

Feedback View of Oscillators

➤ An oscillator may be viewed as a “badly-designed” negative-feedback amplifier—so badly designed that it has a zero or negative phase margin.

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$$

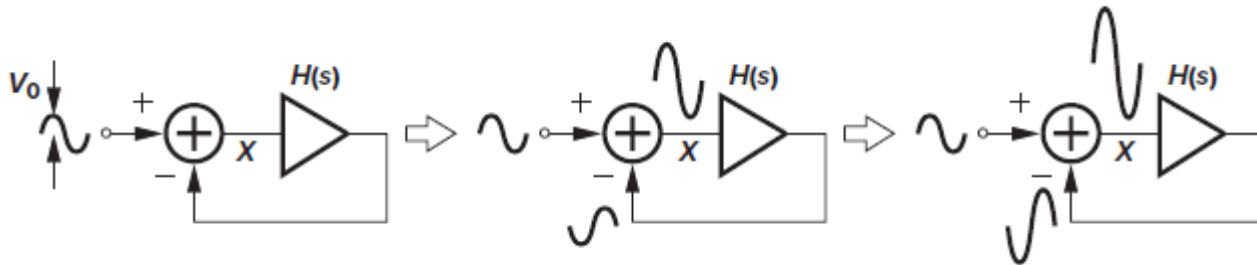
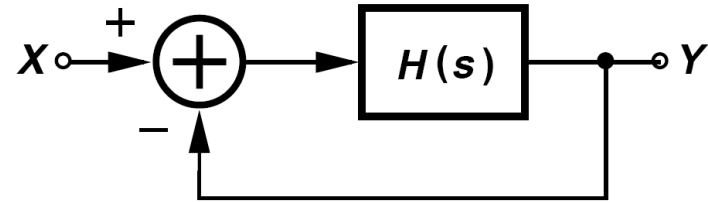


Figure 15.2 Evolution of oscillatory system with time.

In summary, if a negative-feedback circuit has a loop gain that satisfies two conditions:

$$|H(j\omega_0)| \geq 1$$

$$\angle H(j\omega_0) = 180^\circ$$

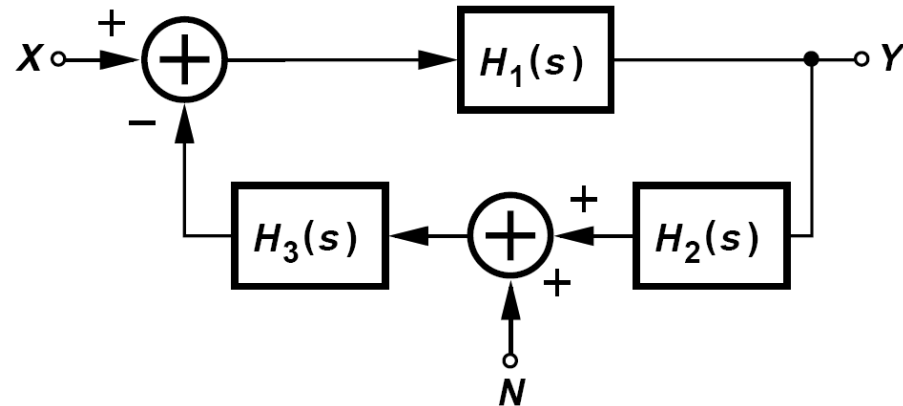
then the circuit may oscillate at ω_0 . Called “Barkhausen criteria,” these conditions are necessary

Feedback View of Oscillators (II)

For the above system to oscillate, must the noise at ω_1 appear at the input?

No, the noise can be anywhere in the loop. For example, consider the system shown in figure below, where the noise N appears in the feedback path. Here,

$$Y(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)H_3(s)}X(s) + \frac{H_1(s)H_3(s)}{1 + H_1(s)H_2(s)H_3(s)}N(s).$$



Thus, if the loop transmission, $H_1H_2H_3$, approaches -1 at ω_1 , N is also amplified indefinitely.

Y/X in the Vicinity of $\omega = \omega_1$

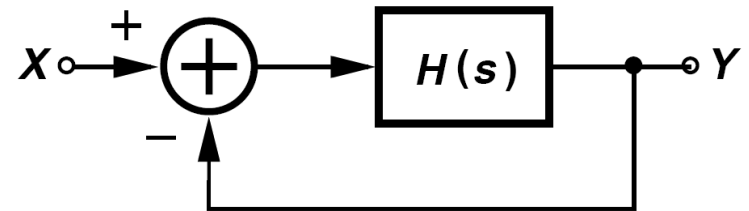
Derive an expression for Y/X in figure below in the vicinity of $\omega = \omega_1$ if $H(j\omega_1) = -1$.

