Lecture 7: DFT & FFT
Practice Exercises

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1 Coding Exercises

Question 1.1

Consider the discrete-time signal

$$x[n] = \cos(0.4\pi n), \qquad 0 \le n < 64.$$

Write Python code using `numpy` and `matplotlib` to:

1. Generate the samples $x[n]$.
2. Compute the DFT using `np.fft.fft`.
3. Plot $x[n]$ using `plt.stem`.
4. Plot the magnitude spectrum $|X[k]|$ versus DFT bin index k.
5. Plot the magnitude spectrum versus discrete-time angular frequency Ω in rad/sample.

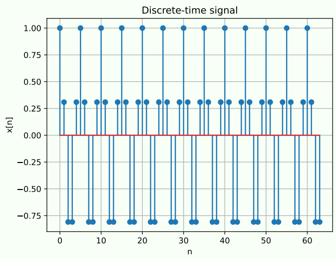
Solution (Code)

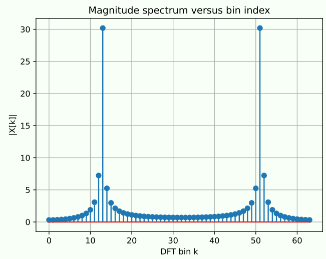
```python
import numpy as np
import matplotlib.pyplot as plt

N = 64
n = np.arange(N)

Omega0 = 0.4 * np.pi
x = np.cos(Omega0 * n)

X = np.fft.fft(x)

k = np.arange(N)
Omega = 2 * np.pi * k / N

plt.figure()
plt.stem(n, x)
plt.xlabel("n")
plt.ylabel("x[n]")
plt.title("Discrete-time signal")
plt.grid(True)

plt.figure()
plt.stem(k, np.abs(X))
plt.xlabel("DFT bin k")
plt.ylabel("|X[k]|")
plt.title("Magnitude spectrum versus bin index")
plt.grid(True)

plt.figure()
```

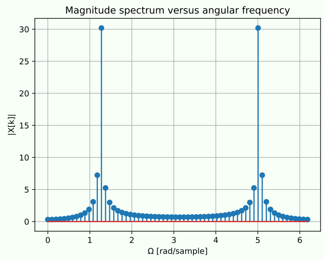
```python
plt.stem(Omega, np.abs(X))
plt.xlabel(r"$\Omega$ [rad/sample]")
plt.ylabel("|X[k]|")
plt.title("Magnitude spectrum versus angular frequency")
plt.grid(True)
plt.show()
```

Solution

Plots:







Answers:
The signal is

$$x[n] = \cos(0.4\pi n).$$

The discrete-time angular frequency is

$$\Omega_0 = 0.4\pi \text{ rad/sample}.$$

The DFT bin spacing is

$$\Delta\Omega = \frac{2\pi}{N}.$$

For $N = 64$,

$$\Delta\Omega = \frac{2\pi}{64} = \frac{\pi}{32}.$$

The expected bin location is

$$k_0 = \frac{\Omega_0}{\Delta\Omega} = \frac{0.4\pi}{\pi/32} = 12.8.$$

Since k_0 is not an integer, the frequency does not fall exactly on a DFT bin. Therefore, the magnitude spectrum shows spectral leakage.
The DFT magnitude should have large values around bins $k = 13$ and $k = 64 - 13 = 51$, corresponding to the positive and negative frequency components of the cosine.

Question 1.2

Consider the continuous-time signal

$$x(t) = \cos(2\pi \cdot 10t).$$

The signal is sampled using sampling frequency

$$f_s = 80 \text{ Hz}.$$

Write Python code using `numpy` and `matplotlib` to:

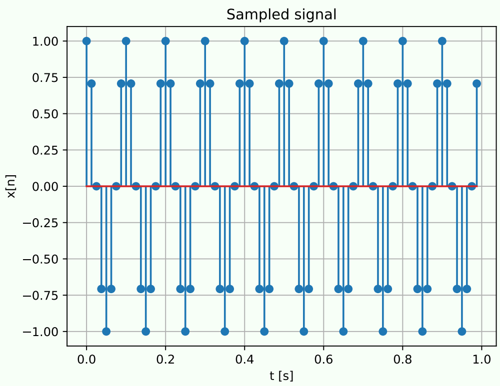
1. Generate 1 second of samples.
2. Compute the FFT using `np.fft.fft`.
3. Plot the magnitude spectrum versus bin index k.
4. Plot the magnitude spectrum versus frequency f in Hz.
5. Plot the magnitude spectrum versus angular frequency ω in rad/s.

