(1.8) $x[n] = \{3, 1, -1, 1\}$

1) $X[k] = \sum_{n=0}^{3} x[n] W_4^{kn}$, $W_4 = -j$

$W_N = e^{-j\frac{2\pi}{N}} \Rightarrow W_4 = e^{-j\frac{\pi}{2}}$

$k=0 \rightarrow X[0] : 3+1-1+1 = 4$

$k=1 \rightarrow X[1] : 3 + 1 \cdot W_4 + (-1) W_4^2 + 1 \cdot W_4^3$

$= 3 - j + 1 + j$

$W_4 = -j$

$W_4^2 = -1, W_4^4 = 1, W_4^6 = -1$

$k=2 \rightarrow X[2] : 3 + 1 \cdot W_4^2 + (-1) W_4^4 + 1 \cdot W_4^6$

$= 3 - 1 - 1 - 1 = 0$

$k=3 \rightarrow X[3] : 3 + W_4^3 - W_4^6 + W_4^9$

$= 3 + j + 1 - j = 4$

$W_4^3 = j, W_4^6 = -1, W_4^9 = -j$

$X[k] = \{4, 4, 0, 4\}$

2) $x[n] = \frac{1}{4} \sum_{k=0}^{3} X[k] W_4^{-kn}$ $X[2] = 0$

$x[n] = \frac{1}{4}\left(4 + 4 W_4^{-n} + 4 W_4^{-3n}\right)$

$n=0 \rightarrow x[0] = \frac{1}{4}(4+4+4) = 3$

$n=1 \rightarrow x[1] = \frac{1}{4}(4 + 4j + 4(-j)) = \frac{4}{4} = 1$

$n=2 \rightarrow x[2] = \frac{1}{4}(4 + 4(-1) + 4(-1)) = -1$

$n=3 \rightarrow x[3] = \frac{1}{4}(4 + 4(-j) + 4j) = 1$

$W_4^{-1} = j, W_4^{-3} = -j$

$W_4^{-2} = -1, W_4^{-6} = -1$

$W_4^{-3} = -j, W_4^{-9} = j$

$x[n] = \{3, 1, -1, 1\}$

(EXTRA) parseval: $\sum_{n=0}^{3} |x[n]|^2 = \frac{1}{4} \sum_{k=0}^{k=3} |X[k]|^2$

$E_x = 3^2 + 1^2 + (-1)^2 + 1^2 = 12$ $\quad E_E = \frac{1}{4}(4^2 + 4^2 + 0^2 + 4^2) = 12$

(2.2) $f_0 = 10 Hz$

$\omega_0 = 2\pi f_0 = 20\pi$ rad/s

$T_s = \frac{1}{80} \Rightarrow \Omega_0 : \omega_0 T_s = 20\pi \frac{1}{80} = \frac{\pi}{4}$ rad/s

$K_0 = N \frac{f_0}{f_s} = 80 \frac{10}{80} = 10$

(2.6) $f_0 = 20 Hz$

$\omega_0 = 2\pi f_0 = 40\pi$ (rad/s)

$\Omega_0 = \omega_0 T_s = 40\pi \frac{1}{50} = 0.8\pi$ (rad/sample)

$= 2\pi \frac{f_0}{f_s} = 2\pi \frac{20}{50}$