# Lecture 2 & 3: Introduction to Systems
# Practice Exercises

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## 1 Practice Questions

**Question 1.1**

Consider the two systems

$$\mathcal{T}_1\{x(t)\} = 2x(t) \qquad \text{and} \qquad \mathcal{T}_2\{x(t)\} = x(t-1).$$

They are connected in **cascade**.

1. Find the overall system when the order is

$$x(t) \longrightarrow \mathcal{T}_1 \longrightarrow \mathcal{T}_2 \longrightarrow y(t).$$

2. Find the overall system when the order is

$$x(t) \longrightarrow \mathcal{T}_2 \longrightarrow \mathcal{T}_1 \longrightarrow y(t).$$

3. Are the two cascade connections equivalent?

**Question 1.2**

Let the systems

$$\mathcal{T}_1\{x(t)\} = 3x(t) \qquad \text{and} \qquad \mathcal{T}_2\{x(t)\} = \frac{d}{dt}x(t)$$

be connected in **parallel**.
Based on the parallel interconnection, answer the following:

1. Write the expressions for the two branch outputs $y_1(t)$ and $y_2(t)$.
2. Derive the overall output $y(t)$.
3. If the input is

$$x(t) = \sin t,$$

compute the final output explicitly.

**Question 1.3**

Two systems are defined as

$$\mathcal{T}_1\{x[n]\} = 2x[n] \qquad \text{and} \qquad \mathcal{T}_2\{x[n]\} = x[n-1].$$

They are connected in **parallel** (addition).

1. Write the expressions for the two branch outputs $y_1[n]$ and $y_2[n]$.

2. Derive the overall system output $y[n]$.

3. If the input signal is
$$x[n] = \delta[n],$$
compute the resulting output signal.

### Question 1.4

Consider the following **feedback system**.
Forward system:
$$\mathcal{T}_1\{x[n]\} = x[n]$$
Feedback system:
$$\mathcal{T}_2\{x[n]\} = 0.5\,x[n].$$
The system follows the feedback structure
$$b[n] = x[n] - v[n]$$
$$y[n] = \mathcal{T}_1\{b[n]\}$$
$$v[n] = \mathcal{T}_2\{y[n]\}.$$

1. Write $y[n]$ in terms of $b[n]$.
2. Express $v[n]$ in terms of $y[n]$.
3. Derive an equation relating $y[n]$ directly to $x[n]$.
4. Solve for the overall system input–output relation.

### Question 1.5

Consider the continuous-time system
$$y(t) = 3x(t) - x(t-2).$$
Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless (static) or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

### Question 1.6

Consider the continuous-time system
$$y(t) = (x(t))^2.$$
Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic

4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.7**

Consider the continuous-time system

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau.$$

Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.8**

Consider the continuous-time system

$$y(t) = t\, x(t).$$

Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.9**

Consider the discrete-time system

$$y[n] = x[n] + x[n-1].$$

Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.10**

Consider the discrete-time system

$$y[n] = x[n] + x[n-1].$$

Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.11**

Consider the discrete-time system

$$y[n] = x[n+1].$$

Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.12**

Consider the discrete-time system

$$y[n] = n\,x[n].$$

Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.13**

Consider the continuous-time system

$$y(t) = x(t+1) + x(t-1).$$

Classify the system according to the following properties:

1. Deterministic or stochastic

2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.14**

Consider the discrete-time system
$$y[n] = x[2n].$$
Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.15**

Consider the discrete-time system
$$y[n] = 2x[n] + 1.$$
Classify the system according to the following properties:

1. Deterministic or stochastic
2. SISO or MIMO
3. Memoryless or dynamic
4. Causal, anti-causal, or non-causal
5. Linear or non-linear
6. Time-invariant or time-varying

**Question 1.16**

Consider the continuous-time system described by the differential equation
$$\frac{dy(t)}{dt} + 2y(t) = 3x(t).$$
Answer the following:

1. What is the order of the system?
2. Is the system linear or non-linear?
3. Is the system time-invariant or time-varying?
4. Does the system have memory?
5. Is the system causal?

### Question 1.17

Consider the discrete-time system described by the difference equation

$$y[n] - 0.5\,y[n-1] = x[n] + 2x[n-1].$$

Answer the following:

1. What is the order of the system?
2. Is the system linear or non-linear?
3. Is the system time-invariant or time-varying?
4. Does the system have memory?
5. Is the system causal?

