Lecture 1: Introduction to Signals
Practice Exercises

Konstantinos Chatzilygeroudis
costashatz@upatras.gr

19 February 2026

1 Practice Questions

Question 1.1

A signal is described by:

- its **domain** (input): time can be **continuous** ($t \in \mathbb{R}$) or **discrete** ($n \in \mathbb{Z}$),
- its **codomain** (output): amplitude can be **continuous** (real-valued) or **discrete** (quantized levels).

Based on this, classify each of the following as **analog**, **discrete-time (sampled)**, **quantized**, or **digital**. For each one, explicitly state: (i) whether time is continuous/discrete and (ii) whether amplitude is continuous/discrete.

1. $x_1(t) = 2\cos(t)$
2. $x_2[n] = 2\cos(0.3n)$
3. $x_3(t) = Q_\Delta\{2\cos(t)\}$ where $Q_\Delta\{x\} = \Delta \cdot \text{round}(x/\Delta)$ and $\Delta = 0.25$
4. $x_4[n] = Q_\Delta\{2\cos(0.3n)\}$ with $\Delta = 0.25$

Question 1.2

Let $x(t) = 2\cos(t)$ and define the uniform quantizer

$$Q_\Delta\{x\} = \Delta \cdot \text{round}\left(\frac{x}{\Delta}\right), \qquad \Delta = 0.5.$$

1. What is the range of $x(t)$ (minimum and maximum)?
2. List all possible output values of $x_Q(t) = Q_\Delta\{x(t)\}$ (i.e., all quantization levels that can appear).
3. What is the maximum possible quantization error $|x_Q(t) - x(t)|$?

Question 1.3

Start from the continuous-time signal $x(t) = 2\cos(t)$.

1. Define a sampled signal $x_s[n] = x(nT_s)$ with sampling period $T_s = \pi/4$. Write $x_s[n]$ explicitly as a function of n.
2. Is $x_s[n]$ discrete-time or continuous-time? Is its amplitude discrete or continuous?
3. Now quantize: $x_D[n] = Q_\Delta\{x_s[n]\}$ with $\Delta = 0.25$. Is $x_D[n]$ digital? Explain.

We define $Q_\Delta\{x\} = \Delta \cdot \text{round}\left(\frac{x}{\Delta}\right)$.

Question 1.4

Consider the continuous-time signal

$$x(t) = \begin{cases} 3, & -2 \le t < 1, \\ 0, & \text{otherwise.} \end{cases}$$

1. Express $x(t)$ in a single equation (hint: use elementary signals).
2. Verify your expression by checking the value of $x(t)$ in the three regions: (i) $t < -2$, (ii) $-2 \le t < 1$, (iii) $t \ge 1$.

Question 1.5

Let

$$x(t) = \begin{cases} t-1, & 1 \le t < 4, \\ 0, & \text{otherwise.} \end{cases}$$

1. Express $x(t)$ in one equation (hint: use elementary signals).
2. Expand your expression into a form that uses only the unit step ($u(\cdot)$) and polynomials in t.

Question 1.6

Let

$$x(t) = \begin{cases} e^{-t}, & 0 \le t < 2, \\ 2e^{-t}, & 2 \le t < 5, \\ 0, & \text{otherwise.} \end{cases}$$

1. Express $x(t)$ in one single equation.
2. Evaluate your expression for $t \in (-1, 0)$, $t \in (1, 2)$, $t \in (3, 5)$, and $t > 5$.

Question 1.7

Express the following discrete-time signal in a single equation:

$$x[n] = \begin{cases} 4, & -1 \le n \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

Question 1.8

Let

$$x[n] = \begin{cases} n, & 0 \le n \le 5, \\ 0, & \text{otherwise.} \end{cases}$$

1. Express $x[n]$ in a single equation.
2. Compute $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$.

Question 1.9

Consider the continuous-time signal

$$x(t) = 2\cos(5t).$$

A continuous time signal is periodic if there exists $T > 0$ such that $x(t) = x(t+T)$ for all t.