We can approximate $H(j\omega)$ by the first two terms in its Taylor series:

$$H[j(\omega_1 + \Delta\omega)] \approx H(j\omega_1) + \Delta\omega \frac{dH(j\omega_1)}{d\omega}$$

Since $H(j\omega_1) = -1$, we have

$$\begin{aligned} \frac{Y}{X}[j(\omega_1 + \Delta\omega)] &= \frac{H(j\omega_1) + \Delta\omega \frac{dH(j\omega_1)}{d\omega}}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}} \\ &\approx \frac{H(j\omega_1)}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}} \\ &\approx \frac{-1}{\Delta\omega \frac{dH(j\omega_1)}{d\omega}} \end{aligned}$$

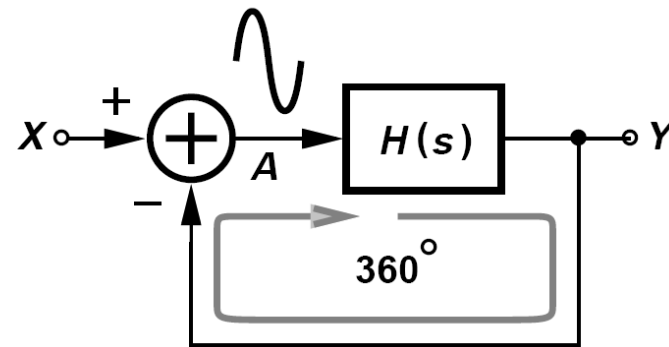
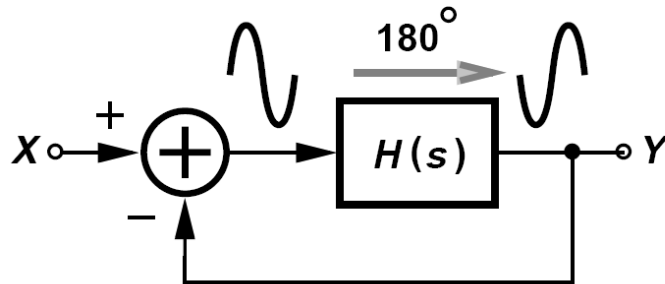


As expected, $Y/X \rightarrow \infty$ as $\Delta\omega \rightarrow 0$, with a “sharpness” proportional to $dH/d\omega$.

Barkhausen's Criteria

$$|H(s = j\omega_1)| = 1$$

$$\angle H(s = j\omega_1) = 180^\circ$$

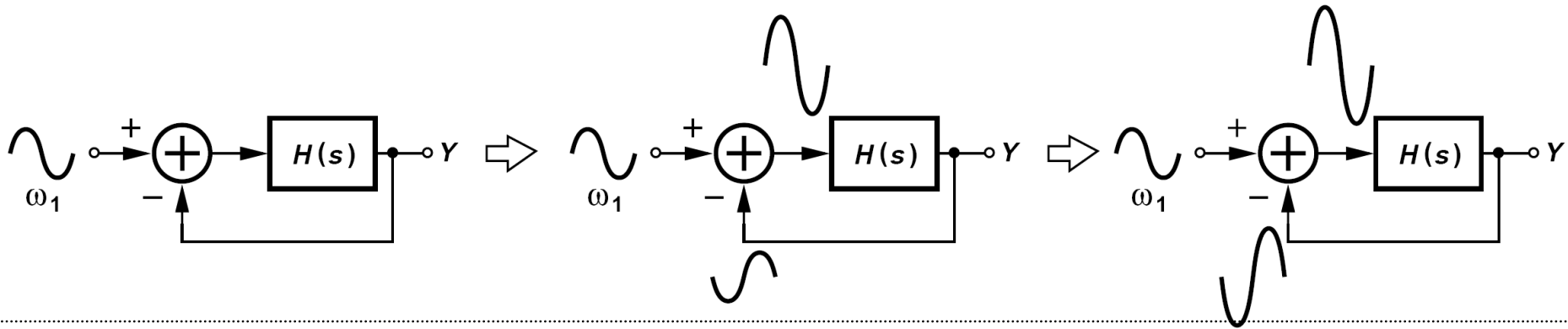


➤ For the circuit to reach steady state, the signal returning to A must exactly coincide with the signal that started at A. We call $\angle H(j\omega_1)$ a “frequency-dependent” phase shift to distinguish it from the 180° phase due to negative feedback.