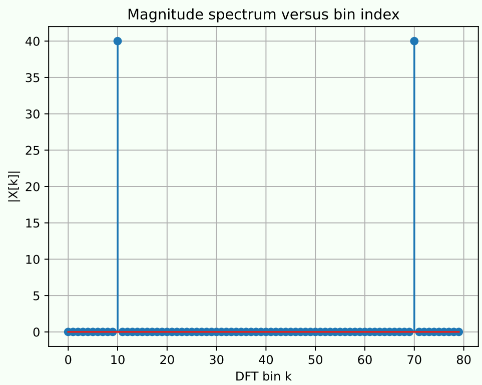
Solution (Code)

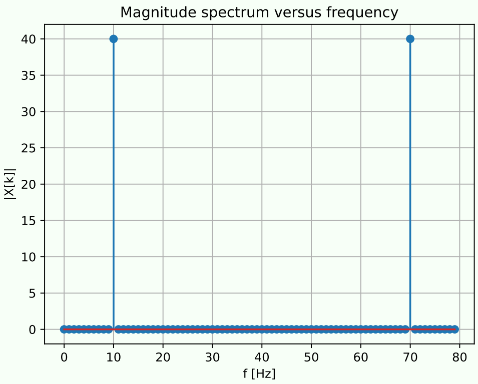
```
import numpy as np
import matplotlib.pyplot as plt

fs = 80
Ts = 1 / fs
duration = 1.0

f0 = 10
t = np.arange(0, duration, Ts)

x = np.cos(2 * np.pi * f0 * t)

N = len(x)
X = np.fft.fft(x)

k = np.arange(N)
f = k * fs / N
omega = 2 * np.pi * f

plt.figure()
plt.stem(t, x)
plt.xlabel("t [s]")
plt.ylabel("x[n]")
plt.title("Sampled signal")
plt.grid(True)

plt.figure()
plt.stem(k, np.abs(X))
plt.xlabel("DFT bin k")
plt.ylabel("|X[k]|")
plt.title("Magnitude spectrum versus bin index")
plt.grid(True)

plt.figure()
plt.stem(f, np.abs(X))
plt.xlabel("f [Hz]")
plt.ylabel("|X[k]|")
plt.title("Magnitude spectrum versus frequency")
plt.grid(True)

plt.figure()
plt.stem(omega, np.abs(X))
plt.xlabel(r"$\omega$ [rad/s]")
plt.ylabel("|X[k]|")
plt.title("Magnitude spectrum versus angular frequency")
plt.grid(True)
plt.show()
```

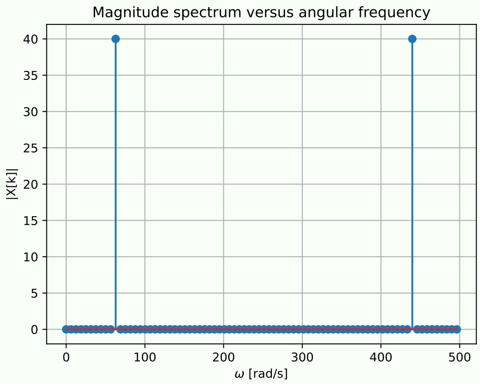
Solution

Plots:









Answers:
The continuous-time signal frequency is

$$f_0 = 10 \text{ Hz}.$$

The continuous-time angular frequency is

$$\omega_0 = 2\pi f_0 = 20\pi \text{ rad/s}.$$

The sampling frequency is

$$f_s = 80 \text{ Hz}.$$

The discrete-time angular frequency is

$$\Omega_0 = \omega_0 T_s.$$

Since

$$T_s = \frac{1}{80},$$

we get

$$\Omega_0 = 20\pi \cdot \frac{1}{80} = \frac{\pi}{4} \text{ rad/sample}.$$

Equivalently,

$$\Omega_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{10}{80} = \frac{\pi}{4}.$$

