ΠΑΝΕΠΙΣΤΗΜΙΟ ΠΑΤΡΩΝ
UNIVERSITY OF PATRAS

LECTURE 2- *PRODUCTION, TECHNOLOGY AND COST FUNCTIONS (USING LINEAR PROGRAMMING TO ESTIMATE EFFICIENCY)*-EFFICIENCY AND PRODUCTIVITY MEASUREMENT

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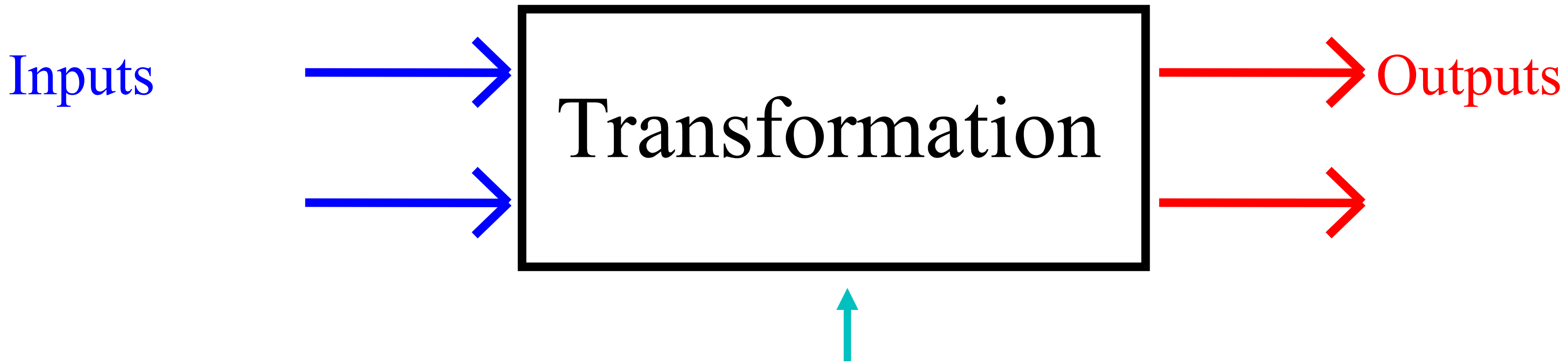
Literature Review

- Efficiency concepts developed by Farrell (1957); Fare, Grosskpof and Lovell (1985;1994); Lovell (1993).
- Debreu (1951) and Koopmans (1951) defined simple measures of efficiency.

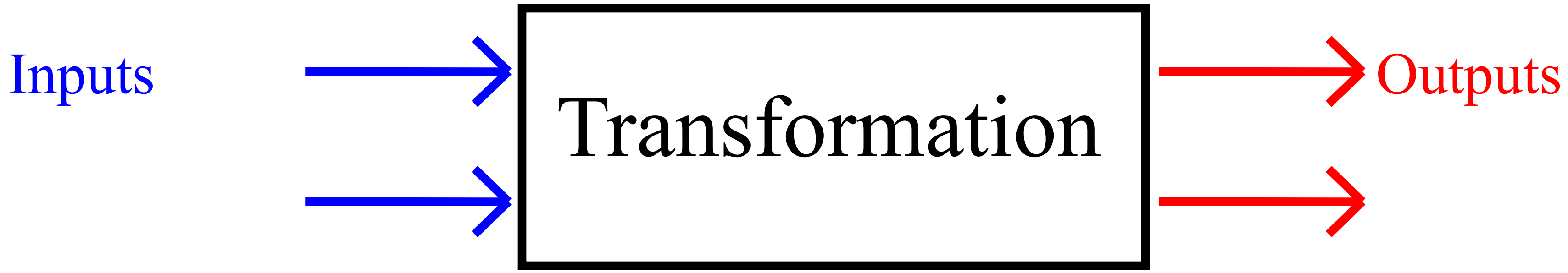
THE FUNDAMENTAL VIEW OF THE PROBLEM



The units to be assessed transform *inputs* into *outputs*

The basic requirement is to compare the Decision Making Units (DMUs) on the levels of outputs they secure relative to their input levels.

MEASURES OF COMPARATIVE EFFICIENCY



In a given operating context the measure of efficiency is normally one of:

- The distance between observed and maximum possible output for given inputs (output efficiency);

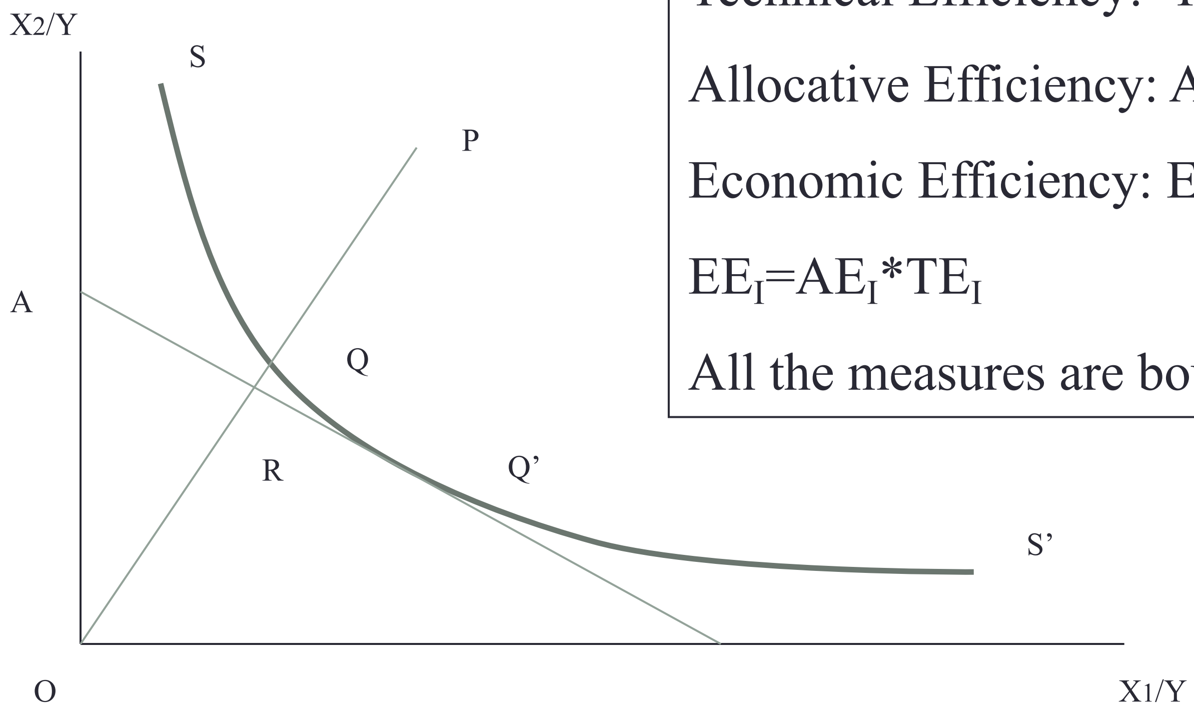
- The distance between observed and minimum possible input for given outputs (input efficiency);
- Remember that inputs and outputs are freely disposable

Efficiency Measures

- Using the distance functions defined so far, we can define (Fare, Grosskopf and Lovell, 1994):
  - *Technical efficiency*
  - *Allocative efficiency*
  - *Economic efficiency*
- A firm is said to be *technically efficient* if it operates on the frontier of the production technology
- A firm is said to be *allocatively efficient* if it makes efficient allocation in terms of choosing optimal input and output combinations.
- A firm is said to be *economically efficient* if it is both technically and allocatively efficient.
- There is also the definition of *scale efficiency* (later on!!)

Input Orientated Measures I

Lets assume a firm which is using two inputs (Labor and Capital) to produce s single output (Y-Total sales).The SS' curve in the following Figure represents the knowledge of the unit isoquants of fully efficient firms.



