8.7 PHASE NOISE

The design of VCOs must deal with trade-offs among tuning range, phase noise, and power dissipation. Our study has thus far focused on the task of tuning. We now turn our attention to phase noise.

8.7.1 Basic Concepts

An ideal oscillator produces a perfectly-periodic output of the form $x(t) = A\cos\omega_c t$. The zero crossings occur at exact integer multiples of $T_c = 2\pi/\omega_c$. In reality, however, the noise of the oscillator devices randomly perturbs the zero crossings. To model this perturbation, we write $x(t) = A\cos[\omega_c t + \phi_n(t)]$, where $\phi_n(t)$ is a small random phase quantity that deviates the zero crossings from integer multiples of T_c. Figure 8.44 illustrates the two waveforms in the time domain. The term $\phi_n(t)$ is called the "phase noise."

The waveforms in Fig. 8.44 can also be viewed from another, slightly different, perspective. We can say that the *period* remains constant if $x(t) = A\cos\omega_c t$ but varies randomly if $x(t) = A\cos[\omega_c t + \phi_n(t)]$ (as indicated by $T_1, \ldots, T_m$ in Fig. 8.44). In other

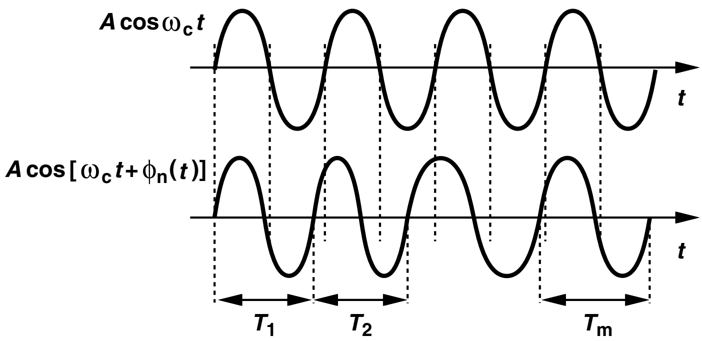

Figure 8.44 *Output waveforms of an ideal and a noisy oscillator.*

words, the *frequency* of the waveform is constant in the former case but varies randomly in the latter. This observation leads to the spectrum of the oscillator output. For $x(t) = A\cos\omega_c t$, the spectrum consists of a single impulse at ω_c [Fig. 8.45(a)], whereas for $x(t) = A\cos[\omega_c t + \phi_n(t)]$ the frequency experiences random variations, i.e., it departs from ω_c occasionally. As a consequence, the impulse is "broadened" to represent this random departure [Fig. 8.45(b)].

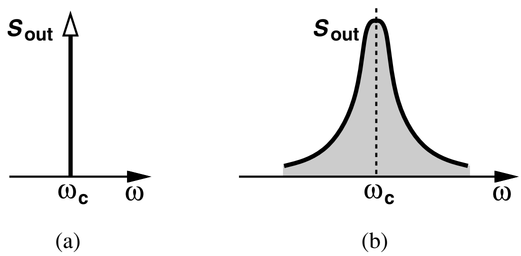

Figure 8.45 *Output spectra of (a) an ideal, and (b) a noisy oscillator.*

Example 8.22

Explain why the broadened impulse cannot assume the shape shown in Fig. 8.46.

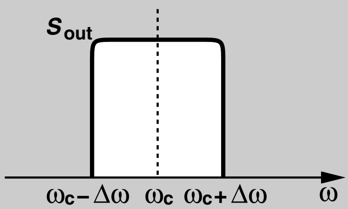

Figure 8.46 *Flat spectrum around oscillation frequency.*

(*Continues*)

Example 8.22 (*Continued*)

Solution:

This spectrum occurs if the oscillator frequency has *equal* probability of appearing anywhere between $\omega_c - \Delta\omega$ and $\omega_c + \Delta\omega$. However, we intuitively expect that the oscillator prefers ω_c to other frequencies, thus spending lesser time at frequencies that are farther from ω_c. This explains the declining phase noise "skirts" in Fig. 8.45(b).

