8.2.2 One-Port View of Oscillators

In the previous section, we considered oscillators as negative-feedback systems that experience sufficient positive feedback at some frequency. An alternative perspective views oscillators as two one-port components, namely, a lossy resonator and an active circuit that cancels the loss. This perspective provides additional insight and is described in this section.

Suppose, as shown in Fig. 8.13(a), a current impulse, $I_0\delta(t)$, is applied to a lossless tank. The impulse is entirely absorbed by C_1 (why?), generating a voltage of I_0/C_1. The charge on C_1 then begins to flow through L_1, and the output voltage falls. When V_{out} reaches zero, C_1 carries no energy but L_1 has a current equal to $L_1 dV_{out}/dt$, which charges C_1 in the opposite direction, driving V_{out} toward its negative peak. This periodic exchange of energy between C_1 and L_1 continues indefinitely, with an amplitude given by the strength of the initial impulse.

Now, let us assume a lossy tank. Depicted in Fig. 8.13(b), such a circuit behaves similarly except that R_p drains and "burns" some of the capacitor energy in every cycle, causing an exponential decay in the amplitude. We therefore surmise that, if an active circuit replenishes the energy lost in each period, then the oscillation can be sustained. In fact, we predict that an active circuit exhibiting an input resistance of $-R_p$ can be attached across the tank to cancel the effect of R_p, thereby recreating the ideal scenario shown in Fig. 8.13(a). Illustrated in Fig. 8.13(c), the resulting topology is called a "one-port oscillator."

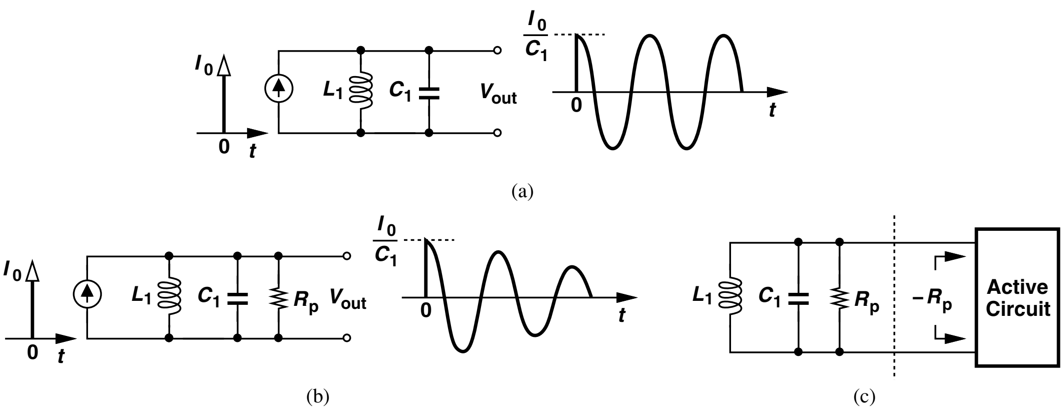

Figure 8.13 *(a) Response of an ideal tank to an impulse, (b) response of a lossy tank to an impulse, (c) cancellation of loss by negative resistance.*

Example 8.8

A student who remembers that loss in a tank results in noise postulates that, if the circuit of Fig. 8.13(c) resembles the ideal lossless topology, then it must also exhibit zero noise. Is that true?

Solution:

No, it is not. Resistance R_p and the active circuit still generate their own (uncorrelated) noise. We return to this point in Section 8.7.