Technical Efficiency: $TE_I = OQ/OP = 1 - QP/OP$

Allocative Efficiency: $AE_I = OR/OQ$

Economic Efficiency: $EE_I = OR/OP = 1 - QP/O$

$EE_I = AE_I * TE_I$

All the measures are bounded between 0 and 1

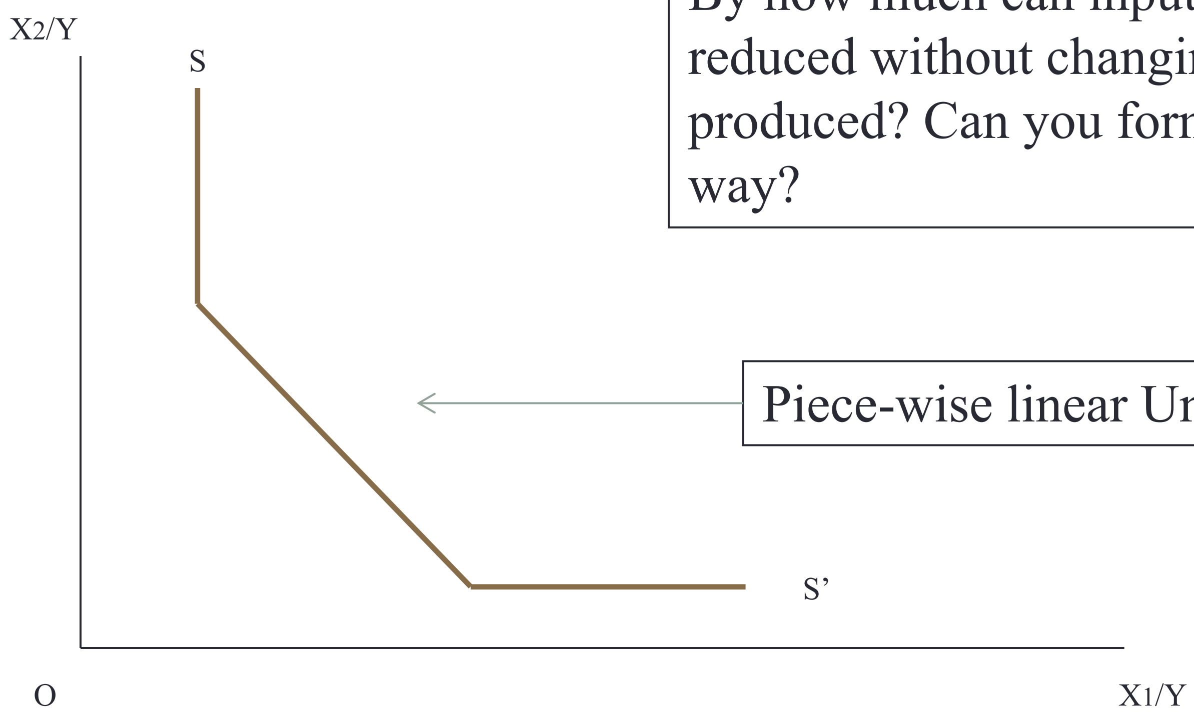
What TE=0.9 means?

Input Orientated Measures II

Farrell (1957) suggests the use of

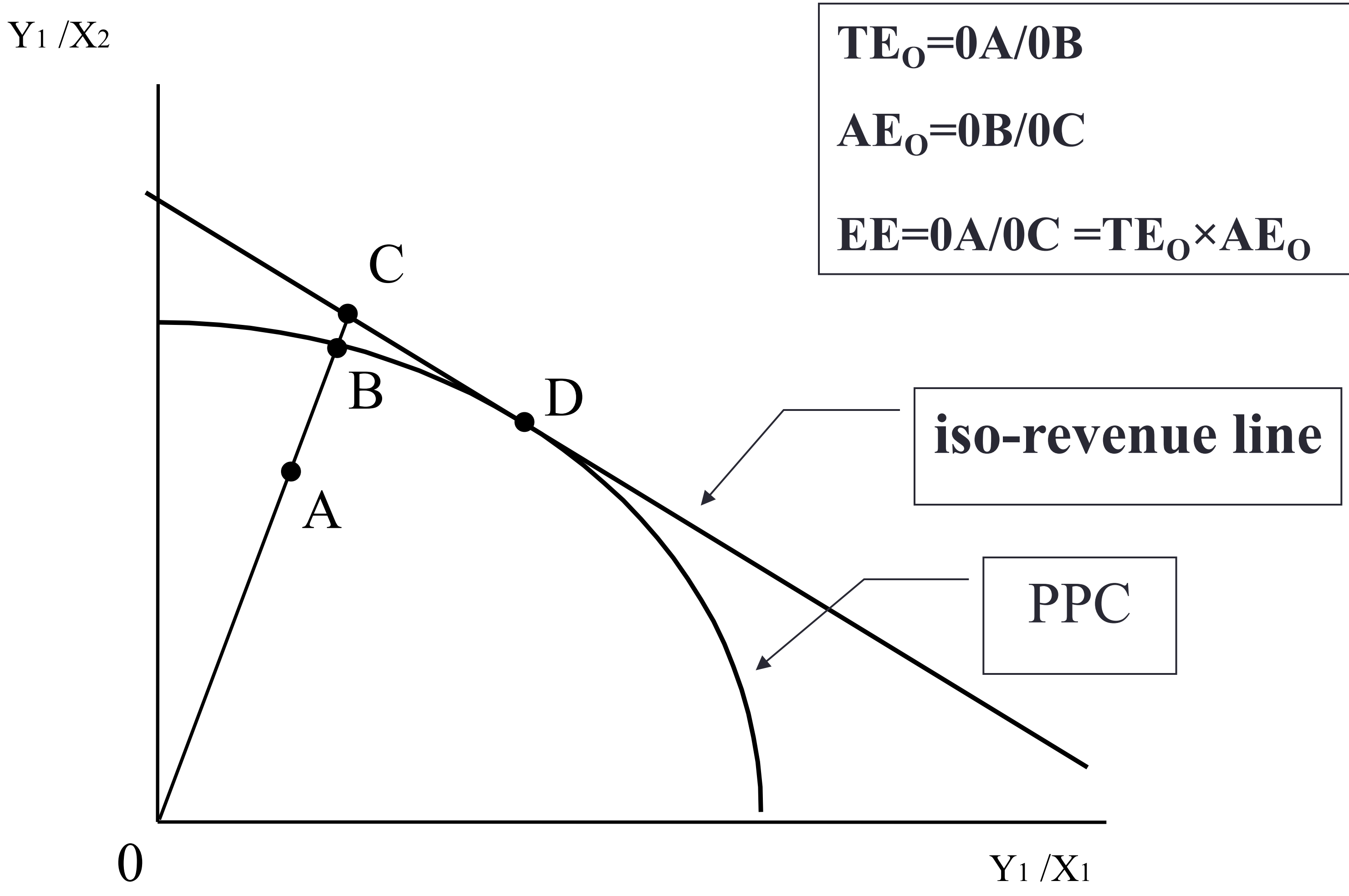
1. a non-parametric piece-wise-linear convex isoquant,
2. A parametric function (Cobb-Douglas)

By how much can input be proportionally reduced without changing the output produced? Can you form it in an another way?



Piece-wise linear Unit Isoquant

Output Orientated Measures I



$TE_O = 0A/0B$

$AE_O = 0B/0C$

$EE = 0A/0C = TE_O \times AE_O$

A Simple Example

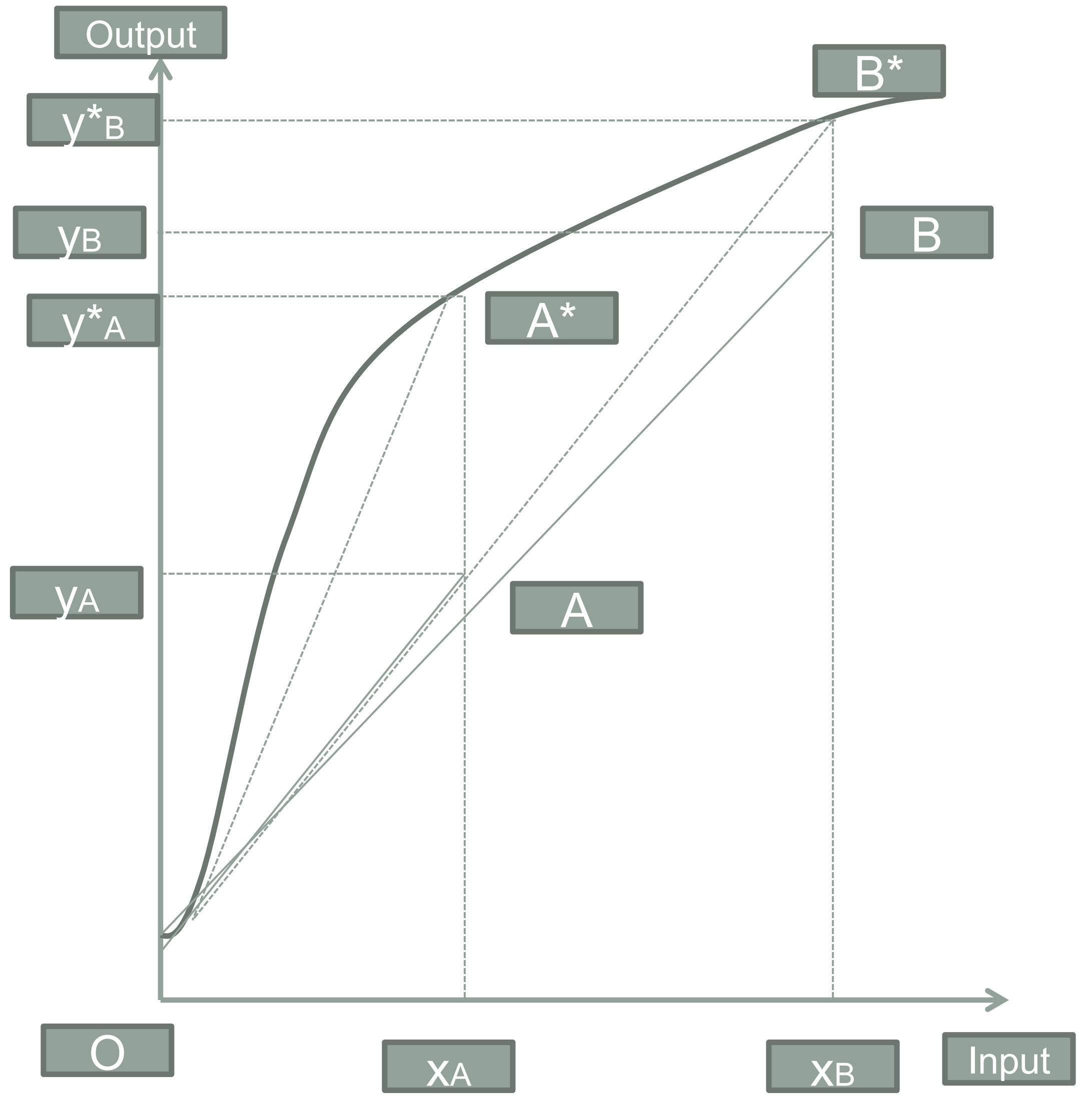
Let us assume two firms A,B with the following quantities