➤ Even though the system was originally configured to have negative feedback, $H(s)$ is so “sluggish” that it contributes an additional phase shift of 180° at ω_1 , thereby creating positive feedback at this frequency.

Significance of $|H(j\omega_1)| = 1$

- For a noise component at ω_1 to “build up” as it circulates around the loop with positive feedback, the loop gain must be at least unity.
- We call $|H(j\omega_1)| = 1$ the “startup” condition.



- What happens if $|H(j\omega_1)| > 1$ and $\angle H(j\omega_1) = 180^\circ$? The growth shown in figure above still occurs but at a faster rate because the returning waveform is amplified by the loop.
- Note that the closed-loop poles now lie in the right half plane.

$$V_X = V_0 + |H(j\omega_0)|V_0 + |H(j\omega_0)|^2V_0 + |H(j\omega_0)|^3V_0 + \dots$$

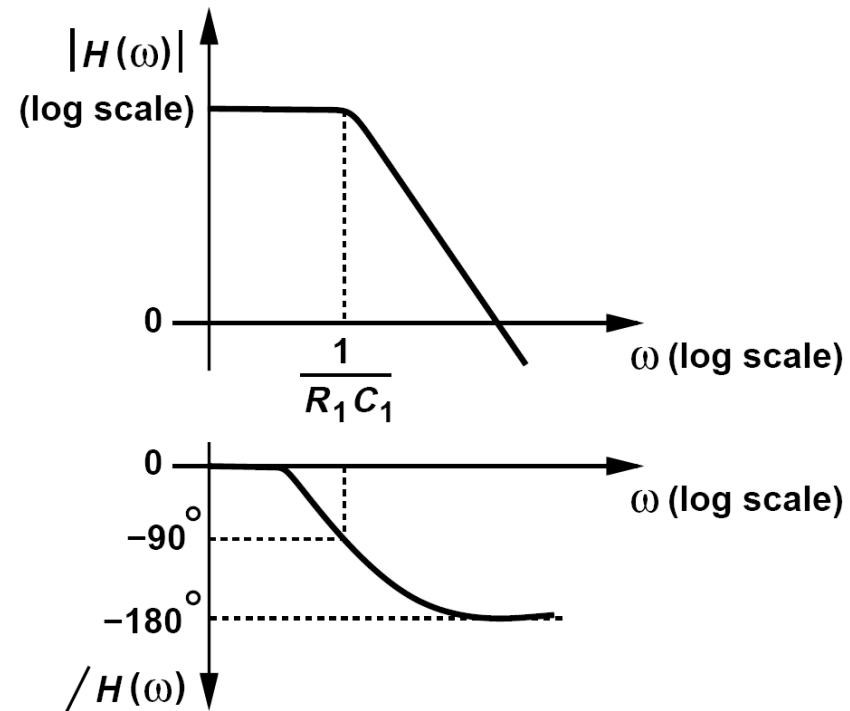
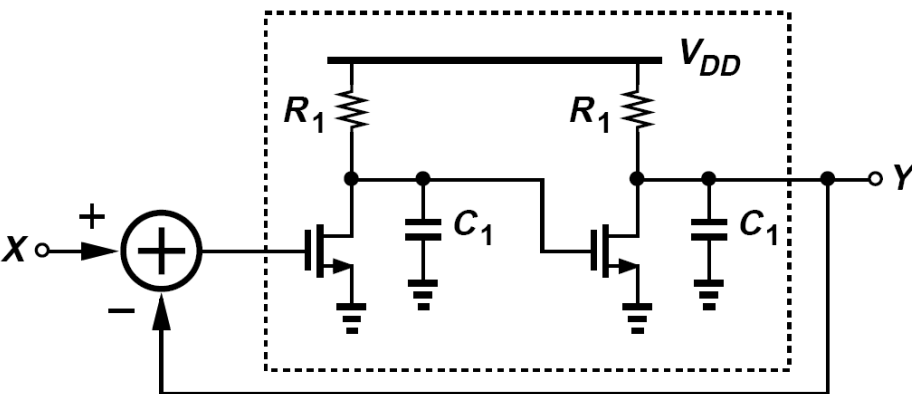
If $|H(j\omega_0)| > 1$, the above summation diverges, whereas if $|H(j\omega_0)| < 1$, then

$$V_X = \frac{V_0}{1 - |H(j\omega_0)|} < \infty$$

Can a Two-Pole System Oscillate? (I)