Our focus on noise in the zero crossings rather than noise on the amplitude arises from the assumption that the latter is removed by hard switching in stages following the oscillator. For example, the switching transistors in an active mixer spend little time near equilibrium, "masking" most of the LO amplitude noise for the rest of the time.

The spectrum of Fig. 8.45(b) can be related to the time-domain expression. Since $\phi_n(t) \ll 1$ rad,

$$x(t) = A\cos[\omega_c t + \phi_n(t)] \tag{8.86}$$

$$\approx A\cos\omega_c t - A\sin\omega_c t \sin[\phi_n(t)] \tag{8.87}$$

$$\approx A\cos\omega_c t - A\phi_n(t)\sin\omega_c t. \tag{8.88}$$

That is, the spectrum of $x(t)$ consists of an impulse at ω_c and the spectrum of $\phi_n(t)$ *translated* to a center frequency of ω_c. Thus, the declining skirts in Fig. 8.45(b) in fact represent the behavior of $\phi_n(t)$ in the frequency domain.

In phase noise calculations, many factors of 2 or 4 appear at different stages and must be carefully taken into account. For example, as illustrated in Fig. 8.47, (1) since $\phi_n(t)$ in Eq. (8.88) is multiplied by $\sin\omega_c t$, its *power* spectral density, $S_{\phi n}$, is multiplied by 1/4 as it is translated to $\pm\omega_c$; (2) A spectrum analyzer measuring the resulting spectrum *folds* the negative-frequency spectrum atop the positive-frequency spectrum, raising the spectral density by a factor of 2.

How is the phase noise quantified? Since the phase noise falls at frequencies farther from ω_c, it must be specified at a certain "frequency offset," i.e., a certain difference with

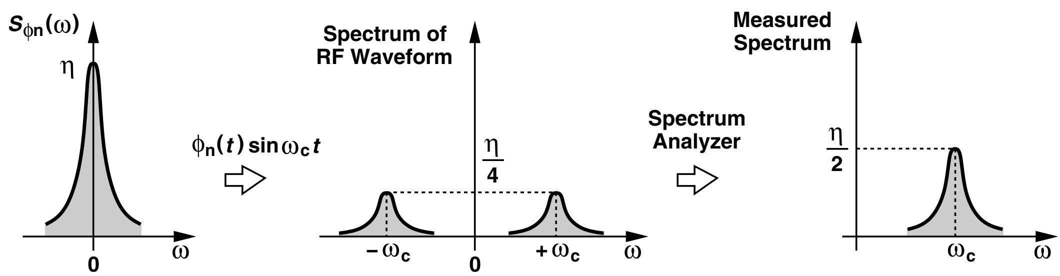

Figure 8.47 *Various factors of 4 and 2 that arise in conversion of noise to phase noise.*

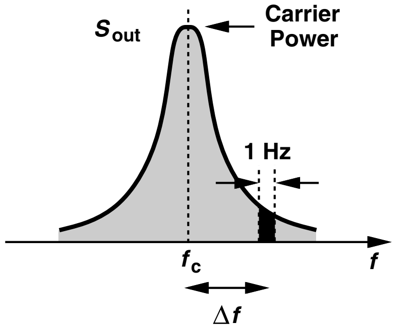

Figure 8.48 *Specification of phase noise.*

respect to ω_c. As shown in Fig. 8.48, we consider a 1-Hz bandwidth of the spectrum at an offset of Δf, measure the power in this bandwidth, and normalize the result to the "carrier power." The carrier power can be viewed as the peak of the spectrum or (more rigorously) as the power given by Eq. (8.86), namely, $A^2/2$. For example, the phase noise of an oscillator in GSM applications must fall below -115 dBc/Hz at 600-kHz offset. Called "dB with respect to the carrier," the unit dBc signifies normalization of the noise power to the carrier power.