1. Show that $x(t)$ is periodic.
2. Find the *fundamental* period T_0 (smallest positive period).

Question 1.10

Let

$$x(t) = 2\cos(2\pi t) + 2\cos\left(2\pi\sqrt{2}\,t\right).$$

1. Compute the individual periods T_1 and T_2.
2. Decide whether $x(t)$ is periodic by checking the ratio T_1/T_2.

Question 1.11

Let

$$x(t) = \cos(4\pi t) + \cos(6\pi t).$$

1. Find the periods T_1 and T_2 of the two cosines.
2. Determine whether the sum is periodic.
3. If it is periodic, find a period T of the sum (smallest positive period if you can).

Question 1.12

Consider the continuous time signal

$$x(t) = \cos(2\pi t) + \cos(4\pi t) + \cos(6\pi t).$$

1. Find the period of each term.
2. Determine whether $x(t)$ is periodic.
3. If it is periodic, find the fundamental period of $x(t)$.

Question 1.13

Consider the continuous time signal

$$x(t) = \cos(2\pi t) + \cos\left(2\pi\sqrt{2}\,t\right) + \cos(4\pi t).$$

Is $x(t)$ periodic? Justify by examining period ratios.

Question 1.14

Consider the discrete-time signal

$$x[n] = \cos\left(\frac{2\pi}{5}n\right) + \cos\left(\frac{2\pi}{6}n\right) + \cos\left(\frac{2\pi}{10}n\right).$$

1. Determine the period of each term.
2. Determine whether the sum is periodic.
3. If so, find the (smallest) period of the sum.

Question 1.15

Consider the continuous time signal

$$x(t) = e^{-2(t-1)}u(t-1).$$

1. Write $x(t)$ as a piecewise-defined function.
2. Determine whether $x(t)$ is causal, anti-causal, or two-sided.
3. State precisely the set of t for which $x(t) \neq 0$.

Question 1.16

Let

$$x(t) = e^{t}u(-t).$$

1. Write $x(t)$ piecewise.
2. Classify it as causal/anti-causal/two-sided.

Question 1.17

Let

$$x(t) = e^{-t}u(t) + e^{t}u(-t).$$

1. Write $x(t)$ piecewise.
2. Determine if it is causal, anti-causal, or two-sided.

Question 1.18

Consider the discrete-time signal

$$x[n] = e^{-2(n-1)}u[n-1].$$

1. Write $x[n]$ as a piecewise-defined sequence.
2. Determine whether $x[n]$ is causal, anti-causal, or two-sided.
3. State precisely the set of integers n for which $x[n] \neq 0$.

Question 1.19

Let

$$x[n] = e^{n}u[-n].$$

1. Write $x[n]$ piecewise.
2. Classify it as causal/anti-causal/two-sided.

Question 1.20

Let

$$x[n] = e^{-n}u[n] + e^{n}u[-n].$$

1. Write $x[n]$ piecewise.
2. Determine if it is causal, anti-causal, or two-sided.

Question 1.21

Let

$$x(t) = 2\cos(t) + t.$$

1. Compute $x(-t)$.
2. Compute the even part $x_e(t) = \frac{1}{2}(x(t) + x(-t))$.
3. Compute the odd part $x_o(t) = \frac{1}{2}(x(t) - x(-t))$.
4. Verify explicitly that $x(t) = x_e(t) + x_o(t)$.

Question 1.22

Let

$$x(t) = e^{-t}u(t).$$

1. Compute $x(-t)$.
2. Compute $x_e(t)$ and $x_o(t)$.
3. For which t is $x_e(t)$ nonzero? For which t is $x_o(t)$ nonzero?

Question 1.23

Define the triangular pulse

$$x(t) = \begin{cases} 1 - \frac{2|t|}{T}, & |t| \leq \frac{T}{2}, \\ 0, & \text{otherwise}, \end{cases} \qquad \text{with } T > 0.$$

1. Show that $x(t)$ is even.
2. Compute $x_e(t)$ and $x_o(t)$ using the formulas and simplify.

Question 1.24

Let $x(t) = e^{-t}u(t)$.

1. Compute the energy $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$ step-by-step.
2. Decide whether $x(t)$ is an energy signal or a power signal.

Question 1.25

Let

$$x(t) = 2e^{-(t-1)}u(t-1).$$

Compute the energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

step-by-step.

Question 1.26

Let

$$x[n] = \left(\frac{1}{2}\right)^n u[n].$$

Compute the energy

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

step-by-step.

Question 1.27

Let $x[n] = e^{-n}u[n]$.

1. Compute the energy $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$ step-by-step.
2. Decide whether $x[n]$ is an energy signal or a power signal.

