Chapter 14

Oscillators

Oscillators are an integral part of many electronic systems. Applications range from clock generation in microprocessors to carrier synthesis in cellular telephones, requiring vastly different oscillator topologies and performance parameters. Robust, high-performance oscillator design in CMOS technology continues to pose interesting challenges. As described in Chapter 15, oscillators are usually embedded in a phase-locked system.

This chapter deals with the analysis and design of CMOS oscillators, more specifically, voltage-controlled oscillators (VCOs). Beginning with a general study of oscillation in feedback systems, we introduce ring oscillators and LC oscillators along with methods of varying the frequency of oscillation. We then describe a mathematical model of VCOs that will be used in the analysis of PLLs in Chapter 15.

14.1 General Considerations

A simple oscillator produces a periodic output, usually in the form of voltage. As such, the circuit has no input while sustaining the output indefinitely. How can a circuit oscillate? Recall from Chapter 10 that negative feedback systems may oscillate, i.e., an oscillator is a badly-designed feedback amplifier![1] Consider the unity-gain negative feedback circuit shown in Fig. 14.1, where

$$\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1+H(s)}. \tag{14.1}$$

As mentioned in Chapter 10, if the amplifier itself experiences so much phase shift at high frequencies that the overall feedback becomes positive, then oscillation may occur. More accurately, if for $s = j\omega_0$, $H(j\omega_0) = -1$, then the closed-loop gain approaches infinity at ω_0. Under this condition, the circuit amplifies its own noise components at ω_0 indefinitely. In fact, as conceptually illustrated in Fig. 14.2, a noise component at ω_0 experiences a total gain of unity and a phase shift of 180°, returning to the subtractor as a negative replica

[1] It is said, "In the high-frequency world, amplifiers oscillate and oscillators don't."

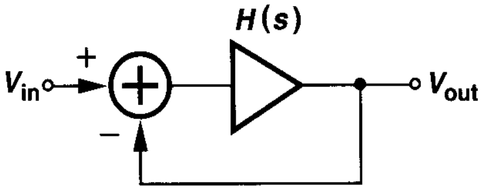

Figure 14.1 Feedback system.

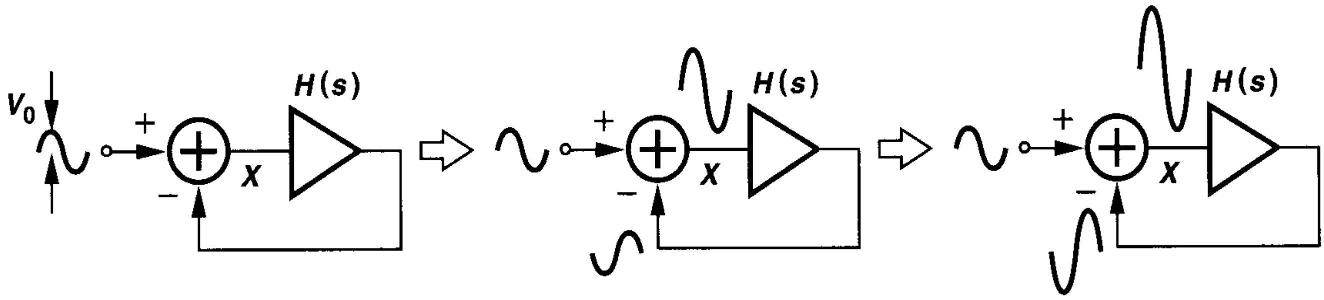

Figure 14.2 Evolution of oscillatory system with time.

of the input. Upon subtraction, the input and the feedback signals give a larger difference. Thus, the circuit continues to "regenerate," allowing the component at ω_0 to grow.