Example 8.23

At high carrier frequencies, it is difficult to measure the noise power in a 1-Hz bandwidth. Suppose a spectrum analyzer measures a noise power of -70 dBm in a 1-kHz bandwidth at 1-MHz offset. How much is the phase noise at this offset if the average oscillator output power is -2 dBm?

Solution:

Since a 1-kHz bandwidth carries $10 \log(1000 \text{ Hz}) = 30$ dB higher noise than a 1-Hz bandwidth, we conclude that the noise power in 1 Hz is equal to -100 dBm. Normalized to the carrier power, this value translates to a phase noise of -98 dBc/Hz.

In practice, the phase noise reaches a constant floor at large frequency offsets (beyond a few megahertz) (Fig. 8.49). We call the regions near and far from the carrier the "close-in" and the "far-out" phase noise, respectively, although the border between the two is vague.

8.7.2 Effect of Phase Noise

To understand the effect of phase noise in RF systems, let us consider the receiver front end shown in Fig. 8.50(a) and study the downconverted spectrum. Referring to the ideal

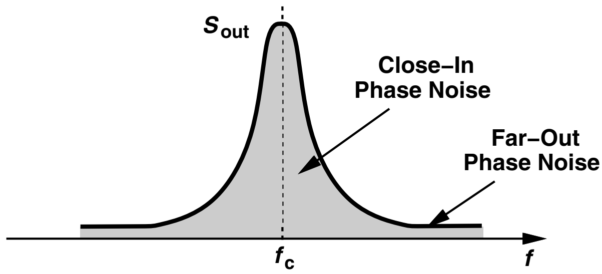

Figure 8.49 *Close-in and far-out phase noise.*

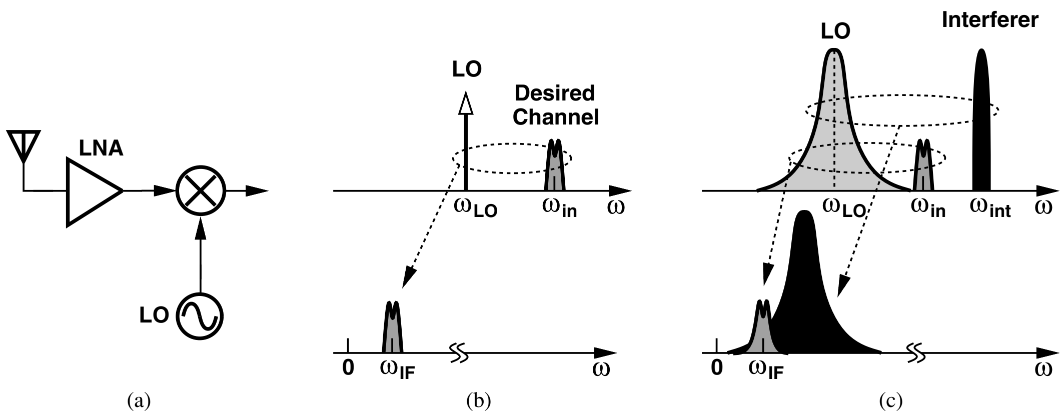

Figure 8.50 *(a) Receive front end, (b) downconversion with an ideal LO, (c) downconversion with a noisy LO (reciprocal mixing).*

case depicted in Fig. 8.50(b), we observe that the desired channel is convolved with the impulse at ω_{LO}, yielding an IF signal at $\omega_{IF} = \omega_{in} - \omega_{LO}$. Now, suppose the LO suffers from phase noise and the desired signal is accompanied by a large interferer. As illustrated in Fig. 8.50(c), the convolution of the desired signal and the interferer with the noisy LO spectrum results in a *broadened* downconverted interferer whose noise skirt corrupts the desired IF signal. This phenomenon is called "reciprocal mixing."

Reciprocal mixing becomes critical in receivers that may sense large interferers. The LO phase noise must then be so small that, when integrated across the desired channel, it produces negligible corruption.