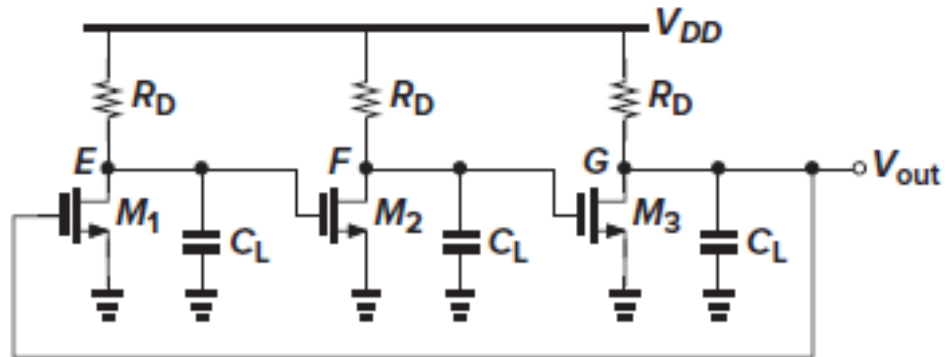
Can a two-pole system oscillate?

Suppose the system exhibits two coincident real poles at ω_p . Figure below (left) shows an example, where two cascaded common-source stages constitute $H(s)$ and $\omega_p = (R_1 C_1)^{-1}$. This system cannot satisfy both of Barkhausen's criteria because the phase shift associated with each stage reaches 90° only at $\omega = \infty$, but $|H(\infty)| = 0$. Figure below (right) plots $|H|$ and $\angle H$ as a function of frequency, revealing no frequency at which both conditions are met. Thus, the circuit cannot oscillate.



Three stage Ring oscillator

$$H(s) = -\frac{A_0^3}{\left(1 + \frac{s}{\omega_0}\right)^3}$$



The circuit oscillates only if the frequency-dependent phase shift equals 180° , i.e., if each stage contributes 60° . The frequency at which this occurs is given by

$$\tan^{-1} \frac{\omega_{osc}}{\omega_0} = 60^\circ \quad (15.7)$$

and hence

$$\omega_{osc} = \sqrt{3}\omega_0 \quad (15.8)$$

The minimum voltage gain per stage must be such that the magnitude of the loop gain at ω_{osc} is equal to unity:

$$\frac{A_0^3}{\left[\sqrt{1 + \left(\frac{\omega_{osc}}{\omega_0}\right)^2}\right]^3} = 1 \quad (15.9)$$

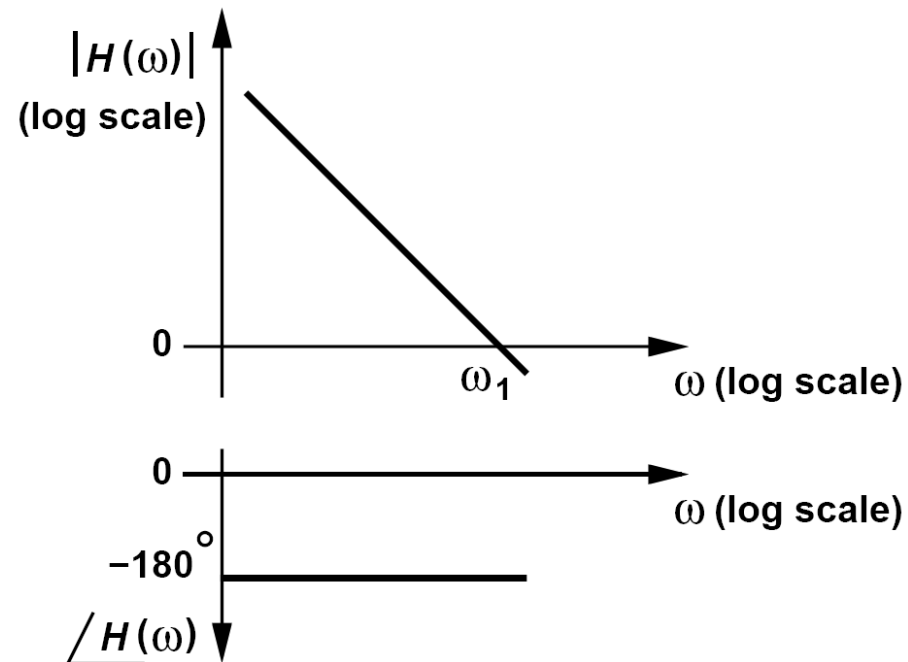
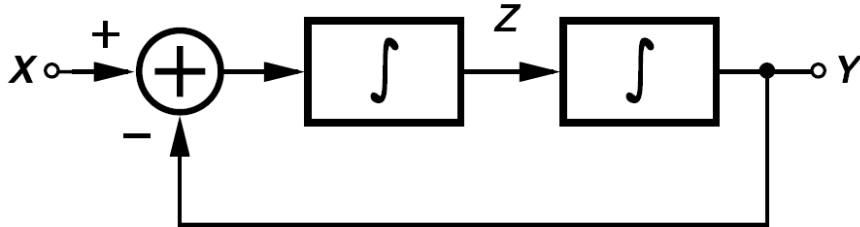
It follows from (15.8) and (15.9) that