How can a circuit present a negative (small-signal) input resistance? Figure 8.14(a) shows an example, where two capacitors are tied from the gate and drain of a transistor to its

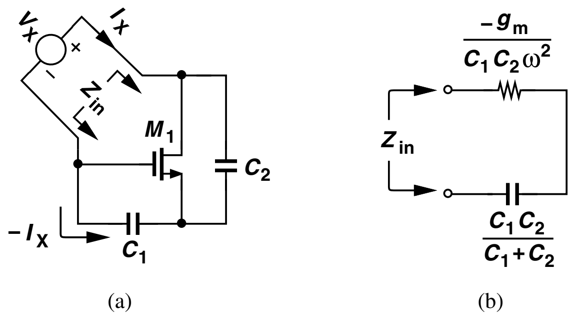

Figure 8.14 *(a) Circuit providing negative resistance, (b) equivalent circuit.*

source. The impedance Z_{in} can be obtained by noting that C_1 carries a current equal to $-I_X$, generating a gate-source voltage of $-I_X/(C_1 s)$ and hence a drain current of $-I_X g_m/(C_1 s)$. The difference between I_X and the drain current flows through C_2, producing a voltage equal to $[I_X + I_X g_m/(C_1 s)]/(C_2 s)$. This voltage must be equal to $V_{GS} + V_X$:

$$-\frac{I_X}{C_1 s} + V_X = \left(I_X + I_X \frac{g_m}{C_1 s}\right)\frac{1}{C_2 s}. \tag{8.22}$$

It follows that

$$\frac{V_X}{I_X}(s) = \frac{1}{C_1 s} + \frac{1}{C_2 s} + \frac{g_m}{C_1 C_2 s^2}. \tag{8.23}$$

For a sinusoidal input, $s = j\omega$,

$$\frac{V_X}{I_X}(j\omega) = \frac{1}{jC_1\omega} + \frac{1}{jC_2\omega} - \frac{g_m}{C_1 C_2 \omega^2}. \tag{8.24}$$

Thus, the input impedance can be viewed as a series combination of C_1, C_2, and a *negative* resistance equal to $-g_m/(C_1 C_2 \omega^2)$ [Fig. 8.14(b)]. Interestingly, the negative resistance *varies* with frequency.

Having found a negative resistance, we can now attach it to a lossy tank so as to construct an oscillator. Since the capacitive component in Eq. (8.24) can become part of the tank, we simply connect an inductor to the negative-resistance port (Fig. 8.15), seeking the condition for oscillation. In this case, it is simpler to model the loss of the inductor by a series resistance, R_S. The circuit oscillates if

$$R_S = \frac{g_m}{C_1 C_2 \omega^2}. \tag{8.25}$$

Under this condition, the circuit reduces to L_1 and the series combination of C_1 and C_2, exhibiting an oscillation frequency of

$$\omega_{osc} = \frac{1}{\sqrt{L_1 \dfrac{C_1 C_2}{C_1 + C_2}}}. \tag{8.26}$$

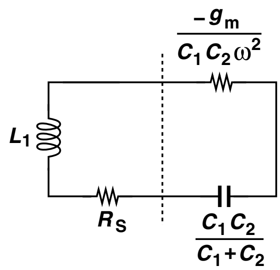

Figure 8.15 *Connection of lossy inductor to negative-resistance circuit.*

Example 8.9

Express the oscillation condition in terms of inductor's parallel equivalent resistance, R_p, rather than R_S.

Solution:

Recall from Chapter 2 that, if $Q > 3$, the series combination can be transformed to a parallel combination and

$$\frac{L_1\omega}{R_S} \approx \frac{R_p}{L_1\omega}. \tag{8.27}$$

Thus,

$$\frac{L_1^2\omega^2}{R_p} = \frac{g_m}{C_1C_2\omega^2}. \tag{8.28}$$

Moreover, we can replace ω^2 with the value given by Eq. (8.26), arriving at the startup condition:

$$g_mR_p = \frac{(C_1 + C_2)^2}{C_1C_2} \tag{8.29}$$

$$= \frac{C_1}{C_2} + \frac{C_2}{C_1} + 2. \tag{8.30}$$

As expected, for oscillation to occur, the transistor in Fig. 8.14(a) must provide sufficient "strength" (transconductance). In fact, (8.30) implies that the minimum allowable g_m is obtained if $C_1 = C_2$. That is, $g_mR_p \geq 4$.