Phase noise also manifests itself in transmitters. Shown in Fig. 8.52 is a scenario where two users are located in close proximity, with user #1 transmitting a high-power signal at f_1 and user #2 receiving this signal and a weak signal at f_2. If f_1 and f_2 are only a few channels apart, the phase noise skirt masking the signal received by user #2 greatly corrupts it even *before* downconversion.

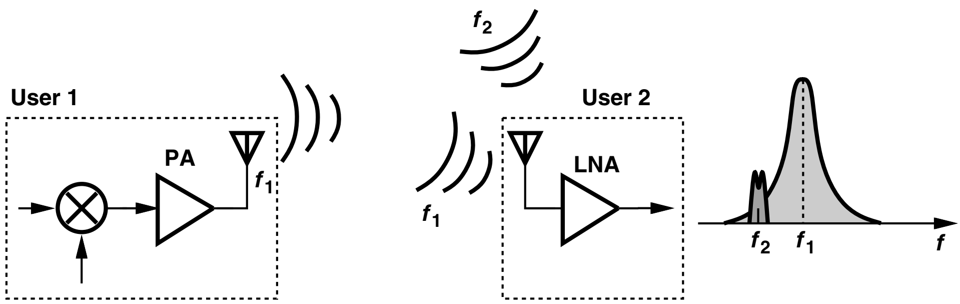

Figure 8.52 *Received noise due to phase noise of an unwanted signal.*

Example 8.25

A student reasons that, if the interferer at f_1 in Fig. 8.52 is so large that its phase noise corrupts the reception by user #2, then it also heavily *compresses* the receiver of user #2. Is this true?

Solution:

Not necessarily. As evidenced by Example 8.24, an interferer, say, 50 dB above the desired signal produces phase noise skirts that are not negligible. For example, the desired signal may have a level of −90 dBm and the interferer, −40 dBm. Since most receivers' 1-dB compression point is well above −40 dBm, user #2's receiver experiences no desensitization, but the phenomenon in Fig. 8.52 is still critical.

The LO phase noise also corrupts phase-modulated signals in the process of upconversion or downconversion. Since the phase noise is indistinguishable from phase (or frequency) modulation, the mixing of the signal with a noisy LO in the TX or RX path corrupts the information carried by the signal. For example, a QPSK signal containing phase noise can be expressed as

$$x_{QPSK}(t) = A\cos\left[\omega_c t + (2k+1)\frac{\pi}{4} + \phi_n(t)\right] \quad k = 0, \cdots, 3 \tag{8.94}$$

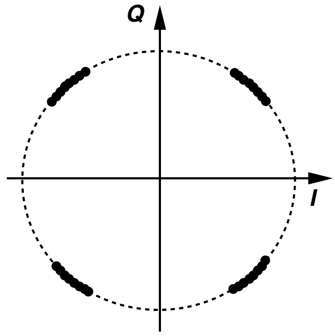

Figure 8.53 *Corruption of a QPSK signal due to phase noise.*

revealing that the amplitude is unaffected by phase noise. Thus, the constellation points experience only random rotation around the origin (Fig. 8.53). If large enough, phase noise and other nonidealities move a constellation point to another quadrant, creating an error.

Example 8.26

Which points in a 16-QAM constellation are most sensitive to phase noise?

Solution:

Consider the four points in the top right quadrant (Fig. 8.54). Points B and C can tolerate a rotation of 45° before they move to adjacent quadrants. Points A and D, on the other hand, can rotate by only $\theta = \tan^{-1}(1/3) = 18.4^\circ$. Thus, the eight outer points near the I and Q axes are most sensitive to phase noise.