$$(x_A, y_A) = (16, 3), (x_B, y_B) = (64, 7)$$

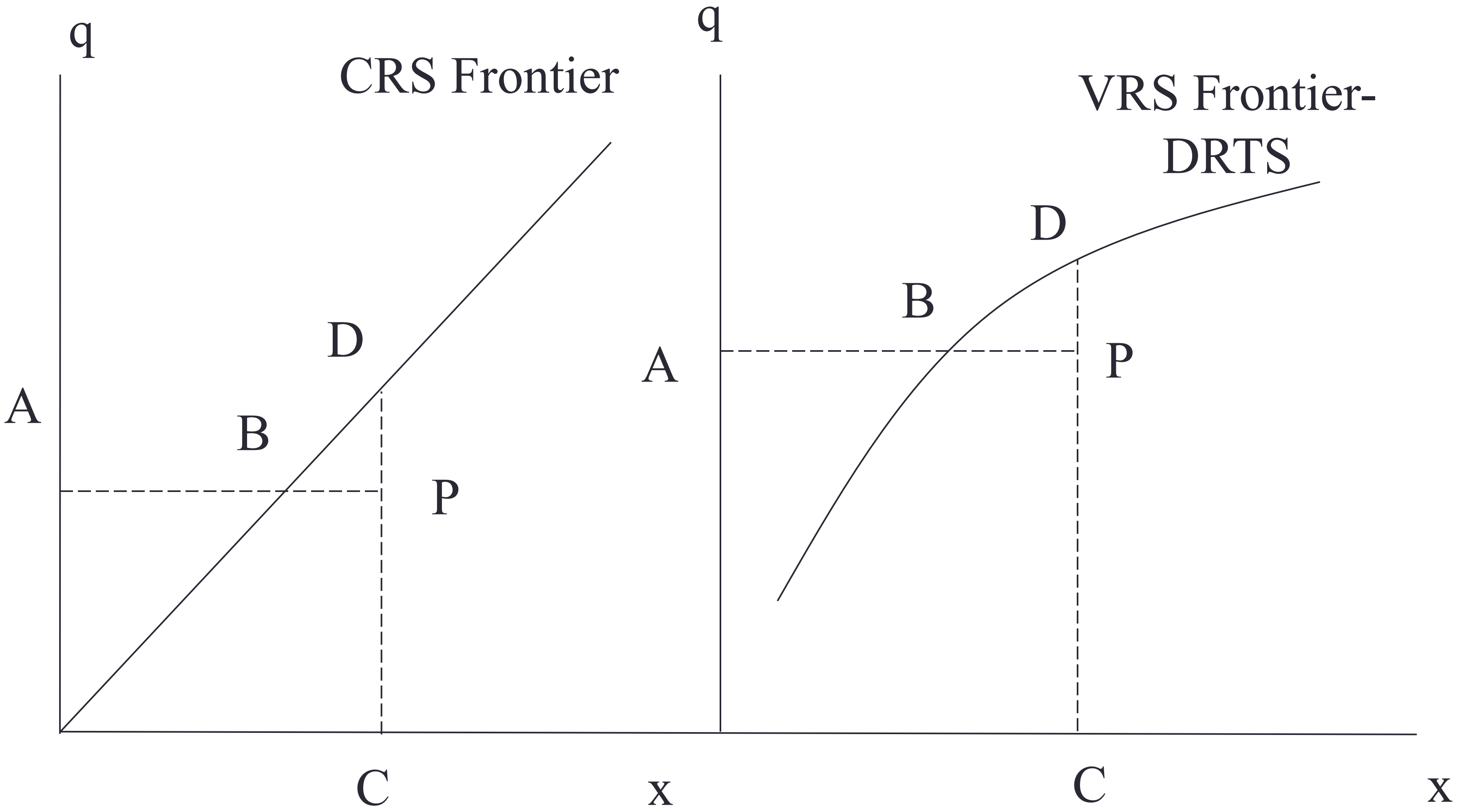
Can you calculate the average productivities and compare

the productivity index of firm A relative to firm B?

How these measures are related with technical efficiency concept?



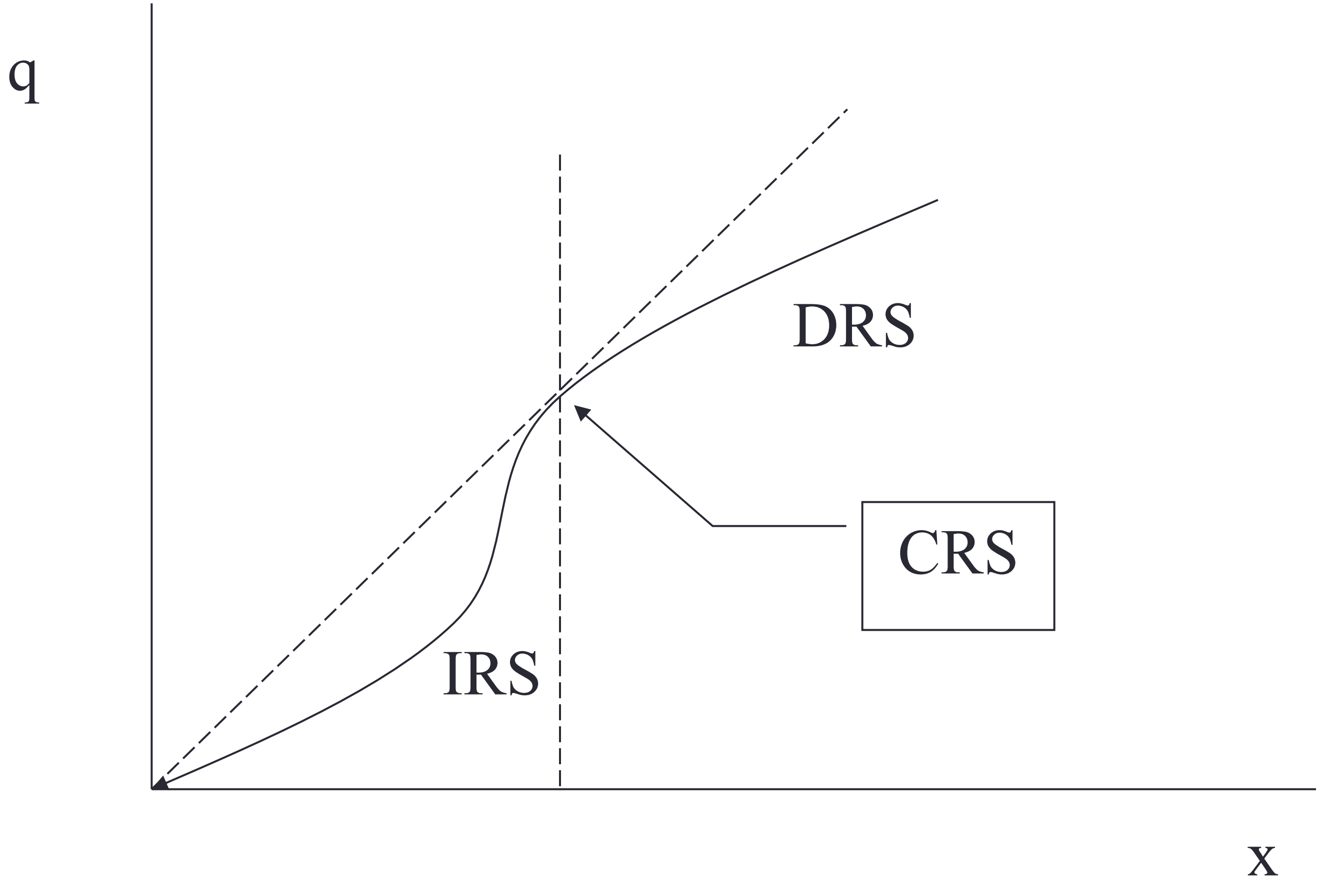
Returns to Scale



Returns to Scale

- A production technology exhibits *constant returns to scale* (CRS) if a $Z\%$ increase in inputs results in $Z\%$ increase in outputs ($\varepsilon = 1$).
- A production technology exhibits *increasing returns to scale* (IRS) if a $Z\%$ increase in inputs results in a more than $Z\%$ increase in outputs ($\varepsilon > 1$).
- A production technology exhibits *decreasing returns to scale* (DRS) if a $Z\%$ increase in inputs results in a less than $Z\%$ increase in outputs ($\varepsilon < 1$).