### Question 1.18

Consider the continuous-time system

$$y(t) = 3x(t).$$

Determine whether the system is **BIBO stable**.
Recall that a system is BIBO stable if every bounded input produces a bounded output.

### Question 1.19

Consider the continuous-time system

$$y(t) = t\,x(t).$$

Determine whether the system is **BIBO stable**.

### Question 1.20

Consider the discrete-time system

$$y[n] = 0.4\,x[n] + 0.2\,x[n-1].$$

Determine whether the system is **BIBO stable**.

### Question 1.21

Consider the discrete-time system

$$y[n] = x[n] + 1.2\,y[n-1].$$

Determine whether the system is **BIBO stable**.

### Question 1.22

Consider the continuous-time system described by

$$\frac{dy(t)}{dt} + y(t) = x(t).$$

Suppose

$$x(t) = 0$$

and the initial condition is

$$y(0) = 3.$$

1. Determine the **zero-input response**.

2. Determine whether a zero-state response exists.

3. Write the total response.

**Question 1.23**

Consider the continuous-time system

$$\frac{dy(t)}{dt} + 2y(t) = x(t).$$

Assume

$$x(t) = 1$$

and the initial condition

$$y(0) = 0.$$

1. Determine the zero-input response.

2. Determine the zero-state response.

3. Write the total response.

**Question 1.24**

Consider the discrete-time system

$$y[n] = x[n] + 0.5\,y[n-1].$$

Suppose

$$x[n] = 0$$

and the initial condition

$$y[-1] = 4.$$

1. Determine the zero-input response.

2. Determine the zero-state response.

3. Write the total response.

**Question 1.25**

Consider the discrete-time system

$$y[n] = x[n] + 0.5\,y[n-1].$$

Assume

$$x[n] = 1$$

and

$$y[-1] = 0.$$

1. Determine the zero-input response.

2. Determine the zero-state response.

3. Compute the first four values of the total response.

**Question 1.26**

Consider the continuous-time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t),$$

with zero initial conditions.
Compute the **impulse response** $h(t)$ of the system.

**Question 1.27**

Consider the discrete-time LTI system

$$y[n] = x[n] + 0.5\,y[n-1],$$

with zero initial conditions.
Compute the **impulse response** $h[n]$ of the system.

**Question 1.28**

Compute the convolution

$$y(t) = (x * h)(t)$$

for

$$x(t) = u(t)$$

$$h(t) = e^{-t}u(t).$$

**Question 1.29**

Compute the convolution

$$y(t) = (x * h)(t)$$

for

$$x(t) = u(t) - u(t-2)$$

$$h(t) = u(t).$$

**Question 1.30**

Compute

$$y(t) = (x * h)(t)$$

for

$$x(t) = e^{-t}u(t)$$

$$h(t) = u(t).$$

**Question 1.31**

Compute

$$y(t) = (x * h)(t)$$

for

$$x(t) = \delta(t-1)$$

$$h(t) = e^{-t}u(t).$$

**Question 1.32**

Compute the convolution

$$y[n] = (x * h)[n]$$

for

$$x[n] = \{1, 2, 1\}, \quad n = 0, 1, 2$$

$$h[n] = \{1, 1\}, \quad n = 0, 1.$$

**Question 1.33**

Compute

$$y[n] = (x * h)[n]$$

for

$$x[n] = \{1, 1, 1\}$$

$$h[n] = \{1, -1\}.$$

**Question 1.34**

Compute the convolution

$$y[n] = (x * h)[n]$$

for

$$x[n] = u[n]$$

$$h[n] = (0.5)^n u[n].$$

**Question 1.35**

Compute

$$y[n] = (x * h)[n]$$

for

$$x[n] = \delta[n-2]$$

$$h[n] = (0.5)^n u[n].$$

**Question 1.36**

Let

$$x(t) = \delta(t-3)$$

$$h(t) = e^{-t}u(t)$$

Compute

$$y(t) = (x * h)(t).$$

Use convolution properties.