For the oscillation to begin, a loop gain of unity or greater is necessary. This can be seen by following the signal around the loop over many cycles and expressing the amplitude of the subtractor's output in Fig. 14.2 as a geometric series (if $\angle H(j\omega_0) = 180°$):

$$V_X = V_0 + |H(j\omega_0)|V_0 + |H(j\omega_0)|^2 V_0 + |H(j\omega_0)|^3 V_0 + \cdots . \qquad (14.2)$$

If $|H(j\omega_0)| > 1$, the above summation diverges whereas if $|H(j\omega_0)| < 1$, then

$$V_X = \frac{V_0}{1 - |H(j\omega_0)|} < \infty. \qquad (14.3)$$

In summary, if a negative-feedback circuit has a loop gain that satisfies two conditions:

$$|H(j\omega_0)| \geq 1 \qquad (14.4)$$

$$\angle H(j\omega_0) = 180°, \qquad (14.5)$$

then the circuit may oscillate at ω_0. Called "Barkhausen criteria," these conditions are necessary but not sufficient [1]. In order to ensure oscillation in the presence of temperature and process variations, we typically choose the loop gain to be at least twice or three times the required value.

We may state the second Barkhausen criterion as $\angle H(j\omega) = 180°$ or a *total* phase shift of 360°. This should not be confusing: if the system is designed to have a low-frequency negative feedback, it already produces 180° of phase shift in the signal traveling around the loop (as represented by the subtractor in Fig. 14.1), and $\angle H(j\omega) = 180°$ denotes an additional *frequency-dependent* phase shift that, as illustrated in Fig. 14.2, ensures the feedback signal *enhances* the original signal. Thus, the three cases illustrated in Fig. 14.3 are equivalent in terms of the second criterion. We say the system of Fig. 14.3(a) exhibits

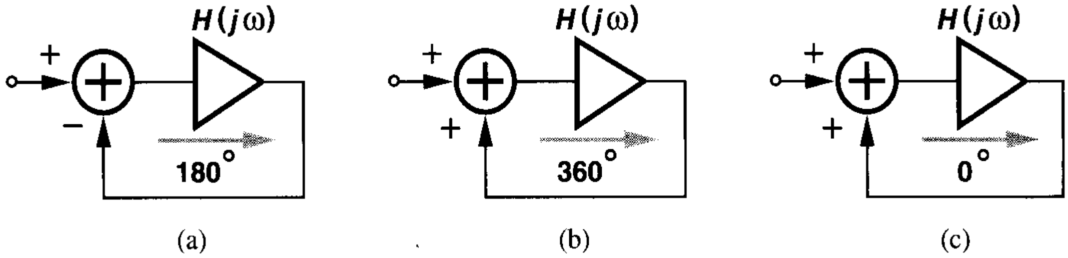

Figure 14.3 Various views of oscillatory feedback system.

a frequency-dependent phase shift of 180° (denoted by the arrow) and a dc phase shift of 180°. The difference between Figs. 14.3(b) and (c) is that the open-loop amplifier in the former contains enough stages with proper polarities to provide a total phase shift of 360° at ω_0 whereas that in the latter produces *no* phase shift at ω_0. Examples of these topologies are presented later in this chapter.

CMOS oscillators in today's technology are typically implemented as "ring oscillators" or "LC oscillators." We study each type in the following sections.

14.2 Ring Oscillators

A ring oscillator consists of a number of gain stages in a loop. To arrive at the actual implementation, we begin by attempting to make a single-stage feedback circuit oscillate.

Example 14.1

Explain why a single common-source stage does not oscillate if it is placed in a unity-gain loop.

Solution

From Fig. 14.4, it is seen that the open-loop circuit contains only one pole, thereby providing a maximum frequency-dependent phase shift of 90° (at a frequency of infinity). Since the common-source stage exhibits a dc phase shift of 180° due to the signal inversion from the gate to the drain, the maximum total phase shift is 270°. The loop therefore fails to sustain oscillation growth.

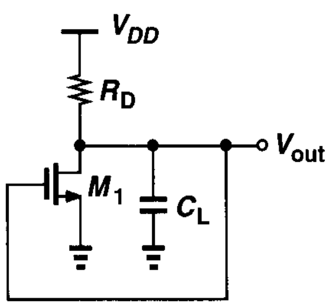

Figure 14.4

The above example suggests that oscillation may occur if the circuit contains multiple stages and hence multiple poles. Indeed, such a topology was considered *undesirable* in Chapter 10 because it led to inadequate phase margin in op amps. We therefore surmise that if the circuit of Fig. 14.4 is modified as shown in Fig. 14.5, then two significant poles appear in the signal path, allowing the frequency-dependent phase shift to approach 180°.