Returns to scale



Economies of scope

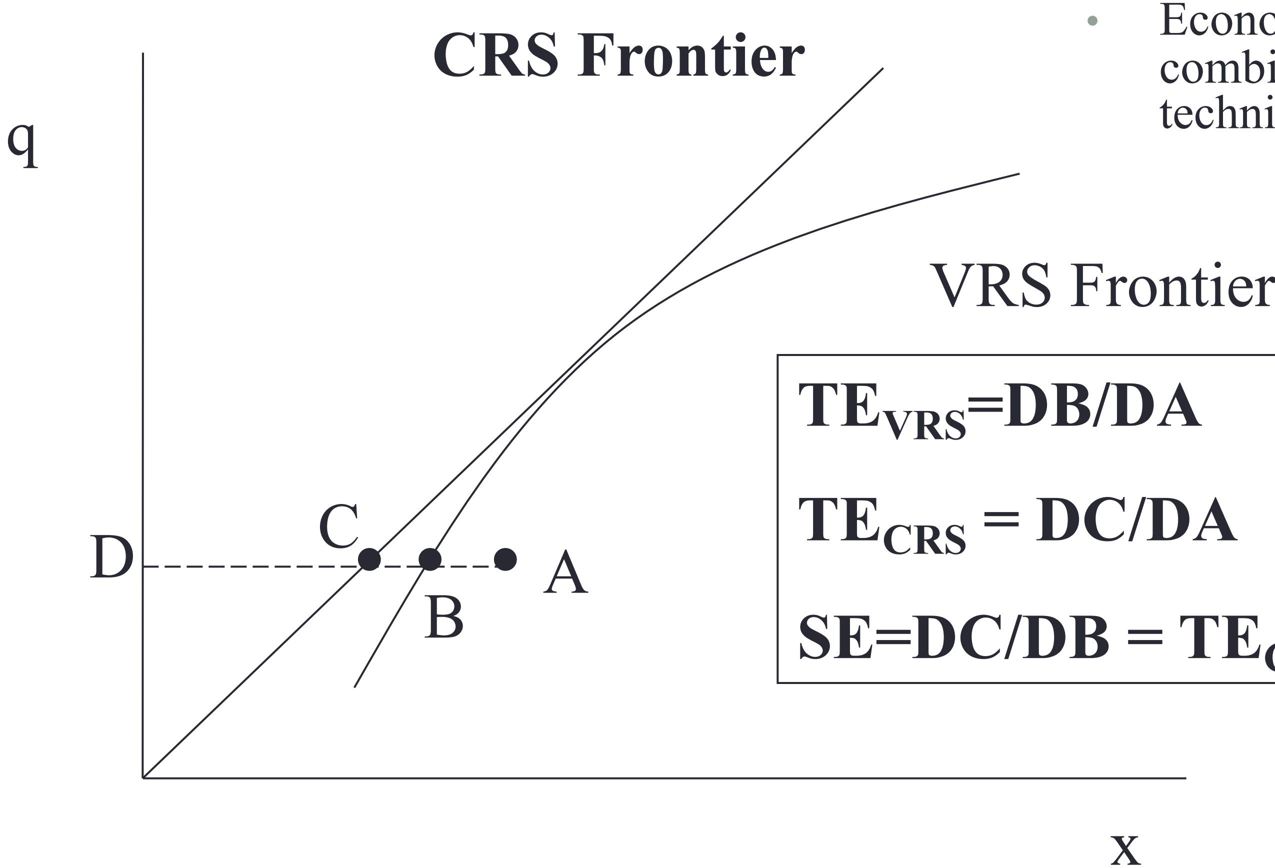
- Is it less costly to produce M different products in one firm versus in M firms?
- One measure of economies of scope is:

$$S = \frac{\sum_{m=1}^{M} c(\mathbf{w}, q_m) - c(\mathbf{w}, \mathbf{q})}{c(\mathbf{w}, \mathbf{q})}$$

- $S > 0$ implies economies of scope – it is better to produce the M outputs in one firm.
- Other measures:
  - product specific measures
  - second derivative measures

Scale Efficiency

- Productive efficiency is the combination of scale and technical.
- Economic efficiency is the combination of scale, technical and allocative.

