(1.9) $x(t) = e^{-\alpha|t|}$, $\alpha > 0$

$$X(w) = \int_{-\infty}^{0} e^{\alpha t} e^{jwt}\,dt + \int_{0}^{\infty} e^{-\alpha t} e^{-jwt}\,dt$$

$$\int_{-\infty}^{0} e^{(\alpha - jw)t}\,dt \left[\frac{e^{(\alpha-jw)t}}{\alpha - jw}\right]_{-\infty}^{0} = \frac{1}{\alpha - jw} \qquad \frac{1}{\alpha + jw}$$

$$X(w) = \frac{1}{\alpha - jw} + \frac{1}{\alpha + jw} = \frac{2\alpha}{\alpha^2 + w^2}$$

(1.24) $x(t) = e^{-\alpha t} u(t) \longrightarrow X(w) = \frac{1}{\alpha + jw}$

$z(t) = e^{-\alpha(t-1)} u(t-1) = x(t-1)$

$x(t - t_0) \longleftrightarrow e^{-jwt_0} X(w)$

$Z(w) = e^{-jw} X(w) = \frac{e^{-jw}}{\alpha + jw}$

$y(t) = \frac{d}{dt} z(t)$

$\frac{d}{dt} z(t) \Longrightarrow jw Z(w)$

$Y(w) = jw \frac{e^{-jw}}{\alpha + jw}$

(1.27) $x(t) = \underbrace{\left(e^{-t} u(t)\right)}_{x_1} * \underbrace{\left(t\,u(t)\right)}_{x_2}$

(F) $x_1 \xrightarrow{F} \frac{1}{1 + jw}$, $x_2 \xrightarrow{F} \frac{1}{(jw)^2}$

$x_1(t) * x_2(t) \xrightarrow{F} X_1 X_2$

$$X(w) = \left(\frac{1}{1 + jw}\right)\left(\frac{1}{jw}\right)^2$$

(1.28) $x[n] = \alpha^n u[n]$, $|\alpha| < 1$

$$X(e^{jw}) = \sum_{n=0}^{\infty} \alpha^n e^{-jwn}$$

$$X(e^{jw}) = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$

$\sum_{n=0}^{\infty} r^n$, $|r| < 1$

$r = \alpha e^{-jw}$

$|r| = |\alpha|\,|e^{-jw}| = |\alpha| < 1$

$\left(\frac{1}{1 - r}\right)$

(*) $\boxed{X(e^{jw}) = \frac{1}{1 - \alpha e^{-jw}} , \ |\alpha| < 1}$

(1.39) $X(e^{jw}) = \frac{1}{1 - \frac{1}{2} e^{-jw}}$

$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn}\,dw$

(*) $\left(\alpha = \frac{1}{2}\right)$ $x[n] = \left(\frac{1}{2}\right)^n u[n]$

(1.33) $x[n] = \delta[n-3] + \delta[n+1]$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n-3] e^{-jwn} + \sum_{n=-\infty}^{\infty} \delta[n+1] e^{-jwn}$$

$\sum \delta[n - n_0] f[n] = f[n_0]$

$\sum \delta[n-3] e^{-jwn} = e^{-j3w}$ $\sum \delta[n+1] e^{-jwn} = e^{jw}$

$X(e^{jw}) = e^{-j3w} + e^{jw}$

(1.40) $X(e^{jw}) = e^{-j3w}$

$\delta[n] \leftrightarrow 1$

$x[n - n_0] \leftrightarrow e^{-jwn_0} X(e^{jw})$

$x[n] = \delta[n-3]$