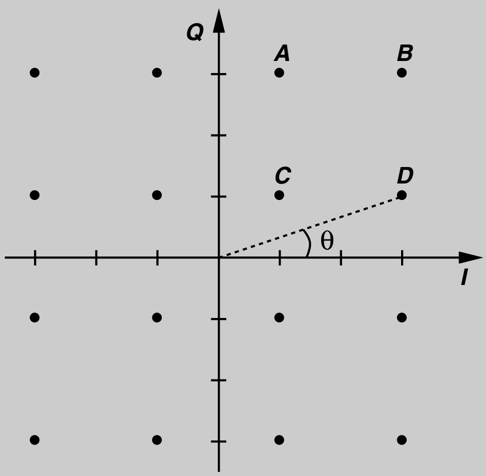

Figure 8.54 *16-QAM constellation for study of effect of phase noise.*

8.7.3 Analysis of Phase Noise: Approach I

Oscillator phase noise has been under study for decades [3]–[17], leading to a multitude of analysis techniques in the frequency and time domains. The calculation of phase noise by hand still remains tedious, but simulation tools such as Cadence's SpectreRF have greatly simplified the task. Nonetheless, a solid understanding of the mechanisms that give rise to phase noise proves essential to oscillator design. In this section, we analyze these mechanisms. In particular, we must answer two important questions: (1) how much and at what point in an oscillation cycle does each device "inject" noise? (2) how does the injected noise produce phase noise in the output voltage waveform?

Q of an Oscillator In Chapters 2 and 7, we derived various expressions for the Q of an LC tank. We know intuitively that a high Q signifies a sharper resonance, i.e., a higher selectivity. Another definition of the Q that is especially well-suited to oscillators is illustrated in Fig. 8.55. Here, the circuit is viewed as a feedback system and the *phase* of the *open-loop* transfer function, $\phi(\omega)$, is examined at the resonance frequency, ω_0. The "open-loop" Q is defined as

$$Q = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|. \tag{8.95}$$

This definition offers an interesting insight if we recall that for steady oscillation, the total phase shift around the loop must be 360° (or zero). Suppose the noise injected by the devices attempts to deviate the frequency from ω_0. From Fig. 8.55, such a deviation translates to a change in the total phase shift around the loop, violating Barkhausen's criterion and forcing the oscillator to return to ω_0. The larger the slope of $\phi(j\omega)$, the greater is this "restoration" force; i.e., oscillators with a high open-loop Q tend to spend less time at frequencies other than ω_0. In Problem 8.10, we prove that this definition of Q coincides with our original definition, $Q = R_p/(L\omega)$, for a CS stage loaded by a second-order tank.

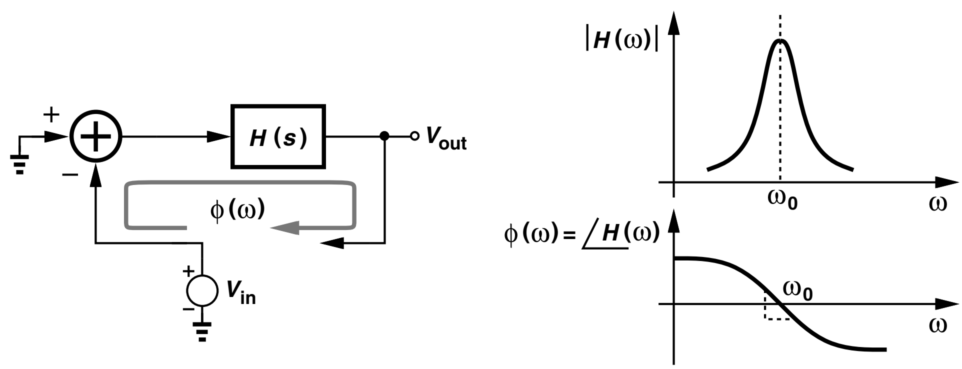

Figure 8.55 *Definition of open-loop Q.*

Example 8.27

Compute the open-loop Q of a cross-coupled LC oscillator.