$$A_0 = 2 \quad (15.10)$$

Can a Two-Pole System Oscillate? (II)

Can a two-pole system oscillate?

But, what if both poles are located at the origin? Realized as two ideal integrators in a loop, such a circuit does oscillate because each integrator contributes a phase shift of -90° at any nonzero frequency. Shown in figure below (right) are $|H|$ and $\angle H$ for this system.



Frequency and Amplitude of Oscillation in Previous Example

The feedback loop of figure above is released at $t = 0$ with initial conditions of z_0 and y_0 at the outputs of the two integrators and $x(t) = 0$. Determine the frequency and amplitude of oscillation.

Assuming each integrator transfer function is expressed as K/s ,

$$\frac{Y}{X}(s) = \frac{K^2}{s^2 + K^2}$$

$$\frac{d^2y}{dt^2} + K^2y = K^2x(t)$$

Substitute x and y ,

$$-A\omega_1^2 \cos(\omega_1 t + \phi_1) + K^2 A \cos(\omega_1 t + \phi_1) = 0$$

$$\omega_1 = K.$$

Interestingly, the circuit automatically finds the frequency at which the loop gain K^2/ω^2 drops to unity.

$$y(0) = A \cos \phi_1 = y_0$$

$$z(0) = \frac{1}{K} \left. \frac{dy}{dt} \right|_{t=0}$$

Chapt

$$= -A \sin \phi_1 = z_0$$

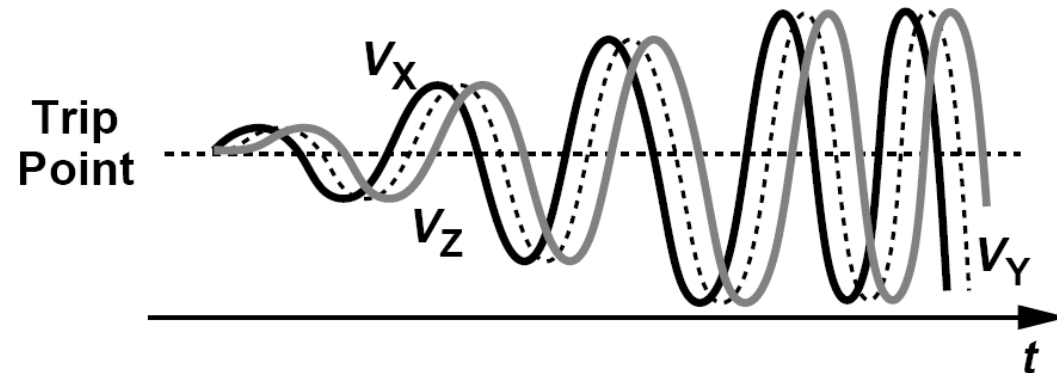
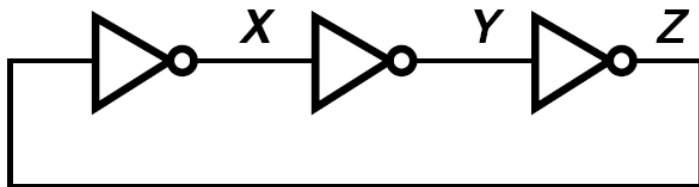


$$\tan \phi_0 = -\frac{z_0}{y_0}$$

$$A = \sqrt{z_0^2 + y_0^2} \quad 9$$

Ring Oscillator

- Other oscillators may begin to oscillate at a frequency at which the loop gain is higher than unity, thereby experiencing an exponential growth in their output amplitude.
- The growth eventually stops due to the saturating behavior of the amplifier(s) in the loop.



- Each stage operates as an amplifier, leading to an oscillation frequency at which each inverter contributes a frequency-dependent phase shift of 60° .