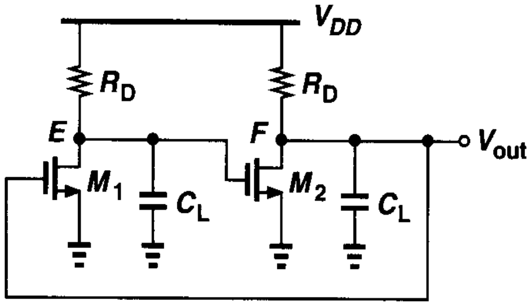

Figure 14.5 Two-pole feedback system.

Unfortunately, this circuit exhibits *positive* feedback near zero frequency due to the signal inversion through each common-source stage. As a result, it simply "latches up" rather than oscillates. That is, if V_E rises, V_F falls, thereby turning M_1 off and allowing V_E to rise further. This may continue until V_E reaches V_{DD} and V_F drops to near zero, a state that will remain indefinitely.

To gain more insight into the oscillation conditions, let us assume an ideal inverting stage (with zero phase shift at all frequencies) is inserted in the loop of Fig. 14.5, providing *negative* feedback near zero frequency and eliminating the problem of latch-up (Fig. 14.6). Does this circuit oscillate? We note that the loop contains only two poles: one at E and another

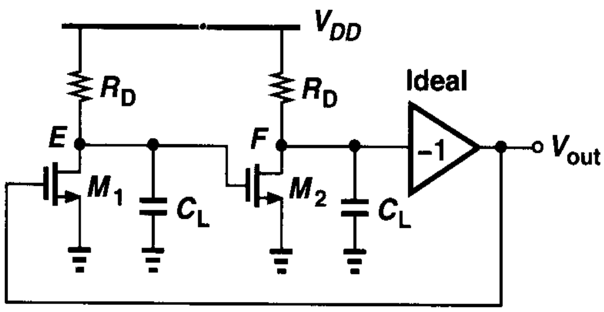

Figure 14.6 Two-pole feedback system with additional signal inversion.

at F. The frequency-dependent phase shift can therefore reach 180°, but at a frequency of infinity. Since the loop gain vanishes at very high frequencies, we observe that the circuit does not satisfy both of Barkhausen's criteria at the same frequency (Fig. 14.7), failing to oscillate.

The foregoing discussion points to the need for greater phase shift around the loop, suggesting the possibility of oscillation if the third inverting stage in Fig. 14.6 contains a pole that contributes significant phase. We then arrive at the topology depicted in Fig. 14.8. If the three stages are identical, the total phase shift around the loop, ϕ, reaches $-135°$ at $\omega = \omega_{p,E} (= \omega_{p,F} = \omega_{p,G})$ and $-270°$ at $\omega = \infty$. Consequently, ϕ equals $-180°$ at $\omega < \infty$, where the loop gain can be still greater than or equal to unity. This circuit indeed oscillates if the loop gain is sufficient and it is an example of a ring oscillator.

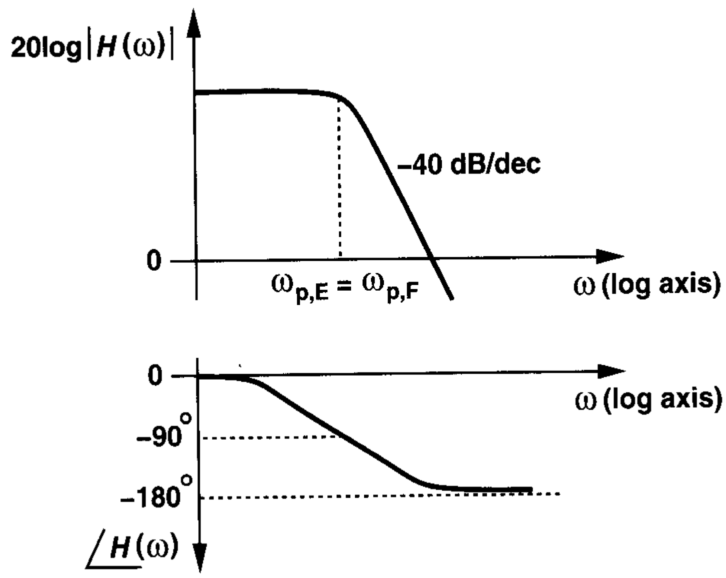

Figure 14.7 Loop gain characteristics of a two-pole system.