Thus the same sinusoid can be described using:

$$f_0 = 10 \text{ Hz},$$

$$\omega_0 = 20\pi \text{ rad/s},$$

$$\Omega_0 = \frac{\pi}{4} \text{ rad/sample}.$$

The frequency axis in Hz is computed as

$$f_k = \frac{k f_s}{N}.$$

The angular-frequency axis in rad/s is

$$\omega_k = 2\pi f_k.$$

Question 1.3

Consider the continuous-time signal

$$x(t) = \cos(2\pi \cdot 20t).$$

The signal is sampled at

$$f_s = 50 \text{ Hz}.$$

Write Python code using `numpy` and `matplotlib` to:

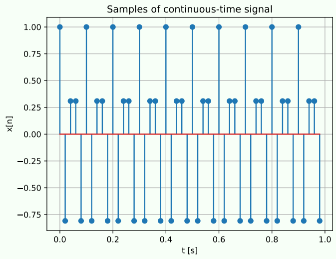
1. Generate samples for 1 second.
2. Compute the FFT of the sampled signal.
3. Plot the spectrum versus frequency in Hz.
4. Plot the same spectrum versus angular frequency in rad/s.
5. Compute the corresponding discrete-time angular frequency in rad/sample.

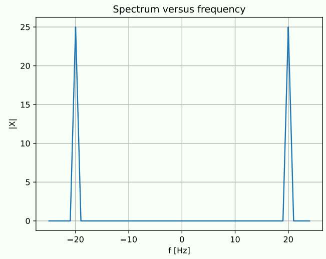
Solution (Code)

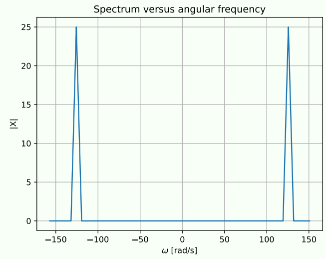
```python
import numpy as np
import matplotlib.pyplot as plt

fs = 50
Ts = 1 / fs
duration = 1.0

f0 = 20
t = np.arange(0, duration, Ts)

x = np.cos(2 * np.pi * f0 * t)

N = len(x)
X = np.fft.fft(x)

freq = np.fft.fftfreq(N, d=Ts)
omega = 2 * np.pi * freq

X_shift = np.fft.fftshift(X)
freq_shift = np.fft.fftshift(freq)
omega_shift = np.fft.fftshift(omega)

plt.figure()
plt.stem(t, x)
plt.xlabel("t [s]")
plt.ylabel("x[n]")
plt.title("Samples of continuous-time signal")
plt.grid(True)

plt.figure()
plt.plot(freq_shift, np.abs(X_shift))
plt.xlabel("f [Hz]")
plt.ylabel("|X|")
plt.title("Spectrum versus frequency")
plt.grid(True)

plt.figure()
plt.plot(omega_shift, np.abs(X_shift))
plt.xlabel(r"$\omega$ [rad/s]")
plt.ylabel("|X|")
plt.title("Spectrum versus angular frequency")
plt.grid(True)

plt.show()
```

Solution

Plots:







Answers:
The continuous-time frequency is

$$f_0 = 20 \text{ Hz}.$$

The continuous-time angular frequency is

$$\omega_0 = 2\pi f_0 = 40\pi \text{ rad/s}.$$

The sampling frequency is

$$f_s = 50 \text{ Hz}.$$

The sampling period is

$$T_s = \frac{1}{50}.$$

The discrete-time angular frequency is

$$\Omega_0 = \omega_0 T_s.$$

Therefore,

$$\Omega_0 = 40\pi \cdot \frac{1}{50} = 0.8\pi \text{ rad/sample}.$$

Equivalently,

$$\Omega_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{20}{50} = 0.8\pi.$$

The frequency axes are related by

$$\omega = 2\pi f.$$

The discrete-time angular frequency is related to continuous-time angular frequency by

$$\Omega = \omega T_s.$$

Thus:

$$f \text{ is measured in Hz},$$

$$\omega \text{ is measured in rad/s},$$

$$\Omega \text{ is measured in rad/sample}.$$