8.3 CROSS-COUPLED OSCILLATOR

In this section, we develop an LC oscillator topology that, owing to its robust operation, has become the dominant choice in RF applications. We begin the development with a feedback system, but will discover that the result also lends itself to the one-port view described in Section 8.2.2.

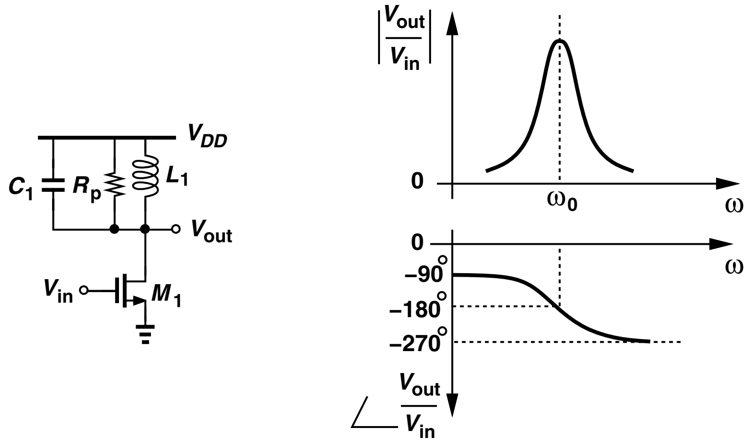

Figure 8.16 *(a) Tuned amplifier, (b) frequency response.*

We wish to build a negative-feedback oscillatory system using "LC-tuned" amplifier stages. Figure 8.16(a) shows such a stage, where C_1 denotes the total capacitance seen at the output node and R_p the equivalent parallel resistance of the tank at the resonance frequency. We neglect C_{GD} here but will see that it can be readily included in the final oscillator topology.

Let us examine the frequency response of the stage. At very low frequencies, L_1 dominates the load and

$$\frac{V_{out}}{V_{in}} \approx -g_m L_1 s. \tag{8.31}$$

That is, $|V_{out}/V_{in}|$ is very small and $\angle(V_{out}/V_{in})$ remains around -90° [Fig. 8.16(b)]. At the resonance frequency, ω_0, the tank reduces to R_p and

$$\frac{V_{out}}{V_{in}} = -g_m R_p. \tag{8.32}$$

The phase shift from the input to the output is thus equal to -180°. At very high frequencies, C_1 dominates, yielding

$$\frac{V_{out}}{V_{in}} \approx -g_m \frac{1}{C_1 s}. \tag{8.33}$$

Thus, $|V_{out}/V_{in}|$ diminishes and $\angle(V_{out}/V_{in})$ approaches $+90^\circ$ $(=-270^\circ)$.

Can the circuit of Fig. 8.16(a) oscillate if its input and output are shorted? As evidenced by the open-loop magnitude and phase plots shown in Fig. 8.16(b), no frequency satisfies Barkhausen's criteria; the total phase shift fails to reach 360° at any frequency.

Upon closer examination, we recognize that the circuit provides a phase shift of 180° with possibly adequate gain ($g_m R_p$) at ω_0. We simply need to increase the phase shift to 360°, perhaps by inserting another stage in the loop. Illustrated in Fig. 8.17(a), the idea is to cascade two identical LC-tuned stages so that, at resonance, the total phase shift around the

