(1.23) $\omega_{sb} = 0.25\pi$

$\omega_p = 0.45\pi$

$A_p = 1dB$

$A_s = 30dB$, $T = 1$

1) $\Omega = 2\tan\left(\frac{\omega}{2}\right) \longrightarrow \Omega_{SB} = 2\tan\left(\frac{0.25\pi}{2}\right) = 2\tan(0.125\pi) \simeq 0.82$

$\longrightarrow \Omega_p = 2\tan\left(\frac{0.45\pi}{2}\right) = 2\tan(0.225\pi) \simeq 0.259$

2) $0 \le \Omega \le \Omega_{sb}$: ζώνη διέλευσης

$\Omega \ge \Omega_p$. ζώνη αποκοπής

$\Omega_L = \frac{\Omega_c}{\Omega} \sim S_L = \frac{\Omega_c}{S}$

3) $$M \ge \frac{\log_{10}\left(\frac{10^{As/10}-1}{10^{Ap/10}-1}\right)}{2\log_{10}\left(\frac{\Omega_p}{\Omega_{SB}}\right)}$$

$A_p = 1$, $A_s = 30$

$\Rightarrow 10^{As/10} - 1 = 10^3 - 1 = 999$

$\Rightarrow 10^{Ap/10} - 1 = 10^{1/10} - 1 \simeq 0.259$

$\frac{\Omega_p}{\Omega_{Sb}} = \frac{2\tan(0.125\pi)}{2\tan(0.225\pi)} \sim 2.062$

$$M \ge \frac{\log_{10}(999/0.259)}{2\log_{10}(2.062)} \Rightarrow$$

$$M \ge \frac{\log_{10}(3858.6)}{2 \cdot 0.314} \simeq 5.71 \Rightarrow \boxed{M = 6}$$

3) $$|H_{HP}(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_c}{\Omega}\right)^{2M}} \Rightarrow$$

$$\Rightarrow |H_{HP}(j\Omega_p)|^2 = 10^{-Ap/10}$$

$$\frac{1}{1+\left(\frac{\Omega_c}{\Omega_p}\right)^{2M}} = 10^{-Ap/10} \Rightarrow \left(\frac{\Omega_c}{\Omega_p}\right)^{2M} = 10^{Ap/10} - 1$$

$$\Rightarrow \Omega_c = \Omega_p \cdot \left(10^{Ap/10} - 1\right)^{2M}$$

$$= 1.7082\,(0.259)^{1/12} \simeq 1.52$$

$\rightarrow 0.82 < 1.52 < 1.70 \rightarrow \Omega_{SB} < \Omega_c < \Omega_p.$

4) $M = 6 \rightarrow$

$B_6(s) = s^6 + 3.8637s^5 + 7.4641s^4 + 9.1416s^3 + 7.4641s^2 + 3.8637s + 1.$

$H_{LP} = \frac{1.}{B_6(S_L)} \longrightarrow S_L = \frac{\Omega_c}{S}$

$\Rightarrow H_{HP}(s) = H_{LP}\left(\frac{\Omega_c}{s}\right)$

$$H_{HP}(s) = \frac{s^6}{s^6 + 3.8637\Omega_c s^5 + 7.4641\,\Omega_c^2 s^4 + 9.1416\Omega_c^3 s^3 + 7.4641\,\Omega_c^4 s^2 + 3.8637\Omega_c^5 s + \Omega_c^6}$$

$\Omega_c \simeq 1.52$

$\Omega_c^2 \simeq 0.33$

$\Omega_c^3 \simeq 3.56$

$\Omega_c^4 \simeq 5.44$

$\Omega_c^5 \simeq 0.31$, $\Omega_c^6 \simeq 12.7$

$$H_{HP}(s) = \frac{s^6}{s^6 + 5.9019s^5 + 17.415s^4 + 32.5885s^3 + 40.6373s^2 + 32.1244s + 12.7037}$$