Question 1.28

Let

$$x[n] = 2e^{-(n-1)}u[n-1].$$

Compute the energy

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

step-by-step.

Question 1.29

Let

$$x(t) = 1 + 2\cos(3t).$$

Compute the average power P.

Question 1.30

Let the continuous time rectangular pulse be

$$x(t) = \begin{cases} 2, & |t| \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

1. Compute its energy E.
2. Compute its average power P.

Question 1.31

Let $x(t) = 2\cos(3t)$.

1. Compute the average power P.
2. Compute the RMS value x_{rms}.

Question 1.32

Let

$$x(t) = 2\cos(2t) + 3\sin(5t).$$

Compute the average power P of $x(t)$.

Question 1.33

Let $x[n] = 2\cos(3n)$.

1. Compute the average power P.
2. Compute the RMS value x_{rms}.

Question 1.34

Let

$$x(t) = 2\cos(4t) + 3\sin(4t).$$

1. Rewrite $x(t)$ as a single sinusoid $A\cos(4t - \phi)$ for suitable A, ϕ.
2. Compute the power P of $x(t)$.

Question 1.35

Let

$$x[n] = 2\cos\left(\frac{\pi}{3}n\right).$$

1. Show that $x[n]$ is periodic and find its fundamental period N_0.
2. Compute its average power using one period:

$$P = \frac{1}{N_0}\sum_{n=0}^{N_0-1} |x[n]|^2.$$

Question 1.36

Let $x[n] = 2\cos(3n)$.

1. Compute the average power P.
2. Compute the RMS value x_{rms}.

Question 1.37

Let

$$x(t) = e^{(-1+j2)t}u(t).$$

1. Compute $|x(t)|$ explicitly.
2. Compute the energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

3. Decide whether $x(t)$ is an energy signal or a power signal.

Question 1.38

Let

$$x[n] = (0.8)^n e^{j\pi n/4}u[n].$$

1. Compute $|x[n]|$.
2. Compute the energy

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2.$$

Question 1.39

Let

$$x(t) = 3e^{j(4t+\pi/3)}.$$

1. Compute $|x(t)|^2$.
2. Compute the average power P.

Question 1.40

Let

$$x[n] = e^{j\pi n/3} + 2e^{-j\pi n/3}.$$

1. Compute $|x[n]|^2$ explicitly.

2. Compute the average power

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2.$$

2 Coding Exercises

Question 2.1

Let $x(t) = 2\cos(t)$. On the interval $t \in [0, 4\pi]$, plot on the same figure:

$$x(t), \quad x(t-\pi), \quad x(2t), \quad -\tfrac{1}{2}x(t).$$

Then answer:

1. Which curves are time-shifted? By how much and in which direction?
2. Which curve is time-compressed? How can you tell from the number of oscillations?
3. Which curve is amplitude-inverted and attenuated? Explain from the formula.

Use 3000 points in the interval $[0, 4\pi]$ for a smooth plot.

Question 2.2

Define the discrete time signal

$$x[n] = 2n\cos\left(\frac{\pi}{4}n\right), \qquad n = -20, \dots, 20.$$

Plot (using a stem plot) the following sequences on separate subplots or separate figures:

$$x[n], \quad x[n-3], \quad x[-n], \quad 2x[n].$$

Then answer:

1. Which operation corresponds to delaying by 3 samples?
2. Which operation corresponds to time reversal?

Question 2.3

Let $x(t) = 2\cos(t)$ on $t \in [0, 2\pi]$ and define $x_Q(t) = Q_\Delta\{x(t)\}$ with $\Delta = 0.25$.

1. Plot $x(t)$ and $x_Q(t)$ on the same axes.
2. Plot the quantization error $e(t) = x_Q(t) - x(t)$.
3. From the error plot, estimate $\max_t |e(t)|$ and compare to the theoretical bound $\Delta/2$.

Use 2000 points in the interval $[0, 2\pi]$ for a smooth plot.

$$Q_\Delta\{x\} = \Delta \cdot \mathrm{round}(x/\Delta)$$

Question 2.4

Consider a reference power $P_1 = 1$ (arbitrary units). For each power ratio

$$\frac{P_2}{P_1} \in \{0.5,\ 2,\ 10,\ 100\},$$

compute the corresponding dB value using

$$\text{dB} = 10\log_{10}\left(\frac{P_2}{P_1}\right).$$

Then answer:

1. Which ratio is closest to +3 dB? Explain numerically.
2. Verify that +10 dB corresponds to ×10 in power.
3. What dB value corresponds to halving the power?