**Question 1.37**

Let

$$x[n] = u[n]$$

$$h[n] = (0.5)^n u[n]$$

Given that

$$(x * h)[n] = 2(1 - (0.5)^{n+1})u[n],$$

compute

$$y[n] = (x * h)[n-2].$$

Use convolution properties.

**Question 1.38**

Let

$$x[n] = \delta[n-1]$$

$$h[n] = (0.5)^n u[n]$$

Compute

$$y[n] = (x * h)[n]$$

using convolution properties.

**Question 1.39**

Let

$$x[n] = \{1, 2, 1\}, \quad n = 0, 1, 2$$

$$y[n] = \{1, 1\}, \quad n = 0, 1$$

Compute the cross-correlation

$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]y[n+m].$$

**Question 1.40**

Let

$$x[n] = \begin{cases} 1 & n = 0, 1, 2 \end{cases}$$

$$y[n] = \begin{cases} 1 & n = -1, 0 \end{cases}$$

Compute $R_{xy}[m]$.

**Question 1.41**

Let

$$x[n] = \{1, 2, 1\}, \quad n = 0, 1, 2$$

Compute the autocorrelation

$$R_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]x[n+m].$$

**Question 1.42**

Let

$$x(t) = u(t)$$

$$y(t) = u(t)$$

Compute

$$R_{xy}(\tau).$$

**Question 1.43**

Let

$$x(t) = u(t) - u(t-2)$$

$$y(t) = u(t) - u(t-1)$$

Compute the cross-correlation.

**Question 1.44**

Let

$$x(t) = u(t) - u(t-1)$$

Compute the autocorrelation $R_{xx}(\tau)$.

## 2 Coding Exercises

**Question 2.1**

Let

$$x(t) = u(t) - u(t-2), \qquad h(t) = u(t) - u(t-1).$$

Approximate the continuous-time convolution

$$y(t) = (x * h)(t)$$

by sampling the signals on the interval $t \in [-1, 4]$ using 4000 equally spaced points.
Use `np.convolve` and the approximation

$$y(t) \approx \Delta t \cdot \texttt{convolve}(x, h),$$

where $\Delta t$ is the sampling step. Continuous convolution is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$, which can be approximated by a Riemann sum when the signals are sampled with step $\Delta t$. Thus $y(t_n) \approx \sum_k x[k]h[n-k]\Delta t = \Delta t\,\mathrm{conv}(x, h)$, giving $y(t) \approx \Delta t\,\mathrm{np.convolve}(x, h)$.
Plot on the same figure:

$$x(t), \quad h(t), \quad y(t).$$

Then answer:

1. What is the support of $x(t)$ and $h(t)$?
2. Over what interval is the convolution nonzero?
3. What is the qualitative shape of $y(t)$?

**Question 2.2**

Let

$$x(t) = e^{-t}u(t), \qquad h(t) = u(t) - u(t-3).$$

Approximate the continuous-time convolution

$$y(t) = (x * h)(t)$$

on the interval $t \in [-1, 8]$ using 5000 points.
Use `np.convolve` and the approximation

$$y(t) \approx \Delta t \cdot \texttt{convolve}(x, h),$$

where $\Delta t$ is the sampling step. Continuous convolution is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$, which can be approximated by a Riemann sum when the signals are sampled with step $\Delta t$. Thus $y(t_n) \approx \sum_k x[k]h[n-k]\Delta t = \Delta t\,\mathrm{conv}(x, h)$, giving $y(t) \approx \Delta t\,\mathrm{np.convolve}(x, h)$.
Plot on the same figure:

$$x(t), \quad h(t), \quad y(t).$$

Then answer:

1. Why is $y(t) = 0$ for $t < 0$?
2. For large $t$, does the output keep growing, saturate, or decay?
3. Which signal acts like a finite moving window in the convolution?

**Question 2.3**

Let

$$x(t) = u(t) - u(t-2), \qquad y(t) = u(t-1) - u(t-3).$$

Approximate the continuous-time cross-correlation

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t+\tau)\,dt$$

by sampling both signals on the interval $t \in [-1, 5]$ using 4000 points. Continuous correlation is $R_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\,y(\tau+t)\,d\tau$, which can be approximated by a Riemann sum when the signals are sampled with step $\Delta t$. Thus $R_{xy}[n] \approx \sum_k x[k]\,y[k+n]\Delta t = \Delta t\,\mathrm{corr}(x, y)$, giving $R_{xy} \approx \Delta t\,\mathrm{scipy.signal.correlate}(x, y)$.
Plot on the same figure:

$$x(t), \quad y(t), \quad R_{xy}(\tau).$$

Then answer:

1. At what lag is the correlation maximum?
2. Why does the maximum occur there?
3. What is the shape of the correlation curve?

