



ΠΑΝΕΠΙΣΤΗΜΙΟ
ΠΑΤΡΩΝ
UNIVERSITY OF PATRAS

Special Topics in Business Economics

Introduction to Malmquist Index Productivity and to Olley Pakes Method

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- A production function is an equation that expresses the relationship between the quantity of inputs (such as labour and capital) used and the amount of output (product) obtained.
- Productivity is the ratio of what is produced (output) to what is required to produce it (input). Usually this ratio is in the form of an average.
- Productivity indexes.
 - TFP, which is the simplest case.
 - Malmquist Index measures the productivity changes along with time variations and can be decomposed into changes in efficiency and technology.

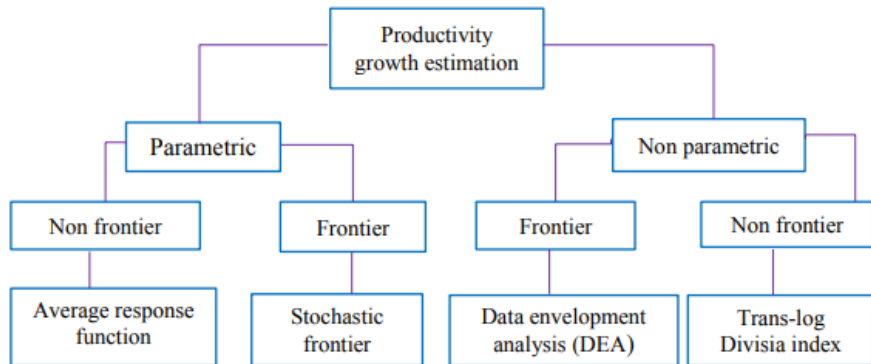
Malmquist Productivity Index

$$M_o(x^{t+1}, y^{t+1}, x^t, y^t) = \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} * \left[\left(\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \right) \left(\frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)} \right) \right]^{1/2}$$

MPI

- The ratio outside the brackets measures the change in relative efficiency between years t and $t+1$.
- The geometric mean of the two ratios inside the brackets captures the shift in technology (innovation) between the two periods evaluated at x^t and x^{t+1} .
- Improvements in productivity yield MPI greater than 1.
- Deterioration in productivity over time is associated with values less than 1.
- Same holds for the two components of Malmquist index.

Productivity Growth



Source: Mahadevan (2002, pp.6)

Productivity Growth

- **Parametric** and **non-parametric** are the two basic methods for productivity estimations.
- The parametric approach often uses the Cobb-Douglas model due to its simplicity.
- Non-parametric approach does not need a functional form assumption of the production process.
- Non frontier assumes that the unit of analysis are technically efficient.
- Frontier steps on the basis of the best performance.

- Olley and Pakes back in 1996 introduced a semi-parametric method that estimates productivity.
- Productivity is often estimated as the deviation between observed output and output predicted by a Cobb–Douglas production function estimated by OLS. Such estimates, however, may suffer from simultaneity and selection biases.

Simultaneity

Simultaneity arises because productivity is known to the profit-maximizing firms when they choose their input levels. Firms will increase their use of inputs as a result of positive productivity shocks. OLS estimation of production functions will yield biased parameter estimates because it does not account for the unobserved productivity shocks.

Selection bias

Selection bias results from the relationship between productivity shocks and the probability of exit from the market. If a firm's profitability is positively related to its capital stock, then a firm with a larger capital stock is more likely to stay in the market despite a low productivity shock than a firm with a smaller capital stock, because the firm with more capital can be expected to produce greater future profits.

Olley-Pakes Method - Assumptions

- 1 Firm decide at the beginning of each period whether to continue participating in the market or not. If the firm exits, it receives a liquidation value Φ . In case the firm does not exit, it chooses variable inputs (labor, material and energy) and a level of investment, I_{it} . The firm also has state variables: a productivity shock, Ω_{it} ; the capital stock, K_{it} ; and the age of the firm, a_{it} .
- 2 Furthermore, expected productivity is a function of current productivity and capital, $E[\Omega_{i,t+1}|\Omega_{it}, K_{it}]$, and that the firm's profit is a function of Ω_{it} and K_{it}
- 3 Productivity shocks evolve according to the distribution

$$p(w_{it+1}|I_{it}) = p(w_{it+1}|w_{it})$$

which means that the distribution $p(w_{it+1}|w_{it})$ defines what the firm knows about the future productivity shocks.