Solution:

We construct the open-loop circuit as shown in Fig. 8.56 and note that $V_{out}/V_X = V_X/V_{in}$ and hence $H(s) = V_{out}/V_{in} = (V_X/V_{in})^2$. Thus, the phase of V_{out}/V_{in} is equal to twice the phase of V_X/V_{in}. Since at $s = j\omega$,

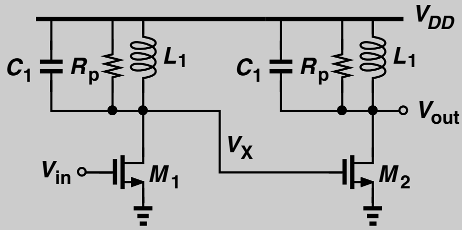

Figure 8.56 *Open-loop model of a cross-coupled oscillator.*

$$\frac{V_X}{V_{in}}(j\omega) = \frac{-jg_m R_p L_1 \omega}{R_p(1 - L_1 C_1 \omega^2) + jL_1\omega}, \tag{8.96}$$

we have

$$\angle H(j\omega) = 2\left[-\frac{\pi}{2} - \tan^{-1}\frac{L_1\omega}{R_p(1 - L_1C_1\omega^2)}\right]. \tag{8.97}$$

Differentiating both sides with respect to ω, calculating the result at $\omega_0 = (\sqrt{L_1C_1})^{-1}$, and multiplying it by $\omega_0/2$, we obtain

$$\left|\frac{\omega_0}{2}\frac{d\angle H(j\omega)}{d\omega}|\right|_{\omega 0} = 2R_pC_1\omega_0 \tag{8.98}$$

$$= 2Q_{tank}, \tag{8.99}$$

where Q_{tank} denotes the Q of each tank. This result is to be expected: the cascade of frequency-selective stages makes the phase transition sharper than that of one stage.

While the open-loop Q indicates how much an oscillator "rejects" the noise, the phase noise depends on three other factors as well: (1) the *amount* of noise that different devices inject, (2) the point in time during a cycle at which the devices inject noise (some parts of the waveform are more sensitive than others), and (3) the output voltage swing (carrier power). We elaborate on these as we analyze phase noise.

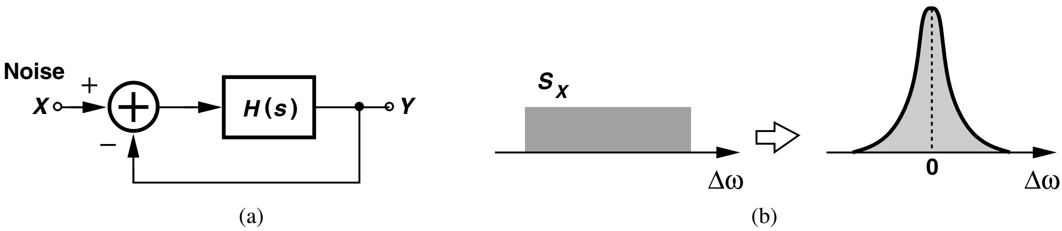

Figure 8.57 *(a) Oscillator model, (b) noise shaping in oscillator.*

Noise Shaping in Oscillators As our first step toward formulating the phase noise, we wish to understand what happens if noise is injected into an oscillatory circuit. Employing the feedback model, we represent the noise as an additive term [Fig. 8.57(a)] and write

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)}. \tag{8.100}$$

In the vicinity of the oscillation frequency, i.e., at $\omega = \omega_0 + \Delta\omega$, we can approximate $H(j\omega)$ with the first two terms in its Taylor series:

$$H(j\omega) \approx H(j\omega_0) + \Delta\omega \frac{dH}{d\omega}. \tag{8.101}$$

If $H(j\omega_0) = -1$ and $\Delta\omega dH/d\omega \ll 1$, then Eq. (8.100) reduces to

$$\frac{Y}{X}(j\omega_0 + j\Delta\omega) \approx \frac{-1}{\Delta\omega \dfrac{dH}{d\omega}}. \tag{8.102}$$