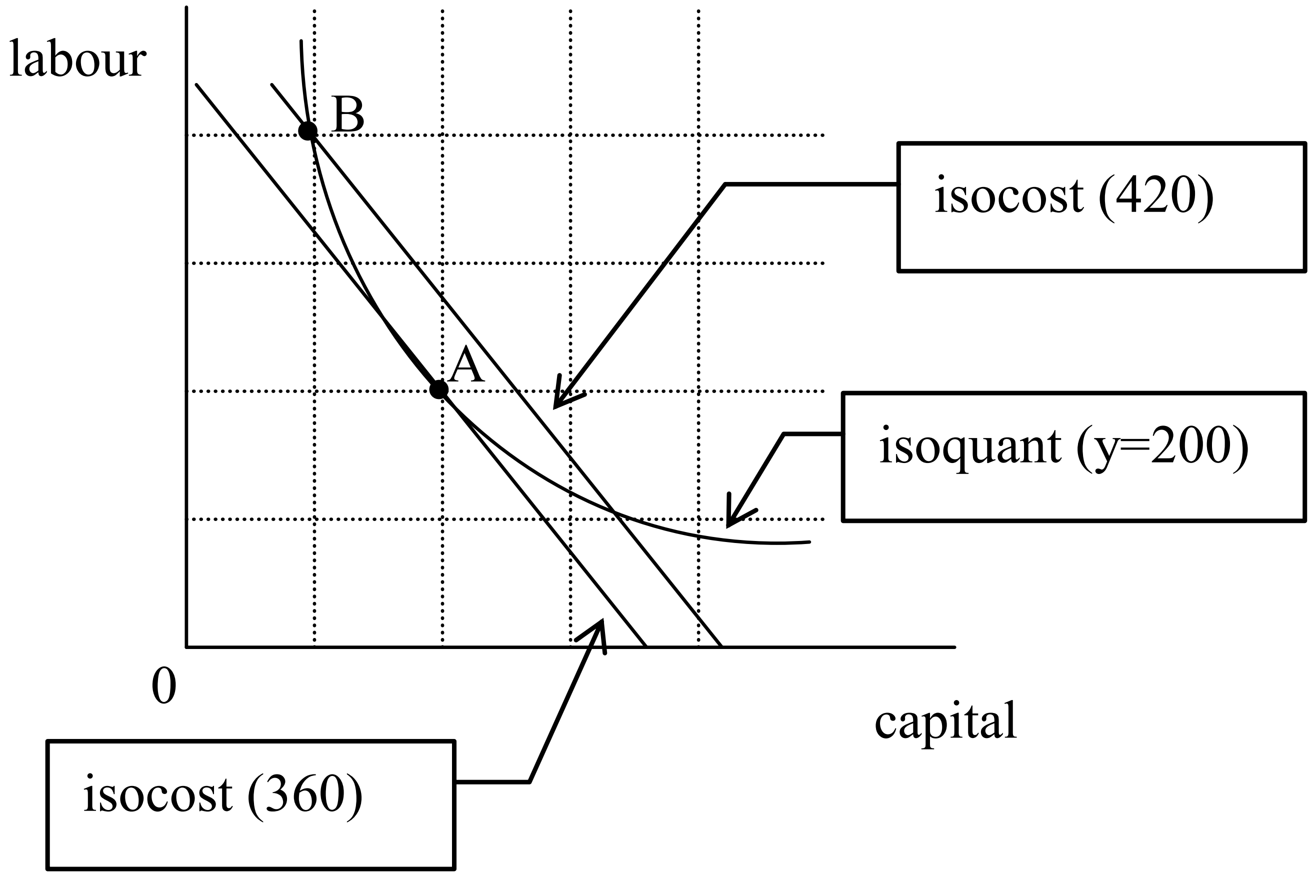


TE_{VRS}=DB/DA

TE_{CRS} = DC/DA

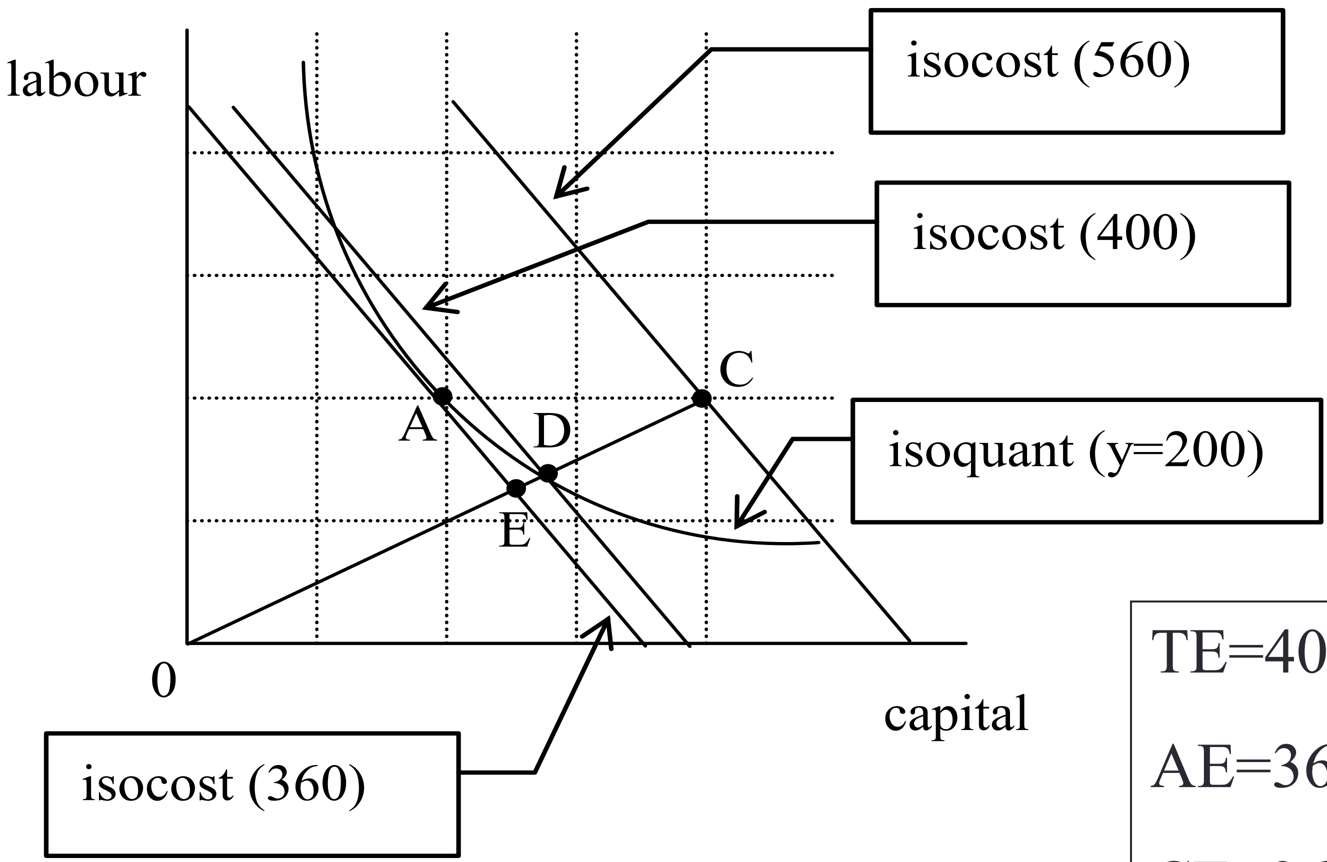
SE=DC/DB = TE_{CRS}/TE_{VRS}

Allocative Efficiency I



AE=360/420=0.86

Allocative Efficiency II



TE=400/560=0.71

AE=360/400=0.9

CE=360/560=0.64

MOST PRODUCTIVE SCALE SIZE

- Starrett (1977) generalize the concept of returns to scale in the context of multi-input, multi-output production function of two vectors $T(x,y)$

$$\sum_{i=1}^{n}\left(\frac{\partial T}{\partial x_i}x_i\right)\frac{dx_i}{x_i}+\sum_{j=1}^{m}\left(\frac{\partial T}{\partial y_j}y_j\right)\frac{dy_j}{y_j}=0$$

- If we assume that all inputs-outputs increase at the same proportional rate α,β respectively we have a local measure of returns to scale $DIR=\frac{\alpha}{\beta}-1=$

$$\frac{\alpha}{\beta}=-\frac{\sum_{i=1}^{n}\left(\frac{\partial T}{\partial x_i}x_i\right)\frac{dx_i}{x_i}}{\sum_{j=1}^{m}\left(\frac{\partial T}{\partial y_j}y_j\right)\frac{dy_j}{y_j}}$$

MOST PRODUCTIVE SCALE SIZE

- Banker defines most productive scale size with reference to $T(x^o, y^o)$ if for any (α, β) satisfying $(\beta x^o, \alpha y^o) \in \mathrm{T}$

  $\frac{\alpha}{\beta} \le 1$

- CRS holds at MPSS.
- Banker also defines returns to scale measure as

$$\rho = \lim_{\beta \to 1} \frac{\alpha(\beta) - 1}{\beta - 1}$$

Methodology

- There are two broad types of method for arriving at measures of comparative efficiency: parametric and non-parametric methods.
- The parametric methods typically hypothesise a functional form and use the data to estimate the parameters of that function. The estimated function is then used to arrive at estimates of the efficiencies of units.
- The non-parametric methods, best known as *Data Envelopment Analysis* (DEA), create virtual units to act as benchmarks for measuring comparative efficiency.

The CRS DEA model

$$MIN_{\theta,\lambda}\theta$$
$$s.t \quad -y_i + Y\lambda \geq 0$$
$$\theta x_i - X\lambda \geq 0$$

y - column vector of outputs,
x - column vector of inputs,
X - input matrix,

Y - output matrix.

q - efficiency score (q<=1).

$\theta < 1$, inefficiency
$\theta = 1$, efficiency

Note: θ is the measure of efficiency, given by the ratio between the weighted average of the outputs (y) produced and the weighted average of the inputs (x) used. See Coelli et al. (1998) for more details. The problem must been solved N times , one for each firm in the sample.

The VRS DEA model

The CRS model can be easily modified to VRS by adding the convexity constraint $n1'\lambda = 1$ that ensures that an inefficient firm is only "benchmarked" against firms of similar size.

$$
\begin{aligned}
& MIN_{\theta,\lambda}\ \theta \\
\text{s. to}\quad & -y_i + Y\lambda \geq 0 \\
& \theta x_i - X\lambda \geq 0 \\
& n1'\lambda = 1 \\
& \lambda \geq 0
\end{aligned}
$$

y - column vector of outputs,
x - column vector of inputs,
X - input matrix,

Y - output matrix.

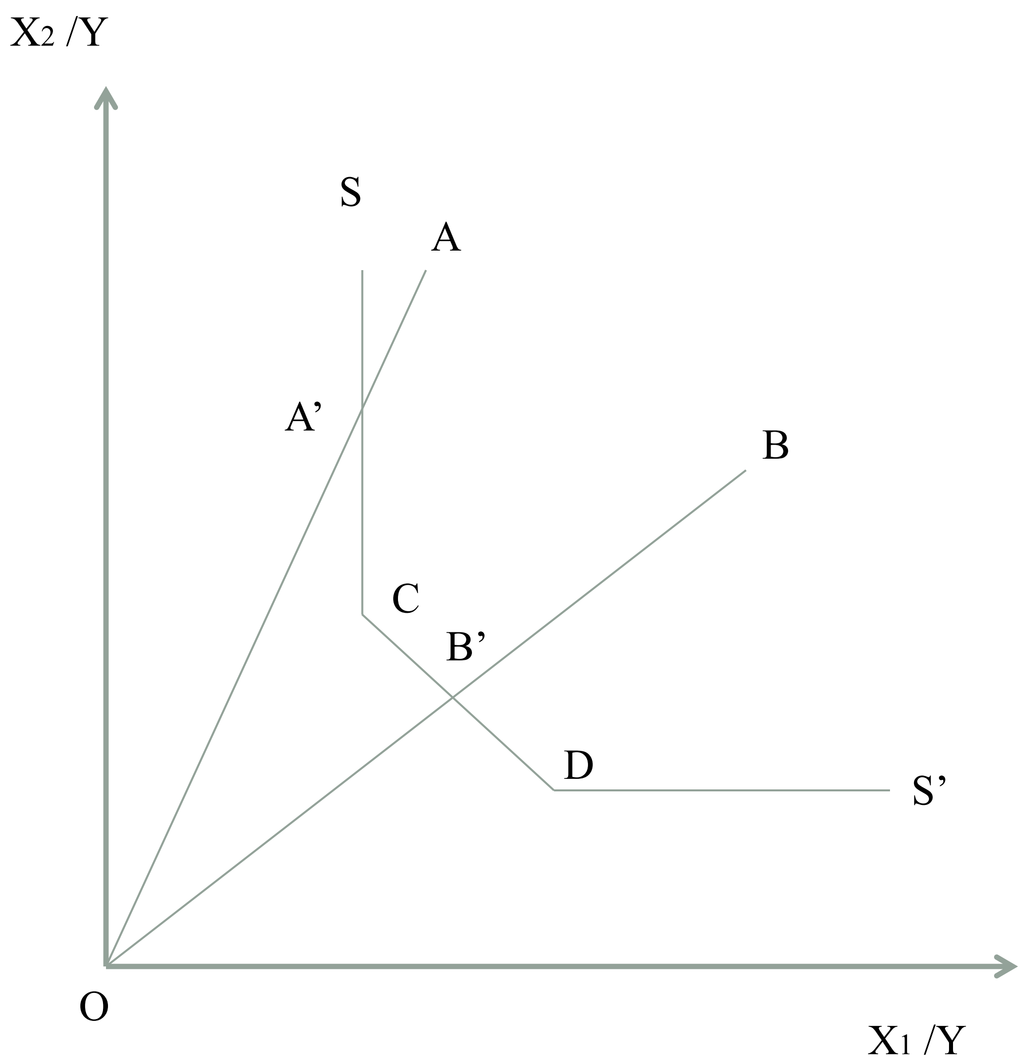
θ - efficiency score (θ<=1).

θ < 1, inefficiency
θ = 1, efficiency

Note: θ is the measure of efficiency, given by the ratio between the weighted average of the outputs (y) produced and the weighted average of the inputs (x) used. See Coelli et al. (1998) for more details.