## Question 2.4

Let

$$x[n] = \{1, 2, 1\}, \qquad h[n] = \{1, 1, -1\}.$$

Implement the discrete-time convolution

$$y[n] = (x * h)[n]$$

**without using** `np.convolve` or `scipy.signal.correlate`.
Use nested loops to compute all output samples, then stem-plot:

$$x[n], \quad h[n], \quad y[n].$$

Then answer:

1. What is the output length?
2. What is the first nonzero output index if both inputs start at $n = 0$?
3. Verify the formula
$$L_y = L_x + L_h - 1.$$

## Question 2.5

Let

$$x[n] = \{1, 0, 2, 1\}, \qquad y[n] = \{1, 1, 0\}.$$

Implement the discrete-time cross-correlation

$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$$

**without using** `np.correlate` or `scipy.signal.correlate`.
Assume both sequences start at $n = 0$. Compute all valid lags, stem-plot the result, and answer:

1. What is the valid lag range?
2. At which lag does the maximum correlation occur?
3. What does that lag mean geometrically?

**Question 2.6**

Let

$$x[n] = \{1, 2, 1\}, \qquad h[n] = \{1, -1\}.$$

Implement the discrete-time convolution manually, and also compute it by calling `np.convolve`. Plot both outputs on the same figure using stem plots and verify that they match exactly.
Then answer:

1. Why is comparing the manual result with `np.convolve` useful?
2. What does the sequence `h[n]={1,-1}` do to a slowly varying signal?
3. Is the output length what you expected from the length formula?

# 3 Proof Questions

**Question 3.1**

Let $x(t)$ and $h(t)$ be two signals. The continuous-time convolution is defined as

$$(x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau.$$

Prove step-by-step that convolution is commutative:

$$x * h = h * x.$$

**Question 3.2**

Consider an LTI system with impulse response $h(t)$.
The output for an input $x(t)$ is

$$y(t) = (x * h)(t).$$

Let the input be

$$x(t) = a\, x_1(t) + b\, x_2(t).$$

Prove that the system is linear, i.e.

$$\mathcal{T}\{ax_1 + bx_2\} = a\mathcal{T}\{x_1\} + b\mathcal{T}\{x_2\}.$$

**Question 3.3**

Let a system be defined by convolution

$$y(t) = (x * h)(t).$$

Prove that this system is time-invariant.

**Question 3.4**

Let an LTI system have impulse response $h(t)$.
Prove that the output for any input $x(t)$ can be written as

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau.$$

**Question 3.5**

Let the autocorrelation of a real signal $x(t)$ be

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt.$$

Prove that

$$R_{xx}(\tau) = R_{xx}(-\tau).$$

**Question 3.6**

The cross-correlation of two real signals is

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t+\tau)dt.$$

Show that cross-correlation can be written as a convolution

$$R_{xy}(\tau) = (x * y^{-})(\tau),$$

where

$$y^{-}(t) = y(-t).$$

**Question 3.7**

Let $x[n]$ and $h[n]$ be two discrete-time signals. The convolution is defined as

$$(x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k].$$

Prove step-by-step that discrete-time convolution is commutative:

$$x * h = h * x.$$

**Question 3.8**

Consider a discrete-time LTI system with impulse response $h[n]$. Its output for an input $x[n]$ is

$$y[n] = (x * h)[n].$$

Prove step-by-step that this system is time-invariant. That is, if the input is shifted by $n_0$,

$$x_1[n] = x[n - n_0],$$

then the output is also shifted by the same amount:

$$y_1[n] = y[n - n_0].$$

**Question 3.9**

Let the cross-correlation of two real-valued discrete-time signals $x[n]$ and $y[n]$ be defined as

$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\, y[n+m].$$

Define

$$y^{-}[n] = y[-n].$$

Prove step-by-step that cross-correlation can be written as a convolution:

$$R_{xy}[m] = (x * y^{-})[m].$$