Introduction to Robotics 2016-7

Consider the first 3 DoF of the modified PUMA 560 arm, shown in Figure 1

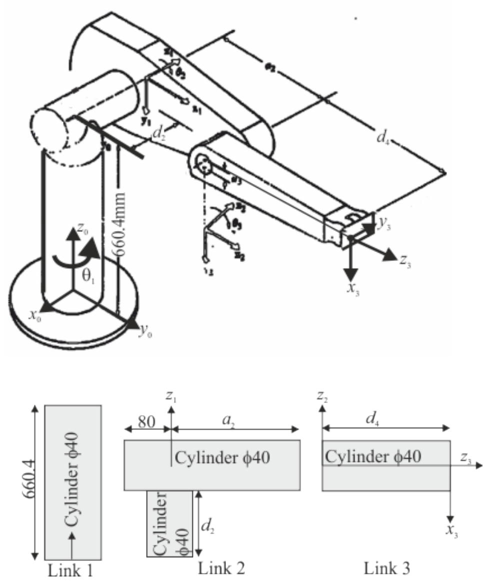

Figure 1 3DoF PUMA 560 robot arm (w./ kinematic parameters)

Note the translation of the 0th-coordinate system at the base of the robot arm, and the translation of 3rd-coordinate system at the tip of the 3rd link (compare them w.r.t. the 1st project). At the bottom of the drawing, the dimensions of each joint are shown; the 1st and 3rd joints have a cylindrical shape while the 2nd joint is composed of the superposition of two cylinders. The circular cross section area of each mentioned cylinder has a diameter of 40mm. The material used in the `filling' of the cylinders has a mass density of $10gr/mm^3$

1. Compute in symbolic form the matrix $A_0^3(\theta_1,\theta_2,\theta_3) = A_0^3\left(\overline{\theta}\right)$
2. Compute the 4×4 inertia matrices J_i of each link $i = 1,2,3$
3. Compute in symbolic form the dynamics of the 3DoF manipulator $D_{3\times 3}\left(\overline{\theta}\right)\ddot{\overline{\theta}} + C_{3\times 1}\left(\overline{\theta},\dot{\overline{\theta}}\right) + G_{3\times 1}\left(\overline{\theta}\right) = \overline{\tau}_{3\times 1}$,
4. Plot $\overline{\tau}(t), t\in[0,12]$ for joint trajectories (expressed in degrees) $\overline{\theta}^d(t) = \begin{bmatrix} A_1\sin\left(\frac{2\pi}{T_1}t\right) \\ A_2\sin\left(\frac{2\pi}{T_2}t\right) \\ 40^\circ + A_3\sin\left(\frac{2\pi}{T_3}t\right)\end{bmatrix}$, where

   $A_1 = 2A_2 = 2A_3 = 80^\circ$ and $T_1 = 2T_2 = 3T_3 = 6$ seconds. Record $\overline{\tau}_{\max} = \begin{bmatrix}\tau_1^{\max} \\ \tau_2^{\max} \\ \tau_3^{\max}\end{bmatrix} = \begin{bmatrix}\max_t \tau_1(t) \\ \max_t \tau_2(t) \\ \max_t \tau_3(t)\end{bmatrix}$
5. Assume the trajectories defined in the previous question as the desired ones. Design a computed torque controller $\overline{\tau}_{3\times 1} = D_{3\times 3}\left(\overline{\theta}\right)\left[\ddot{\overline{\theta}}^d + K_d\left(\dot{\overline{\theta}}^d - \dot{\overline{\theta}}\right) + K_p\left(\overline{\theta}^d - \overline{\theta}\right)\right] + C_{3\times 1}\left(\overline{\theta},\dot{\overline{\theta}}\right) + G_{3\times 1}\left(\overline{\theta}\right)$ that when applied to the robot forces its joints to track these desired angles for $\overline{\theta}(0) = \begin{bmatrix}-160^\circ \\ -225^\circ \\ -45^\circ\end{bmatrix}$. Assume that K_d, K_p are diagonal positive matrices. Select their diagonal elements so that you have a satisfactory response, while the maximum torque applied to the motors does not exceed $2\overline{\tau}_{\max}$