The VRS DEA model-Digging more I

- Slack
- Define efficiency for A,B firms.
- Is the point A’ a efficient point?
- One could reduce the amount of X2 used by the CA’ and produce the same output (input slack)



The VRS DEA model-Digging more II

Firm	y	x_1	x_2	x_1/y	x_2/y
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

- Input Slack equal to zero→ $Y\lambda - y_i = 0$
- Output Slack equal to zero→ $\theta x_i - X\lambda = 0$

http://www.uq.edu.au/economics/cepa/deap.php

The VRS DEA model-Digging more III

Firm	y	x_1	x_2	x_1/y	x_2/y
1	1	2	5	2	5
2	2	2	4	1	2
3	3	6	6	2	2
4	1	3	2	3	2
5	2	6	2	3	1

- Let us now for firm 3 see the LP problem!!

$$MIN_{\theta,\lambda}\theta$$
$$s.t \ -y_i + Y\lambda \geq 0$$
$$\theta x_i - X\lambda \geq 0$$

$\longrightarrow$

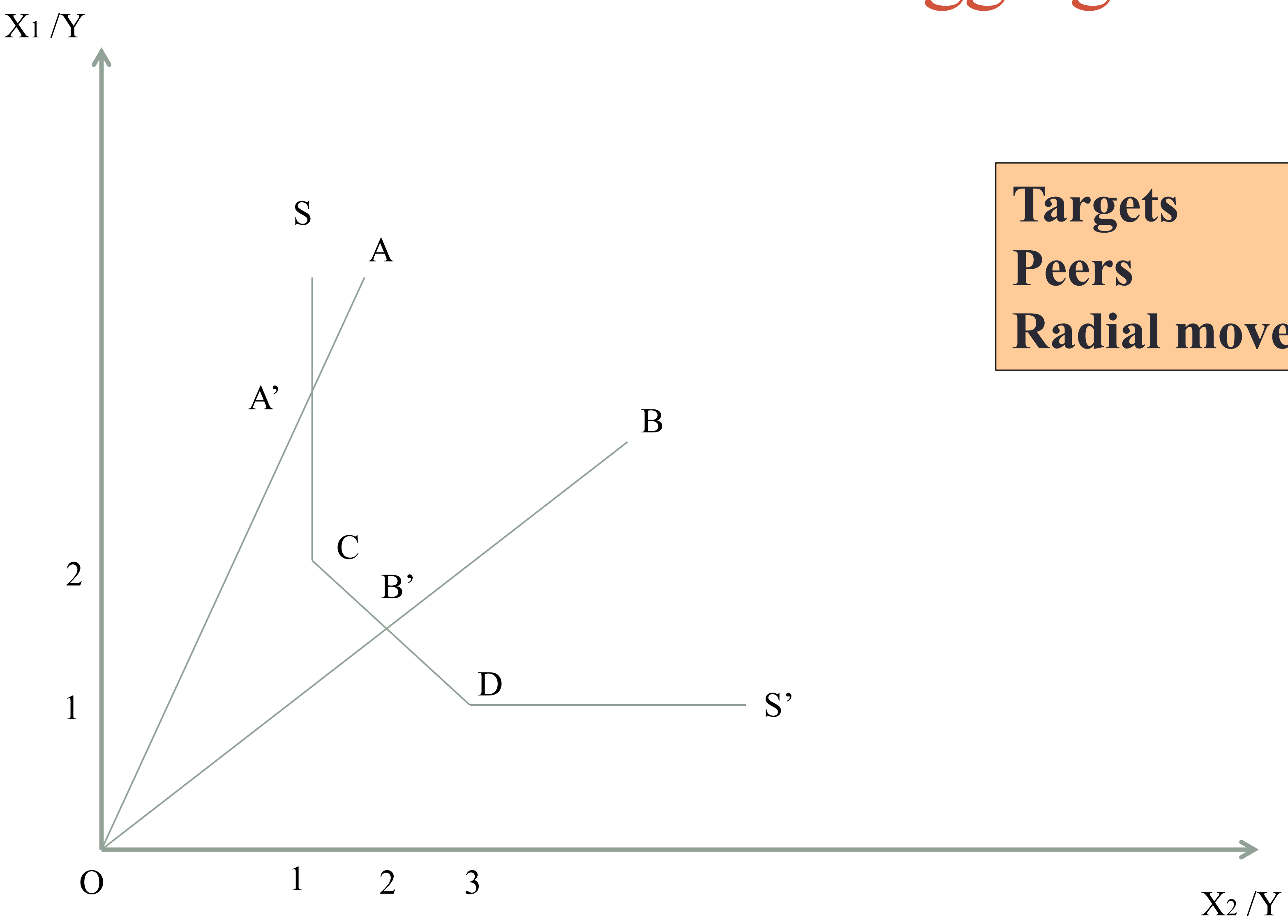
$$\min_{\theta,\lambda}$$
$$s.t \ -y_3 + (y_1\lambda_1 + y_2\lambda_2 + y_3\lambda_3 + y_4\lambda_4 + y_5\lambda_5) \geq 0$$
$$\theta x_{13} - (x_{11}\lambda_1 + x_{12}\lambda_2 + x_{13}\lambda_3 + x_{14}\lambda_4 + x_{15}\lambda_5) \geq 0$$
$$\theta x_{23} - (x_{21}\lambda_1 + x_{22}\lambda_2 + x_{23}\lambda_3 + x_{24}\lambda_4 + x_{25}\lambda_5) \geq 0$$
$$\lambda \geq 0$$
$$\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)'$$

The VRS DEA model-Digging more IV

Firm	θ	λ_1	λ_2	λ_3	λ_4	λ_5	IS_1	IS_2	OS
1	0.5	-	0.5	-	-	-	-	0.5	-
2	1	-	1	-	-	-	-	-	-
3	0.833	-	1	-	-	0.5	-	-	-
4	0.714	-	0.214	-	-	0.286	-	-	-
5	1	-	-	-	-	1	-	-	-

- Can you explain now the values of θ, λ?
- What the value of technical efficiency say to us?
- Which firms are the peers of firm 3?
- Which firms are also the targets for firm 3?

The VRS DEA model-Digging more V



Targets
Peers
Radial movement

DEA results output

EG2-out - Σημειωματάριο

Αρχείο Επεξεργασία Μορφή Προβολή Βοήθεια

```
 FIRM BY FIRM RESULTS:


Results for firm:     1
Technical efficiency = 1.000
Scale efficiency     = 0.500  (irs)
 PROJECTION SUMMARY:
  variable           original        radial         slack      projected
                        value      movement      movement          value
 output     1           1.000         0.000         0.000          1.000
 input      1           2.000         0.000         0.000          2.000
 LISTING OF PEERS:
  peer    lambda weight
    1       1.000


Results for firm:     2
Technical efficiency = 0.625
Scale efficiency     = 0.800  (irs)
 PROJECTION SUMMARY:
  variable           original        radial         slack      projected
                        value      movement      movement          value
 output     1           2.000         0.000         0.000          2.000
 input      1           4.000        -1.500         0.000          2.500
 LISTING OF PEERS:
  peer    lambda weight
    1       0.500
    3       0.500


Results for firm:     3
Technical efficiency = 1.000
Scale efficiency     = 1.000  (crs)
 PROJECTION SUMMARY:
```

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How do we measure efficiency?

- Depends upon the type of data available for the measurement purpose.
- Three types:
  - Observed input and output data for a given firm over two periods or data for a few firms at a given point of time;
  - Observed input and output data for a large sample of firms from a given industry (cross-sectional data)
  - Panel data on a cross-section of firms over time
- In the first case measurement is limited to productivity measurement based on restrictive assumptions.

Overview of Methods

- index numbers (IN)
  - Price and quantity index numbers used in aggregation (eg. Tornqvist, Fisher)
- data envelopment analysis (DEA)
  - non-parametric, linear programming
- stochastic frontier analysis (SFA)
  - parametric, econometric