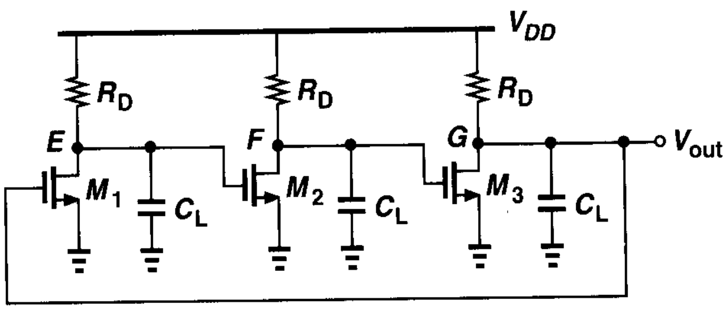

Figure 14.8 Three-stage ring oscillator.

It is instructive to calculate the minimum voltage gain per stage in Fig. 14.8 that is necessary for oscillation. Neglecting the effect of the gate-drain overlap capacitance and denoting the transfer function of each stage by $-A_0/(1+s/\omega_0)$, we have for the loop gain:

$$H(s) = -\frac{A_0^3}{(1+\frac{s}{\omega_0})^3}. \tag{14.6}$$

The circuit oscillates only if the frequency-dependent phase shift equals 180°, i.e., if each stage contributes 60°. The frequency at which this occurs is given by

$$\tan^{-1}\frac{\omega_{osc}}{\omega_0} = 60^\circ \tag{14.7}$$

and hence:

$$\omega_{osc} = \sqrt{3}\omega_0. \tag{14.8}$$

The minimum voltage gain per stage must be such that the magnitude of the loop gain at ω_{osc} is equal to unity:

$$\frac{A_0^3}{\left[\sqrt{1+(\frac{\omega_{osc}}{\omega_0})^2}\right]^3} = 1. \tag{14.9}$$

It follows from (14.8) and (14.9) that

$$A_0 = 2. \tag{14.10}$$

In summary, a three-stage ring oscillator requires a low-frequency gain of 2 per stage, and it oscillates at a frequency of $\sqrt{3}\omega_0$, where ω_0 is the 3-dB bandwidth of each stage.

Let us now examine the waveforms at the three nodes of the oscillator of Fig. 14.8. Since each stage contributes a frequency-dependent phase shift of 60° as well as a low-frequency signal inversion, the waveform at each node is 240° (or 120°) out of phase with respect to its neighboring nodes (Fig. 14.9). The ability to generate multiple phases is a very useful property of ring oscillators.

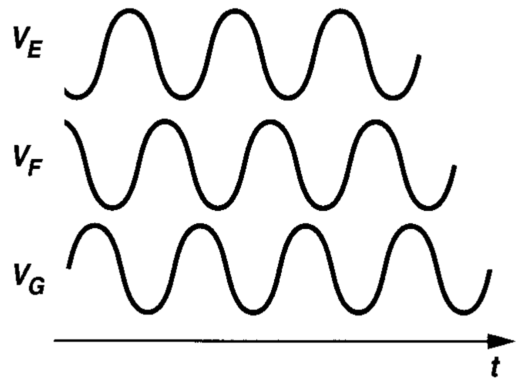

Figure 14.9 Waveforms of a three-stage ring oscillator.

Amplitude Limiting The natural question at this point is: what happens if in the three-stage ring of Fig. 14.8, $A_0 \neq 2$? We know from Barkhausen's criteria that if $A_0 < 2$, the circuit fails to oscillate, but what if $A_0 > 2$? To answer this question, we first model the oscillator by a linear feedback system, as depicted in Fig. 14.10. Note that the feedback is positive (i.e., V_{out} is *added* to V_{in}) because $H(s)$ in Eq. (14.6) already includes the negative polarity resulting from three inversions in the signal path. The closed-loop transfer function is:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\dfrac{-A_0^3}{(1+s/\omega_0)^3}}{1+\dfrac{A_0^3}{(1+s/\omega_0)^3}} \tag{14.11}$$