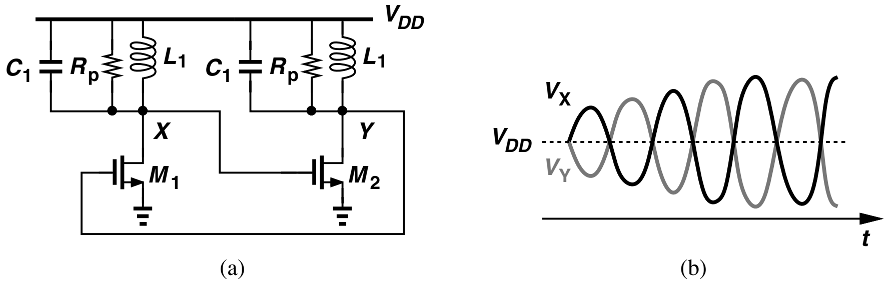

Figure 8.17 *Cascade of two tuned amplifiers in feedback loop.*

loop is equal to 360°. The circuit oscillates if the loop gain is equal to or greater than unity:

$$(g_m R_p)^2 \geq 1. \tag{8.34}$$

Example 8.10

Assuming that the circuit of Fig. 8.17(a) oscillates, plot the voltage waveforms at X and Y.

Solution:

At $t = 0$, $V_X = V_Y = V_{DD}$. As a noise component at ω_0 is amplified and circulated around the loop, V_X and V_Y begin to grow while maintaining a 180° phase difference [Fig. 8.17(b)]. A unique attribute of inductive loads is that they can provide peak voltages above the supply.[1] The growth of V_X and V_Y ceases when M_1 and M_2 enter the triode region for part of the period, reducing the loop gain.

The above circuit can be redrawn as shown in Fig. 8.18(a) and is called a "cross-coupled" oscillator due to the connection of M_1 and M_2. Forming the core of most RF

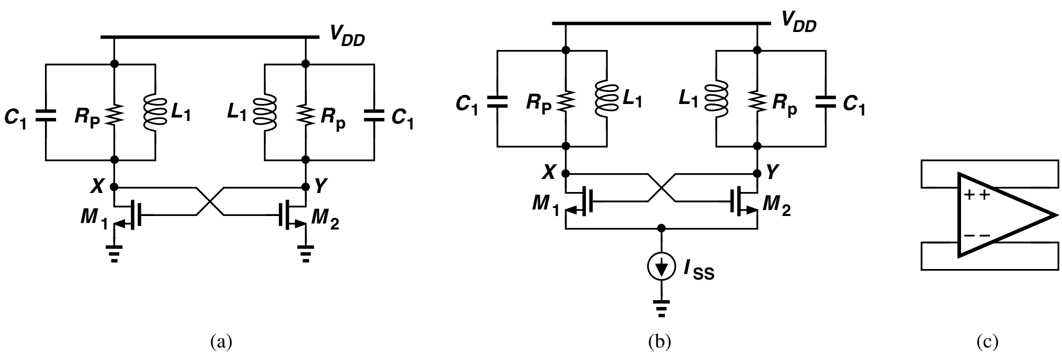

Figure 8.18 *(a) Simple cross-coupled oscillator, (b) addition of tail current, (c) equivalence to a differential pair placed in feedback.*

1. If L_1 has no series resistance, then its average voltage drop must be zero; thus, V_X and V_Y must go above V_{DD} and below V_{DD}.

oscillators used in practice, this topology entails many interesting properties and will be studied from different perspectives in this chapter.

Let us compute the oscillation frequency of the circuit. The capacitance at X includes C_{GS2}, C_{DB1}, and the effect of C_{GD1} and C_{GD2}. We note that (a) C_{GD1} and C_{GD2} are in parallel, and (b) the total voltage change across $C_{GD1} + C_{GD2}$ is equal to twice the voltage change at X (or Y) because V_X and V_Y vary differentially. Thus,

$$\omega_{osc} = \frac{1}{\sqrt{L_1(C_{GS2} + C_{DB1} + 4C_{GD} + C_1)}}. \tag{8.35}$$

Here, C_1 denotes the parasitic capacitance of L_1 plus the input capacitance of the next stage.