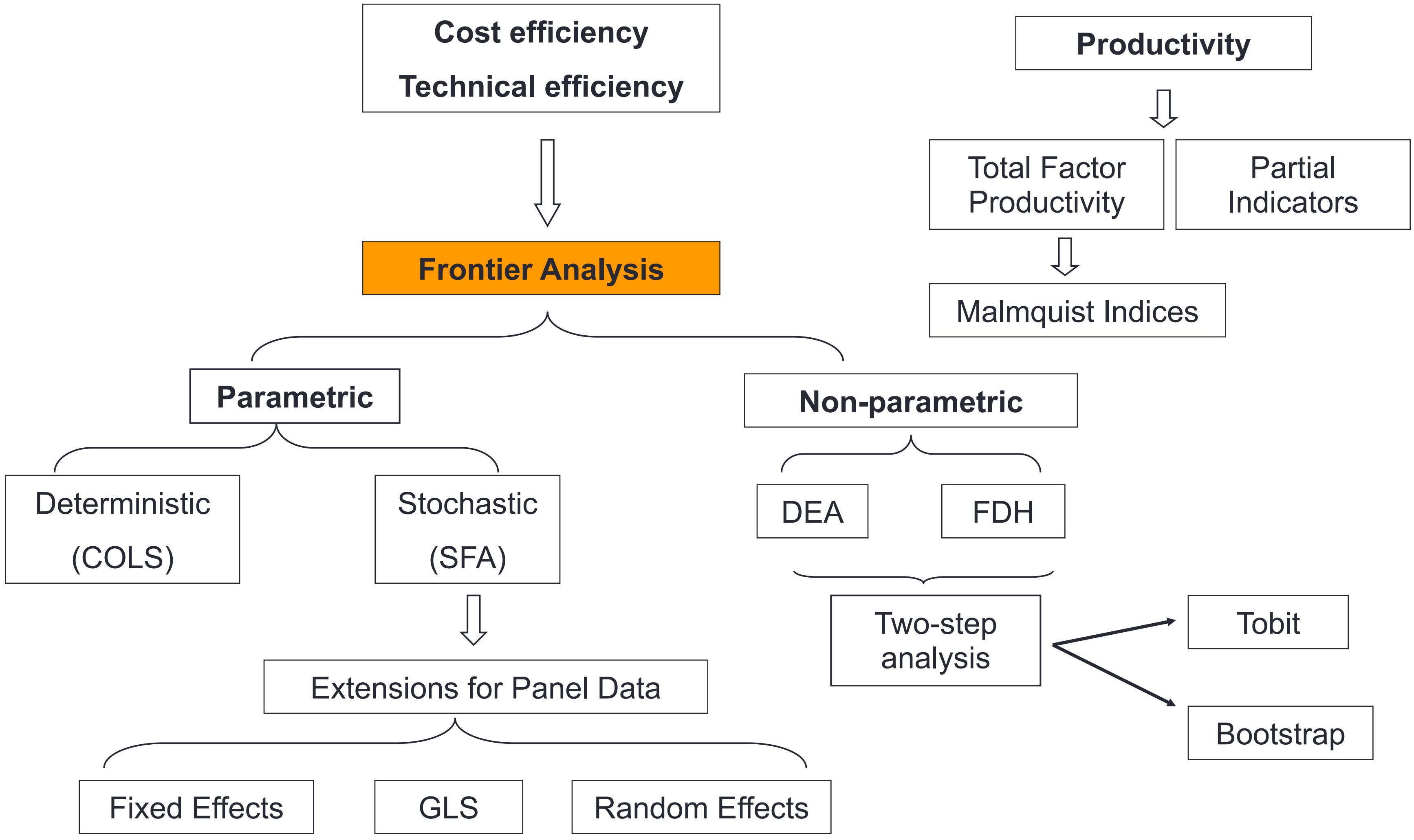
Relative merits of Index Numbers

- Advantages:
  - only need 2 observations
  - transparent and reproducible
  - easy to calculate
- Disadvantages:
  - need price information
  - cannot decompose

Relative merits of Frontier Methods

- DEA advantages:
  - no need to specify functional form or distributional forms for errors
  - easy to accommodate multiple outputs
  - easy to calculate
- SFA advantages:
  - attempts to account for data noise
  - can conduct hypothesis tests

Examples of possible methods

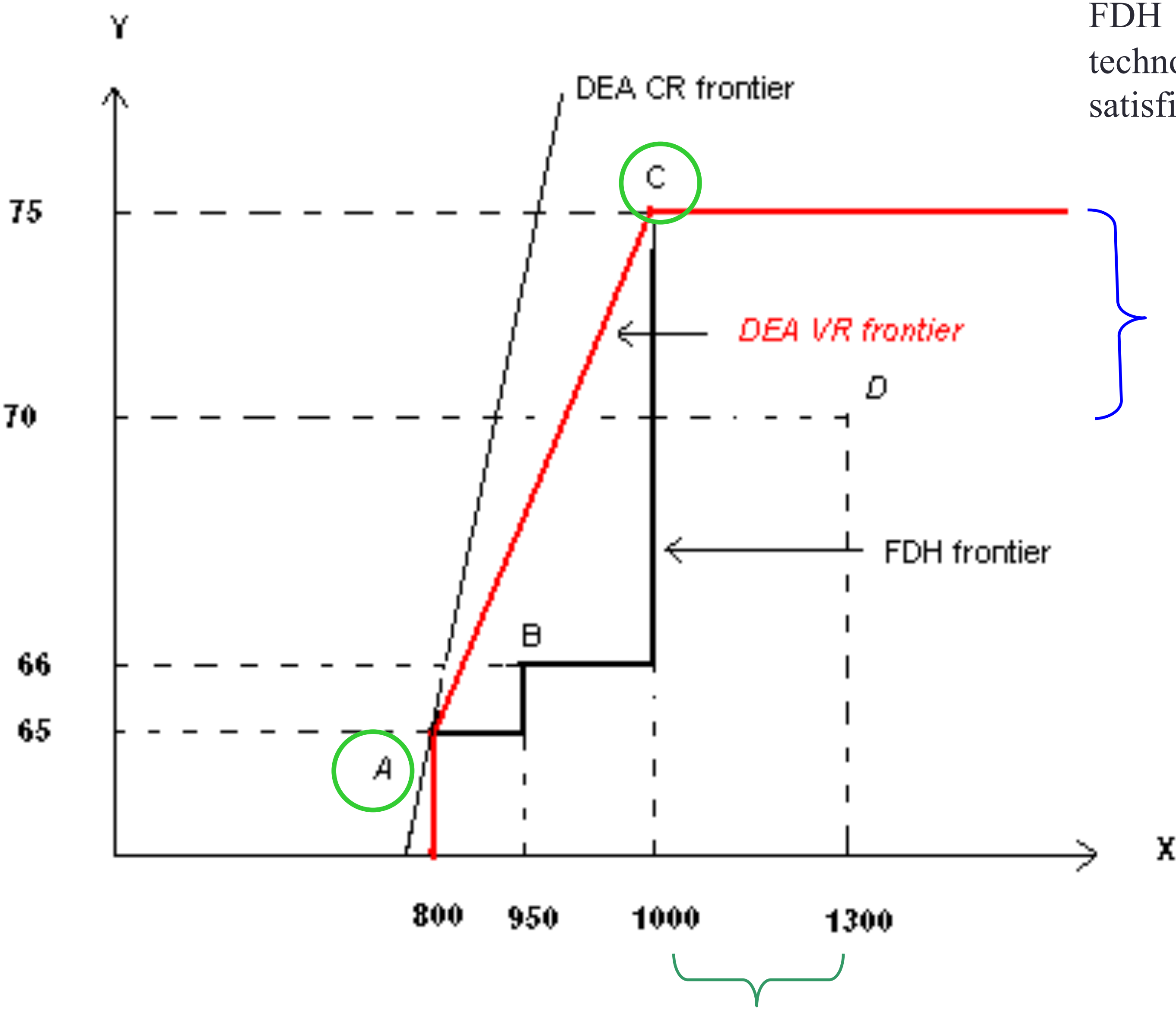


DATA ENVELOPMENT ANALYSIS

- Data Envelopment Analysis (DEA) measures the relative efficiencies of organizations with multiple inputs and multiple outputs. The organizations are called the decision-making units, or DMUs.
- DEA assigns weights to the inputs and outputs of a DMU that give it the best possible efficiency. It thus arrives at a weighting of the relative importance of the input and output variables that reflects the emphasis that appears to have been placed on them for that particular DMU.
- At the same time, though, DEA then gives all the other DMUs the same weights and compares the resulting efficiencies with that for the DMU of focus.
- If the focus DMU looks at least as good as any other DMU, it receives a maximum efficiency score. But if some other DMU looks better than the focus DMU, the weights having been calculated to be most favorable to the focus DMU, then it will receive an efficiency score less than maximum.

DEA and FDH (Deprints et al., 1984)

