(1.1) $x(t) = 3\cos(40\pi t) + 2\sin(90\pi t)$, $F_s = 100\,Hz$

1) $x[n]$

$x[n] = x(nT_s)$ $\xrightarrow{T_s = \frac{1}{F_s}}$

$x[n] = 3\cos(40\pi n T_s) + 2\sin(90\pi n T_s)$

$= 3\cos\left(\frac{40\pi n}{100}\right) + 2\sin\left(\frac{90\pi n}{100}\right)$

$x[n] = 3\cos(0.4\pi n) + 2\sin(0.9\pi n)$

2) $x[n] = 3\cos(\omega_1 \pi n) + 2\sin(\omega_2 \pi n)$

$\Rightarrow \omega_1 = 0.4$, $\omega_2 = 0.9$

3) $[-\pi, \pi] \rightarrow$ ΟΧΙ αλλαγής

4) ομοίως (2)

(1.2) $x[n] = \cos\left(\frac{3\pi}{4} n\right)$, $0 \le n \le 7$

1) $\cos(\omega_0 n) = \frac{1}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right)$

$\omega_0 = \frac{3\pi}{4}$, $\omega_k = \frac{2\pi k}{8}$, 8-point DFT

$\frac{\pi k}{4} = \frac{3\pi}{4} \Rightarrow \boxed{k = 3}$

$-\frac{3\pi}{4} \rightarrow$ αρνητικό $\Rightarrow k = 8 - 3 \Rightarrow \boxed{k = 5}$

$x[n] = \frac{1}{2} e^{j\frac{2\pi}{8} 3n} + \frac{1}{2} e^{j\frac{2\pi}{8} 5n}$ $\left(e^{-j\frac{2\pi}{8} 3n} = e^{j\frac{2\pi}{8} 5n}\right)$

$\sum_{n=0}^{7} e^{j\frac{2\pi}{8} mn} e^{-j\frac{2\pi}{8} kn} = \begin{cases} 8, & k = m \\ 0, & k \neq m \end{cases}$

$X[3] = \frac{8}{2} = 4$

$X[5] = 4$

$X[k] = \begin{cases} 4, & k = 3 \\ 4, & k = 5 \\ 0, & \text{αλλιώς} \end{cases}$

(1.3) $x[n] = \{1, 2, 0, -1\}$

$X[k] = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4} kn}$

$x[0] = 1$
$x[1] = 2$
$x[2] = 0$
$x[3] = -1$

$X[k] = 1 + 2e^{-j\frac{\pi}{2} k} - e^{-j\frac{3\pi}{2} k}$

$k = 0 \rightarrow X[0] = 1 + 2 \cdot 1 - 1 = 2$

$k = 1 \rightarrow X[1] = 1 + 2e^{-j\frac{\pi}{2}} - e^{-j\frac{3\pi}{2}}$

$X[1] = 1 + 2(-j) - j = 1 - 3j$

$e^{-j\frac{\pi}{2}} = -j$, $e^{-j\frac{3\pi}{2}} = j$

$e^{-j\pi} = -1$, $e^{-j3\pi} = -1$

$k = 2 \rightarrow X[2] = 1 + 2e^{-j\pi} - e^{-j3\pi}$

$X[2] = 1 + 2(-1) - (-1) = 0$

$k = 3 \rightarrow X[3] = 1 + 2e^{-j\frac{3\pi}{2}} - e^{-j\frac{9\pi}{2}}$

$e^{j\frac{3\pi}{2}} = j$

$e^{-j\frac{9\pi}{2}} = j$

$X[3] = 1 + 3j$

$X[k] = \{2, 1 - 3j, 0, 1 + 3j\}$

2) $|X[k]| \xrightarrow{k=0} |X[0]| = 2$

$k = 1 \Rightarrow |X[1]| = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

$k = 2 \rightarrow |X[2]| = 0$

$k = 3 \rightarrow |X[3]| = \sqrt{1^2 + 3^2} = \sqrt{10}$

3) $\angle X[k] \rightarrow \angle X[0] = 0$

$\angle X[1] = -\tan^{-1}(3)$

$\angle X[2]$ → ΔΕΝ ΟΡΙΖΕΤΑΙ

$\angle X[3] \rightarrow \tan^{-1}(3)$

4) $\sum_{n=0}^{3} |x[n]|^2 = 1^2 + 2^2 + 0^2 + (-1)^2 = 6$

$\frac{1}{N} \sum_{k=0}^{3} |X[k]|^2$

$\sum_{k=0}^{3} |X[k]|^2 = 2^2 + \left(\sqrt{10}\right)^2 + 0^2 + \left(\sqrt{10}\right)^2 = 24$

$\Rightarrow \frac{1}{4} 24 = 6$