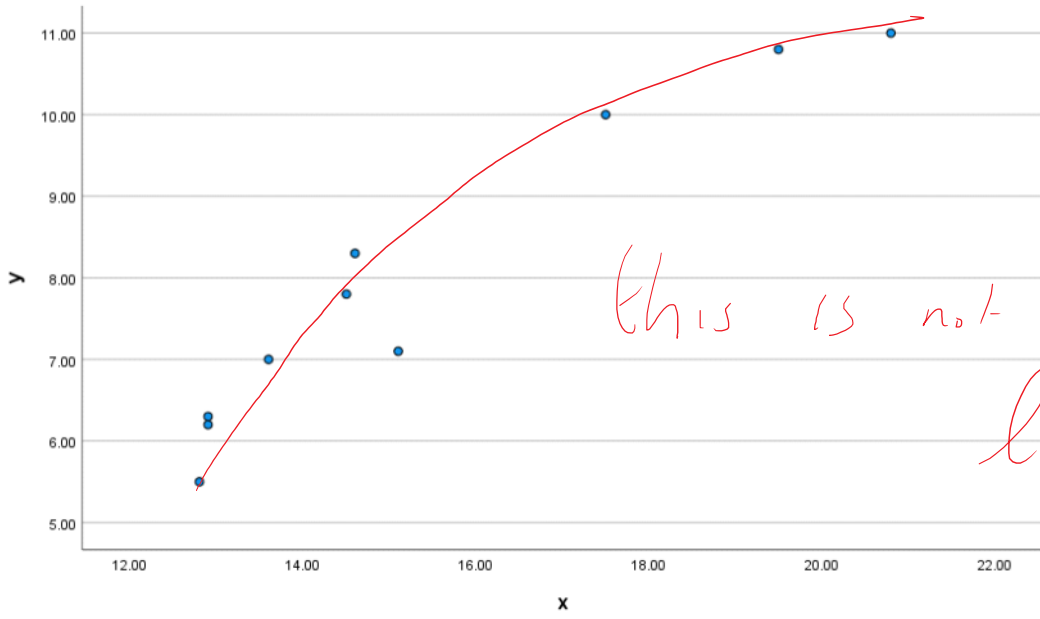
16 times that we used driver 1 with type 2

$$59.3\% = \frac{16}{27}$$

21.3

$$40\% = \frac{16}{40}$$

$$\% = \frac{16}{75}$$



$X \neq Y \rightarrow Y$

this is not a linear relat.

|

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# MLE

Suppose that we have observed

$$x_1, x_2, \dots, x_n$$

from a distribution with probab. function (discrete)

$$f(x; \theta)$$

then the probab. of the observed sample is

$$P(X_1=x_1 \wedge X_2=x_2 \wedge \dots \wedge X_n=x_n)$$

crete)

$$P(X_1=x_1 \wedge X_2=x_2 \wedge \dots \wedge X_n=x_n) \stackrel{\text{ind. dep.}}{=} \dots$$

$$P(X_1=x_1) \cdot P(X_2=x_2) \cdot \dots \cdot P(X_n=x_n)$$

$$= f(x_1; \theta) \cdot f(x_2; \theta) \cdot \dots \cdot f(x_n; \theta)$$

↑ unknown  
 ↑  
 known

↑  
 known

↑  
 known

prob.  
 to  
 observe

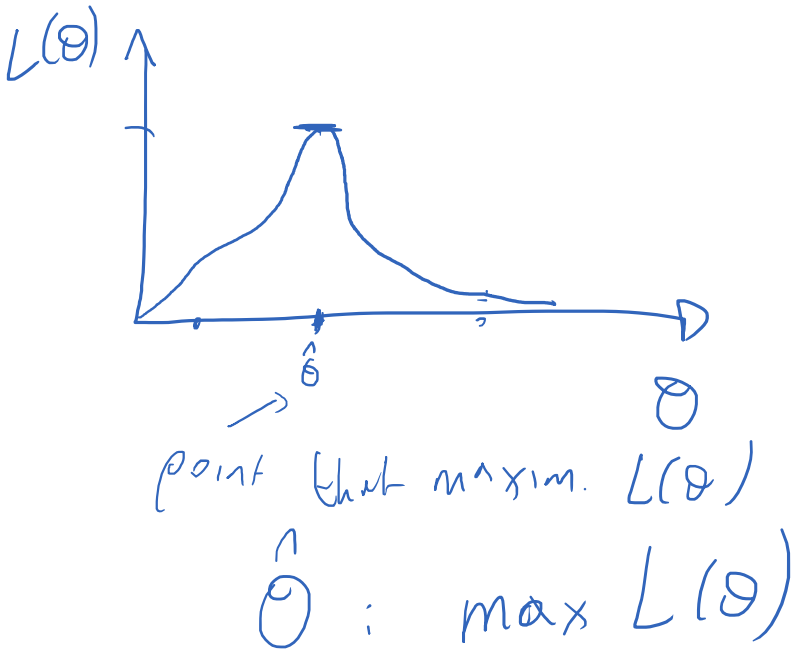
what  
 we have observed

observed values of  
 the sample

$= L(\theta)$

the  
 $\theta$

fun fun  
of  
g



$$L(\theta) = \prod_{i=1}^n f(x_i; \theta)$$

$$l(\theta) = \log L(\theta)$$

$$= \log \prod f$$

$$= \sum \log f$$

log-likelihood

$$l'(\theta) = 0$$

solve  
 mle  $\hat{\theta}$

$(x_i, 0)$

$(x_j, 0)$