The oscillator of Fig. 8.18(a) suffers from poorly-defined bias currents. Since the average V_{GS} of each transistor is equal to V_{DD}, the currents strongly depend on the mobility, threshold voltage, and temperature. With differential V_X and V_Y, we surmise that M_1 and M_2 can operate as a differential pair if they are tied to a tail current source. Shown in Fig. 8.18(b), the resulting circuit is more robust and can be viewed as an inductively-loaded differential pair with positive feedback [Fig. 8.18(c)]. The oscillation amplitude grows until the pair experiences saturation. We sometimes refer to this circuit as the "tail-biased oscillator" to distinguish it from other cross-coupled topologies.

Example 8.11

Compute the voltage swings in the circuit of Fig. 8.18(b) if M_1 and M_2 experience complete current switching with abrupt edges.

Solution:

From Example 8.7, the drain current of each transistor swings between zero and I_{SS}, yielding a peak differential output swing of

$$V_{XY} \approx \frac{4}{\pi} I_{SS} R_p. \tag{8.36}$$

The above-supply swings in the cross-coupled oscillator of Fig. 8.18(b) raise concern with respect to transistor reliability. The instantaneous voltage difference between any two terminals of M_1 or M_2 must remain below the maximum value allowed by the technology. Figure 8.19 shows a "snapshot" of the circuit when M_1 is off and M_2 is on. Each transistor may experience stress under the following conditions: (1) The drain reaches $V_{DD} + V_a$, where V_a is the peak single-ended swing, e.g., $(2/\pi)I_{SS}R_p$, while the gate drops to $V_{DD} - V_a$. The transistor remains off, but its drain-gate voltage is equal to $2V_a$ and its drain-source voltage is greater than $2V_a$ (why?). (2) The drain falls to $V_{DD} - V_a$ while the gate rises to $V_{DD} + V_a$. Thus, the gate-drain voltage reaches $2V_a$ and the gate-source voltage exceeds $2V_a$. We note that both V_{DS1} and V_{GS2} may assume excessively large values. Proper choice of V_a, I_{SS}, and device dimensions avoids stressing the transistors.

The reader may wonder how the inductance value and the device dimensions are selected in the cross-coupled oscillator. We defer the design procedure to after we have studied voltage-controlled oscillators and phase noise (Section 8.8).

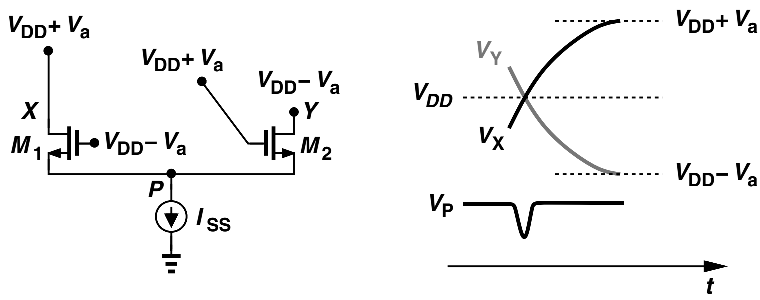

Figure 8.19 *Voltage swings in cross-coupled oscillator.*[2]

Example 8.12

A student claims that the cross-coupled oscillator of Fig. 8.18(b) exhibits no supply sensitivity if the tail current source is ideal. Is this true?

Solution:

No, it is not. The drain-substrate capacitance of each transistor sustains an average voltage equal to V_{DD} (Fig. 8.20). Thus, supply variations modulate this capacitance and hence the oscillation frequency.

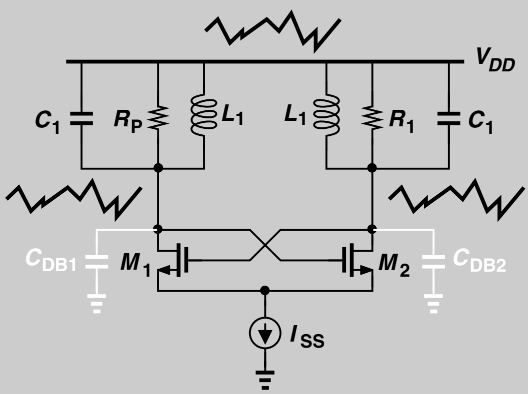

Figure 8.20 *Modulation of drain junction capacitances by V_{DD}.*

While conceived as a feedback system, the cross-coupled oscillator also lends itself to the one-port view described in Section 8.2.2. Let us first redraw the circuit as shown in Fig. 8.21(a) and note that, for small differential waveforms at X and Y, V_N does not change even if it is not connected to V_{DD}. Disconnecting this node from V_{DD} (only for small-signal analysis) and recognizing that the series combination of two identical tanks