$$= \frac{-A_0^3}{(1+s/\omega_0)^3 + A_0^3}. \tag{14.12}$$

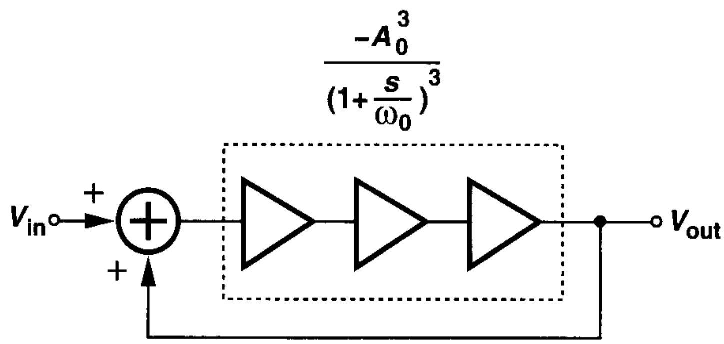

Figure 14.10 Linear model of three-stage ring oscillator.

The denominator of (14.12) can be expanded as:

$$(1+\frac{s}{\omega_0})^3 + A_0^3 = (1+\frac{s}{\omega_0}+A_0)\left[(1+\frac{s}{\omega_0})^2 - (1+\frac{s}{\omega_0})A_0 + A_0^2\right]. \tag{14.13}$$

Thus, the closed-loop system exhibits three poles:

$$s_1 = (-A_0 - 1)\omega_0 \tag{14.14}$$

$$s_{2,3} = [\frac{A_0(1 \pm j\sqrt{3})}{2} - 1]\omega_0. \tag{14.15}$$

Since A_0 itself is positive, the first pole leads to a decaying exponential term: $\exp[(-A_0 - 1)\omega_0 t]$, which can be neglected in the steady state. Figure 14.11 illustrates the locations of the poles for different values of A_0, revealing that for $A_0 > 2$, the two complex poles exhibit a positive real part and hence give rise to a growing sinusoid. Neglecting the effect of s_1, we express the output waveform as

$$V_{out}(t) = a\exp(\frac{A_0 - 2}{2}\omega_0 t)\cos(\frac{A_0\sqrt{3}}{2}\omega_0 t). \tag{14.16}$$

Thus, if $A_0 > 2$, the exponential envelope grows to infinity.

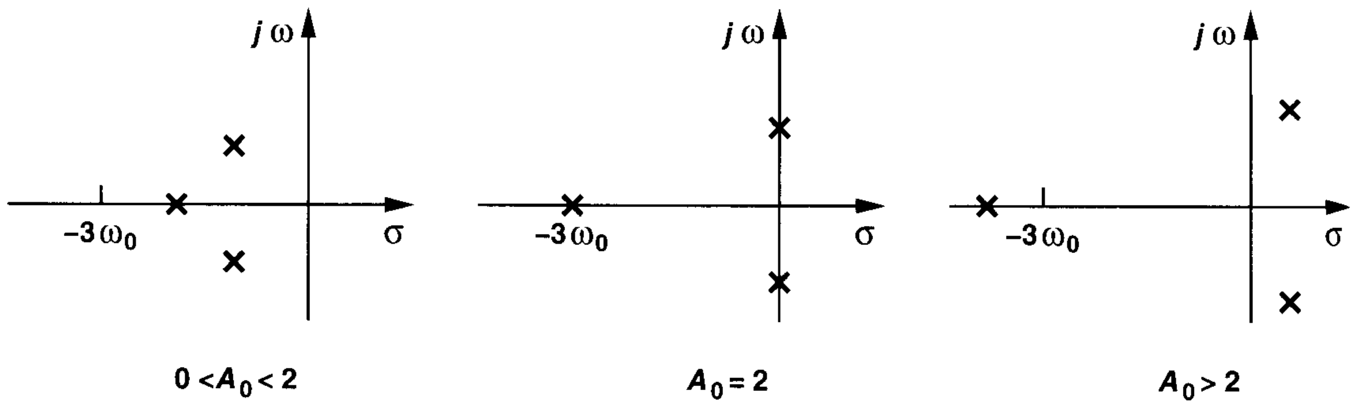

Figure 14.11 Poles of three-stage ring oscillator for various values of gain.