In other words, as shown in Fig. 8.57(b), the noise spectrum is "shaped" by

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{\Delta\omega^2 |\dfrac{dH}{d\omega}|^2}. \tag{8.103}$$

To determine the shape of $|dH/d\omega|^2$, we write $H(j\omega)$ in polar form, $H(j\omega) = |H| \exp(j\phi)$ and differentiate with respect to ω,

$$\frac{dH}{d\omega} = \left(\frac{d|H|}{d\omega} + j|H|\frac{d\phi}{d\omega}\right) \exp(j\phi). \tag{8.104}$$

It follows that

$$\left|\frac{dH}{d\omega}\right|^2 = \left|\frac{d|H|}{d\omega}\right|^2 + \left|\frac{d\phi}{d\omega}\right|^2 |H|^2. \tag{8.105}$$

This equation leads to a general definition of Q [4], but we limit our study here to simple LC oscillators. Note that (a) in an LC oscillator, the term $|d|H|/d\omega|^2$ is much less than

$|d\phi/d\omega|^2$ in the vicinity of the resonance frequency, and (b) $|H|$ is close to unity for steady oscillations. The right-hand side of Eq. (8.105) therefore reduces to $|d\phi/d\omega|^2$, yielding

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{\dfrac{\omega_0^2}{4}\left|\dfrac{d\phi}{d\omega}\right|^2}\frac{\omega_0^2}{4\Delta\omega^2}. \tag{8.106}$$

From (8.95),

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{4Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2. \tag{8.107}$$

Known as "Leeson's Equation" [3], this result reaffirms our intuition that the open-loop Q signifies how much the oscillator rejects the noise.

Example 8.28

A student designs the cross-coupled oscillator of Fig. 8.58 with $2/g_m = 2R_p$, reasoning that the tank now has infinite Q and hence the oscillator produces no phase noise![6] Explain the flaw in this argument. (This circuit is similar to that in Fig. 8.21(b), but with the tank components renamed.)

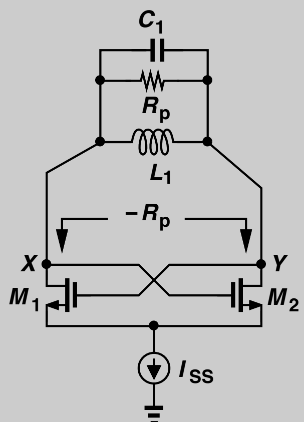

Figure 8.58 *Apparently infinite Q in an oscillator.*

Solution:

The Q in Eq. (8.107) is the *open-loop Q*, i.e., $\omega_0/2$ times the slope of the phase of the *open-loop* transfer function, which was calculated in Example 8.27. The "closed-loop" Q does not carry much meaning.

6. The center tap of L_1 is tied to V_{DD} but not shown.

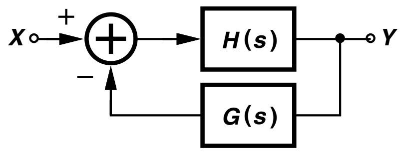

Figure 8.59 *Noise shaping in a general oscillator.*

In Problem 8.11, we prove that, if the feedback path has a transfer function $G(s)$ (Fig. 8.59), then

$$\left|\frac{Y}{X}(j\omega_0 + j\Delta\omega)\right|^2 = \frac{1}{4Q^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2\left|\frac{1}{G(j\omega_0)}\right|^2, \tag{8.108}$$

where the open-loop Q is given by

$$Q = \frac{\omega_0}{2}\left|\frac{d(GH)}{d\omega}\right|. \tag{8.109}$$

Linear Model The foregoing development suggests that the total noise at the output of an oscillator can be obtained according to a number of transfer functions similar to Eq. (8.107) from each noise source to the output. Such an approach begins with a small-signal (linear) model and can account for some of the nonidealities [4]. However, the small-signal model may ignore some important effects, e.g., the noise of the tail current source, or face other difficulties. The following example illustrates this point.