$(S = j\Omega)$ $s = 2\,\frac{1-z^{-1}}{1+z^{-1}}$

$$\Rightarrow H(z) = H_{HP}(s)\Big|_{S = 2\cdot\frac{1-z^{-1}}{1+z^{-1}}}$$

$$H(z) = \frac{b_0 + b_1z^{-1} + \dots + b_6z^{-6}}{1 + a_1z^{-1} + \dots + a_6z^{6}}$$

(1.1) $x[n] = A\cos(w_0 n)$

$P(A=1) = P(A=-1) = \frac{1}{2}$

1) Τυχαία διαδικασία

2) $\mu_x[n] = \mathbb{E}\{x[n]\} = \mathbb{E}\{A\cos(w_0n)\}$

$= \mathbb{E}\{A\}\cos(w_0 n)$

$\mathbb{E}\{A\} = 1 \cdot P(A=1) + (-1)P(A=-1) =$

$= 1\cdot\frac{1}{2} + (-1)\left(\frac{1}{2}\right) = 0.$

$\Rightarrow \boxed{\mu_x[n] = 0}$

3) $R_{xx}[n_1,n_2] = \mathbb{E}\{x[n_1]x[n_2]\}$

$\Rightarrow R_{xx}[n_1,n_2] = \mathbb{E}\{A\cos(w_0n_1)A\cos(w_0n_2)\}$

$= \cancel{\mathbb{E}\{A^2\}}^{1} \cdot \cos(w_0n_1)\cos(w_0n_2)$

$\xrightarrow[A^2=1]{A=\pm1} = \cos(w_0n_1)\cos(w_0n_2)$

$\Rightarrow \mathbb{E}\{A^2\} = 1$

(1.12) $$\hat{r}_{xy}[n_0,n_0] = \frac{1}{N}\sum_{i=1}^{N} x_i[n_0]y_i[n_0]$$

$$\overset{N=4}{=} \frac{1}{4}\left[1\cdot2 + 2\cdot1 + 3\cdot0 + 4(-1)\right] = \frac{1}{4}(2+2+0-4) = 0$$

$$\hat{r}_{xz}[n_0,n_0] = \frac{1}{4}\left[1\cdot0 + 2\cdot1 + 3\cdot2 + 4\cdot3\right] = \frac{1}{4}(0+2+6+12)$$

$$= \frac{20}{4} = 5$$

$$\hat{r}_{yz}[n_0,n_0] = \frac{1}{4}\left[2\cdot0 + 1\cdot1 + 0\cdot2 + (-1)\cdot3\right] = \frac{1}{4}(0+1+0-3) = -\frac{2}{4} = -\frac{1}{2}.$$

$$\Rightarrow \boxed{\begin{aligned}\hat{r}_{xy} &= 0\\ \hat{r}_{xz} &= 5\\ \hat{r}_{yz} &= -\frac{1}{2}\end{aligned}}$$

(1.16) $x[n] = n + w$, $\mathbb{E}\{w\} = 0$

$\mathbb{E}\{w^2\} = \sigma_w^2$

1) $\mu_x[n] = \mathbb{E}\{x[n]\} = \mathbb{E}\{n+w\}$

$= n + \cancel{\mathbb{E}\{w\}}^{0} = n. \rightarrow$ deterministic, non-constant.

$\boxed{\text{NOT WSS}}$

(1.17) $x[n] = nA$

$\mathbb{E}\{A\} = 0$, $\mathbb{E}\{A^2\} = \sigma_A^2.$

1) $\mu_x[n] = \mathbb{E}\{x[n]\} \xrightarrow{x=nA} \mathbb{E}\{nA\}.$

$= n\,\mathbb{E}\{A\} = 0.$

2) $R_{xx}[n,n] = \mathbb{E}\{n^2A^2\} = n^2\,\mathbb{E}\{A^2\}$

$= n^2 \cdot \sigma_A^2$

$\Rightarrow R_{xx}[n,n] = R_{xx}[0].$

$\boxed{R_{xx}[n,n] = n^2\sigma_A^2}$ ~> εξαρτάται από n ~> όχι σταθερό.

$\boxed{\text{όχι WSS}}$