Question 2.5

Let $A_1 = 1$ and consider amplitude ratios

$$\frac{A_2}{A_1} \in \{0.5,\ 2,\ 10\}.$$

1. Compute $\text{dB}_A = 20\log_{10}(A_2/A_1)$ for each ratio.
2. Compute the corresponding power ratio assuming $P \propto A^2$, i.e. $P_2/P_1 = (A_2/A_1)^2$.
3. Compute $\text{dB}_P = 10\log_{10}(P_2/P_1)$ and verify $\text{dB}_A = \text{dB}_P$.

Question 2.6

Dynamic range and relative dB.
Consider the continuous-time signal

$$x(t) = e^{\sigma|t|}\Big(\cos(2\pi f_0 t) + a\cos(2\pi f_1 t)\Big), \qquad \sigma < 0,$$

where frequency f is in Hz.

Its continuous-time Fourier transform (CTFT)[a] is given by: For $\alpha > 0$,

$$\int_{-\infty}^{\infty} e^{-\alpha|t|}e^{-j2\pi ft}\,dt = \frac{2\alpha}{\alpha^2 + (2\pi f)^2}.$$

Using modulation (frequency shift),

$$\mathcal{F}\{e^{-\alpha|t|}\cos(2\pi f_c t)\}(f) = \alpha\left(\frac{1}{\alpha^2 + (2\pi(f-f_c))^2} + \frac{1}{\alpha^2 + (2\pi(f+f_c))^2}\right),$$

where $\alpha = -\sigma > 0$.

Task. Using $\sigma = -8$, $f_0 = 60$ Hz, $f_1 = 200$ Hz, $a = 0.005$, and

$$f \in [0, 1000] \text{ Hz with 4096 points},$$

do the following:

1. Implement a function `X_of_f(f, sigma, f0, f1, a)` that returns $X(f)$ using the formula above.
2. Compute $m(f) = |X(f)|$ and plot $m(f)$ in linear scale.
3. Plot the **relative dB** magnitude

$$m_{\text{dB}}(f) = 20\log_{10}\left(\frac{m(f)}{m_{\max}} + \varepsilon\right), \qquad \varepsilon = 10^{-12}.$$

4. Compute and print

$$m_{\max}, \quad m_{\min}, \quad \Delta_{\text{dB}} = 20\log_{10}\left(\frac{m_{\max}}{m_{\min} + \varepsilon}\right).$$

5. Change a to 0.05. What changes in the relative dB plot?

Use 4096 samples in the $[0, 1000]$ Hz (frequency) interval for smooth plots.

[a] We will learn how to compute this in future lectures

3 Proof Questions

Question 3.1

Let $x(t)$ be any signal with finite energy

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty.$$

Define the time-shifted signal $y(t) = x(t - t_0)$ for some real t_0. Prove step-by-step that $E_y = E_x$.

Question 3.2

Assume $x(t)$ has finite average power

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.$$

Let $y(t) = \alpha x(t)$ for some complex (or real) constant α. Prove that $P_y = |\alpha|^2 P_x$.

Question 3.3

A signal $x(t)$ can be decomposed as $x(t) = x_e(t) + x_o(t)$ where x_e is even and x_o is odd. Prove that this decomposition is unique.

Question 3.4

Let $x(t)$ and $y(t)$ be signals. Recall:

$$x \text{ is even } \Leftrightarrow x(t) = x(-t), \qquad x \text{ is odd } \Leftrightarrow x(t) = -x(-t).$$

Prove the following statements:

1. even + even $\Rightarrow$ even
2. odd + odd $\Rightarrow$ odd
3. even $\times$ even $\Rightarrow$ even
4. odd $\times$ odd $\Rightarrow$ even
5. even $\times$ odd $\Rightarrow$ odd

Question 3.5

Let the function Kronecker delta be:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}.$$

Prove:

1. $u[n] = \sum_{k=-\infty}^{n} \delta[k]$.
2. $\sum_{n=-\infty}^{\infty} x[n]\delta[n - n_0] = x[n_0]$.