- 4 Firms' investment decisions are given by $i_{it} = f_t(k_{it}, w_{it})$ which is strictly monotonic and one can invert the last and get $w_{it} = f_t^{-1}(k_{it}, i_{it})$

Olley-Pakes Method - Production Function

For estimation purposes, we assume Cobb-Douglas Production Function:

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_m m_{it} + \beta_e e_{it} + \beta_k k_{it} + a_{it} + u_{it} \quad (1)$$

$$u_{it} = \Omega_{it} + n_{it} \quad (2)$$

y_{it} is log output for firm i in period t

l_{it} , m_{it} , e_{it} and k_{it} are the log values of labor, material, energy and capital inputs.

a_{it} is the age of the firm.

Ω_{it} a productivity shock that is observed by the decision-maker in the firm.

n_{it} is an unexpected productivity shock that is unobserved by the decision-maker.

Olley-Pakes Method - Production Function

- The firm's decision to invest in further capital, I_{it} depends on Ω , K and α :

$$I_{it} = I(\Omega_{it}, K_{it}, \alpha_{it})$$

Implying that future productivity is increasing in the current productivity shock, thus firms facing a large productivity shock in period t will invest more in period $t+1$.

- **In other words future productivity is strictly increasing with respect to Ω_{it} .**
- (1) becomes:

$$y_{it} = \beta_l I_{it} + \beta_m m_{it} + \beta_e e_{it} + \phi(i_{it}, k_{it}, \alpha_{it}) + n_{it} \quad (3)$$

where $\phi(i_{it}, k_{it}, \alpha_{it}) = \beta_0 + \beta_k k_{it} + \beta_a a_{it} + h(i_{it}, k_{it}, a_{it})$, and $\phi(\cdot)$ gets approximated with a second order polynomial series in age, capital and investment. Coefficient estimators for labor, material and age can be estimated by OLS and will be consistent because $\phi(\cdot)$ controls for unobserved productivity and thus the error term is no longer correlated with the inputs. β_k and β_a are estimated using nonlinear method. (G.M.M.)

Olley-Pakes Method - Overview

- A method for robust estimation of the production function allowing for selection (exit), unobserved permanent differences across firms.
- Requires variable (investment in this case) and the unobserved firm-level and also state variable productivity.
- Exit is also conditioned on the unobserved productivity.

```
olley_pakes(data, formula = y ~ free + capital + proxy + controls, exit =  
NULL, id = "id", time = "year", bootstrap = TRUE, reps = 2, degree =  
c(3, 2), verify = TRUE, maxiter = 100, ...)
```

- *data* A data frame containing the variables of the model.
- *exit* An optional formula with the name of the variable indicator of firm's last period.
- *id* A character with the name of the indicator variable.
- *time* A character with the name of the time variable.
- *bootstrap* An optional logical. If TRUE calculate bootstrap standard errors.

`opreg depvar [if] [in], exit(varname) state(varlist) proxy(varname)
free(varlist) [cvars(varlist) vce(bootstrap, bootstrap_options) level(#)]`

- `exit(varname)` specifies a dummy variable indicating whether firm i exited from the market in year t . A value of 1 indicates the firm exited.
- `state(varlist)` specifies the state variables that appear in the production function. Typical state variables are age and the log of capital.
- `proxy(varname)` specifies the variable that proxies for unobserved productivity. Typically this variable is the log of investment.
- `free(varlist)` specifies the freely variable inputs, such as the logs of materials, energy, and labor.
- `cvars(varlist)` specifies any additional independent variables that will be used in the first and second stages of estimation. Examples include year, size of firm, and region dummy variables.
- `vce(bootstrap, bootstrap_options)` allows specification of options to control the bootstrap process.

- [1] Färe, R., Grosskopf, S., Norris, M., & Zhang, Z. (1994). **Productivity growth, technical progress, and efficiency change in industrialized countries.** *The American economic review*, 66-83.
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- [3] Yasar, M., Raciborski, R., & Poi, B. (2008). **Production function estimation in Stata using the Olley and Pakes method.** *The Stata Journal*, 8(2), 221-231.
- [4] <https://cran.r-project.org/web/packages/estprod/estprod.pdf>