2. The voltage at node P *falls* at the crossings of V_X and V_Y if M_1 and M_2 do not enter the triode region at any point. On the other hand, if each transistor enters the deep triode region in a half cycle, then V_P is low most of the time and *rises* at the crossings at V_X and V_Y.

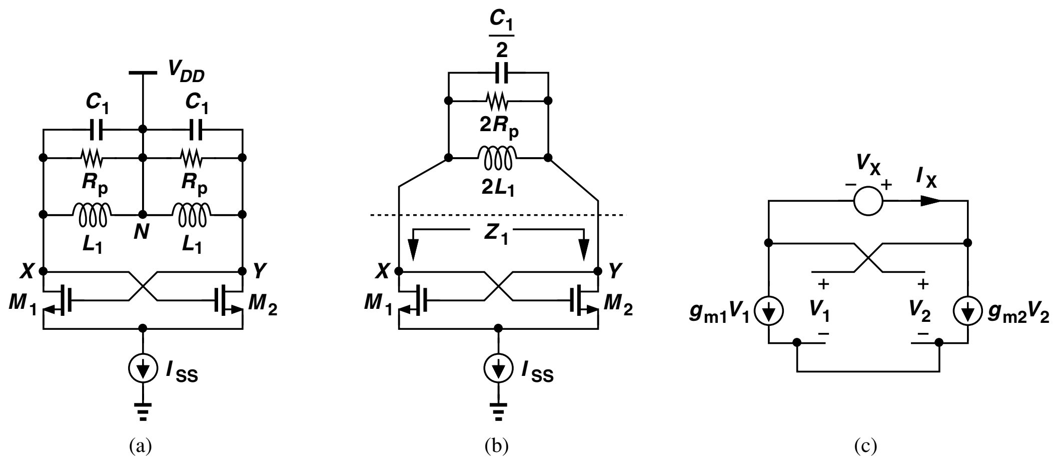

Figure 8.21 *(a) Redrawing of cross-coupled oscillator, (b) load tanks merged, (c) equivalent circuit of cross-coupled pair.*

can be represented as a single tank, we arrive at the circuit depicted in Fig. 8.21(b). We can now view the oscillator as a lossy resonator ($2L_1$, $C_1/2$, and $2R_p$) tied to the port of an active circuit (M_1, M_2, and I_{SS}), expecting that the latter replenishes the energy lost in the former. That is, Z_1 must contain a negative resistance. This can be seen from the equivalent circuit shown in Fig. 8.21(c), where $V_1 - V_2 = V_X$ and

$$I_X = -g_{m1}V_1 = g_{m2}V_2. \tag{8.37}$$

It follows that

$$\frac{V_X}{I_X} = -\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\right), \tag{8.38}$$

which, for $g_{m1} = g_{m2} = g_m$, reduces to

$$\frac{V_X}{I_X} = -\frac{2}{g_m}. \tag{8.39}$$

For oscillation to occur, the negative resistance must cancel the loss of the tank:

$$\frac{2}{g_m} \leq 2R_p \tag{8.40}$$

and hence

$$g_mR_p \geq 1. \tag{8.41}$$

As expected, this condition is identical to that expressed by Eq. (8.34).[3]

3. This topology is also called a "negative-G_m oscillator." This is not quite correct because it does not contain a negative *transconductance* but a negative *conductance.*