FDH non-convex technological set that satisfies disposability

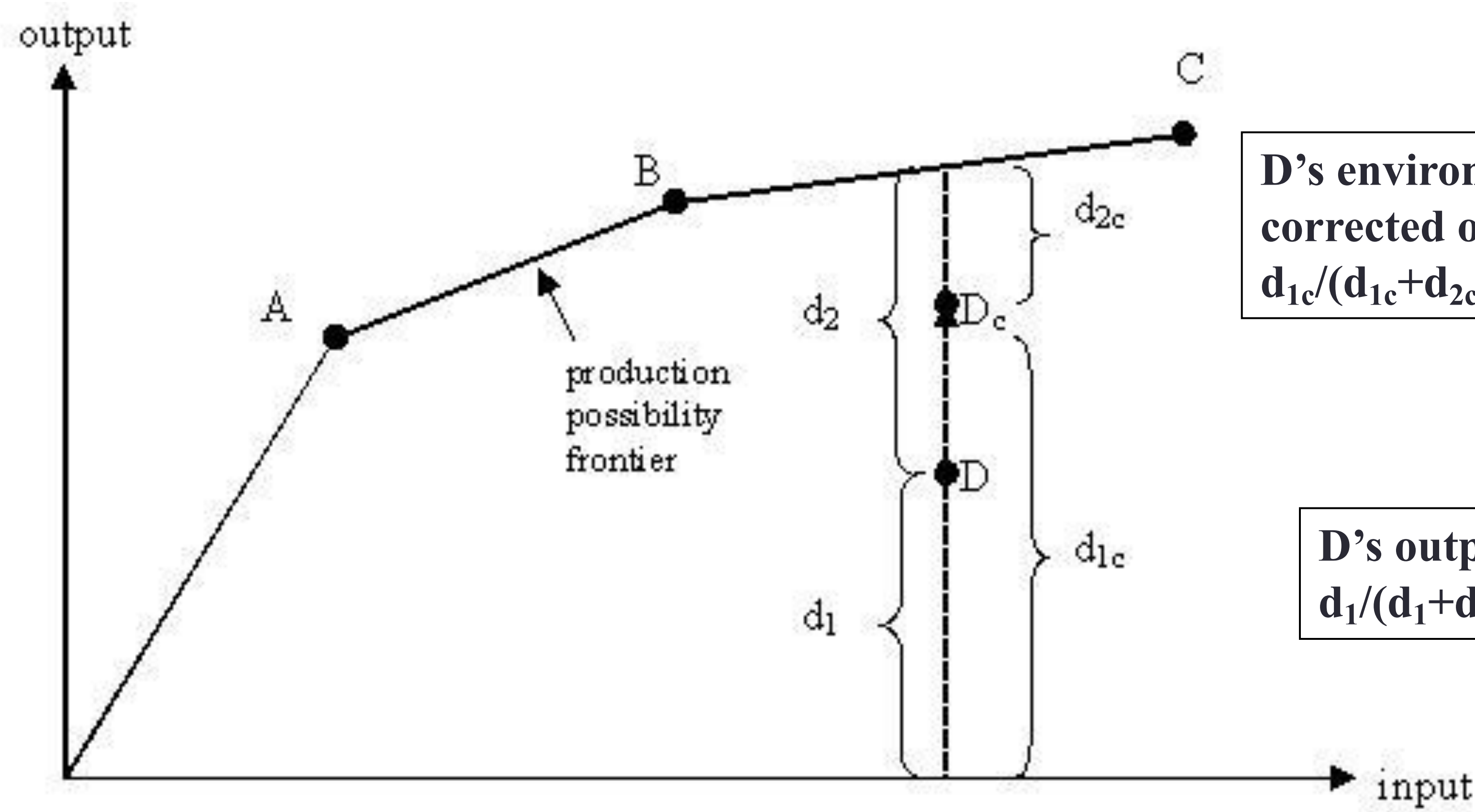


D's output inefficiency

A, C – efficient;
B, D – less efficient.

D's input inefficiency

Non-discretionary inputs and two-step procedure (1)



D's environment corrected output score= $d_{1c}/(d_{1c}+d_{2c})$

D's output score= $d_1/(d_1+d_2)$

$1 > d_{1c}/(d_{1c}+d_{2c}) > d_1/(d_1+d_2)$, the environment corrected score is closer to the frontier.

Category of Variables

- To this point, DEA has been essentially a mathematical process in which the data for input and output are taken as given, without further interpretation with respect to the reality of operations.
- But reality needs to be recognized, so there are several extensions that can be made to the basic DEA model, applicable to any of the variations.
- They fall into seven categories:
  - (1) Discretionary and Non-discretionary Variables
  - (2) Categorical Variables
  - (3)*A priori* restrictions on Weights
  - (4) Relationships between Weights on Variables
  - (5) *A priori* assessments of Efficient Units
  - (6) Substitutability of Variables
  - (7) Discrimination among Efficient Units

Discretionary & Non-discretionary

- In identifying input and output variables, one wants to include all that are relevant to the operation. For example, the level of output is determined not only by what the unit itself does but by the size of the market to which the output is delivered.
- The result, though, is that some relevant variables, such as the size of the market, are not under the control of management. Such variables, called non-discretionary, are in contrast to those that are under management control, called discretionary.
- In assessing efficiency, all variables are considered, but in determining the criterion function to be maximized or minimized, only the discretionary variables are included.

Categorical Variables-Negative

- In the DEA model as so far presented, the variables are treated as essentially quantitative, but sometimes one would like to identify non-quantitative variables, such as ordinal or nominal variables.
- For example, one might like to compare institutions of the same type, such as public or private universities.
- This is accomplished by introducing categorical variables containing numbers for order or identifiers for names.

- *Portela, M. S., Thanassoulis, E., & Simpson, G. (2004). Negative data in DEA: A directional distance approach applied to bank branches. Journal of the Operational Research Society, 55(10), 1111-1121.*

A priori Restrictions on Weights

- In the model as presented, the weights are limited only by the requirements that they be non-negative.
- However, there may be reason to require that weights be further limited.
- For example, it may be felt that a given variable *must* be included in the assessment so its weight must have at least a minimal value greater than zero. This might represent an output that is essential in assessment.
- As another example, a variable may be such a large weight would over-emphasize its *a priori* importance so that there should be an upper limit on the weight. This might represent an output variable that is counter-productive.

A priori assessments of Efficient Units

- Some DMUs may be regarded, based on *a priori* knowledge, as eminently efficient or notoriously inefficient. While one might argue about the validity of such *a priori* judgments, frequently they must be recognized.
- To do so, additional conditions may be imposed upon the choice of weights. For example, the condition $mY_j/nX_j <= 1$ may be replaced by equality for a given DMU which is regarded as eminently efficient.

Substitutability of Variables

- A still unresolved issue is the means for representing substitutability of variables. For example, two input variables may represent two different type of labor which may be, to some extent, substitutable for each other.
- How is such substitutability to be incorporated?
- Let's explore this issue a bit further since, by doing so, we can illuminate some additional perspectives on the basic DEA model.

Substitutability of Variables

- For simplicity in description, consider two input variables and a single output variable that has the same value for all DMUs. The graphic representation of the envelopment surface can now best be presented not in terms of the relationship between output and input, as previously shown, but between the variables of input.
- The two variables are "Professional Staff" and "Non-Professional Staff". The assumption is that they are completely substitutable and that physicians differ only in their "styles" of providing service, represented by the mix of the two means for doing so.
- The "efficient" DMUs are located on the red envelopment surface, which shows the minimums in use of variables.

Strengths & Weaknesses

- Strengths
  - DEA can handle multiple inputs and multiple outputs
  - DEA doesn't require relating inputs to outputs.
  - Comparisons are directly against peers
  - Inputs and outputs can have very different units
- Weaknesses
  - Measurement error can cause significant problems
  - DEA does not measure"absolute" efficiency
  - Statistical tests are not applicable
  - Large problems can be computationally intensive

R PROJECT

- Language and computational environment to make statistical analyzes and data mining.
- https://www.r-project.org/
- It's free and open source.
- Provides a variety of functions for statistical analysis (linear and nonlinear regression, statistical tests, time series analysis temporal, multivariate statistics, design of experiments, etc.).
- Provides functions for the development of various types of graphs, useful in exploratory data analysis and data visualization.
- It is highly extensible.
- Rapid diffusion (2 million users worldwide).

DEA Packages

- Packages dedicated to DEA models:
- FEAR (Frontier Efficiency Analysis with R)

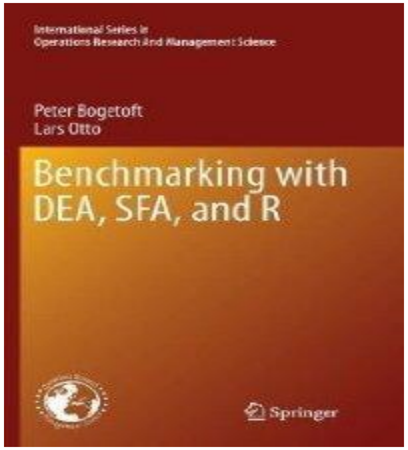
http://www.clemson.edu/economics/faculty/wilson/Software/FEAR/fear.html

- Benchmarking (https://cran.r-project.org/web/packages/Benchmarking/Benchmarking.pdf)
- Frontiles (Partial Efficiency Analysis)-https://cran.r-project.org/web/packages/frontiles/index.html
- Nonparaeff (Non-parametric Frontier Analysis)-https://cran.r-project.org/web/packages/nonparaeff/nonparaeff.pdf
- rDEA https://cran.r-project.org/web/packages/rDEA/index.html
- R operates like a big Library

References

- Use the books-libraries

1. FEAR (http://www.clemson.edu/economics/faculty/wilson/Software/FEAR/fear.html)
2. Benchmarking (BOGETOFT & OTTO, 2011)



Matrices in R

- matr1<-rbind(c(1,2,-1),c(-3,1,5))

matr1

- [,1] [,2] [,3]
- [1,] 1 2 -1
- [2,] -3 1 5

matr2<-cbind(c(1,2,-1),c(-3,1,5))

matr2

- [,1] [,2]
- [1,] 1 -3
- [2,] 2 1
- [3,] -1 5

matr3<-cbind(matr1,matr2)

- Error in cbind(matr1, matr2) :
- number of rows of matrices must match (see arg 2)

matr4<matrix(1:28,nrow=7,ncol=4)

> matr4

- [,1] [,2] [,3] [,4]
- [1,] 1 8 15 22
- [2,] 2 9 16 23
- [3,] 3 10 17 24
- [4,] 4 11 18 25
- [5,] 5 12 19 26
- [6,] 6 13 20 27
- [7,] 7 14 21 28
- >

Reading Data

Upload the package xlsx in R.

- require(xlsx)
- setwd("c:/example")
- data <- read.xlsx("c:/example/RegItal2011.xls", 1)
- data <- read.xlsx("c:/example/RegItal2011.xlsx", 1)
- outputs <- data.frame(data[2])
- inputs<-data.frame(data[c(3,4)])
- N <- dim(data)[1]
- s <- dim(inputs)[2]
- m <- dim(outputs)[2]

Data and Plots

- **Define Dataset and Variables**

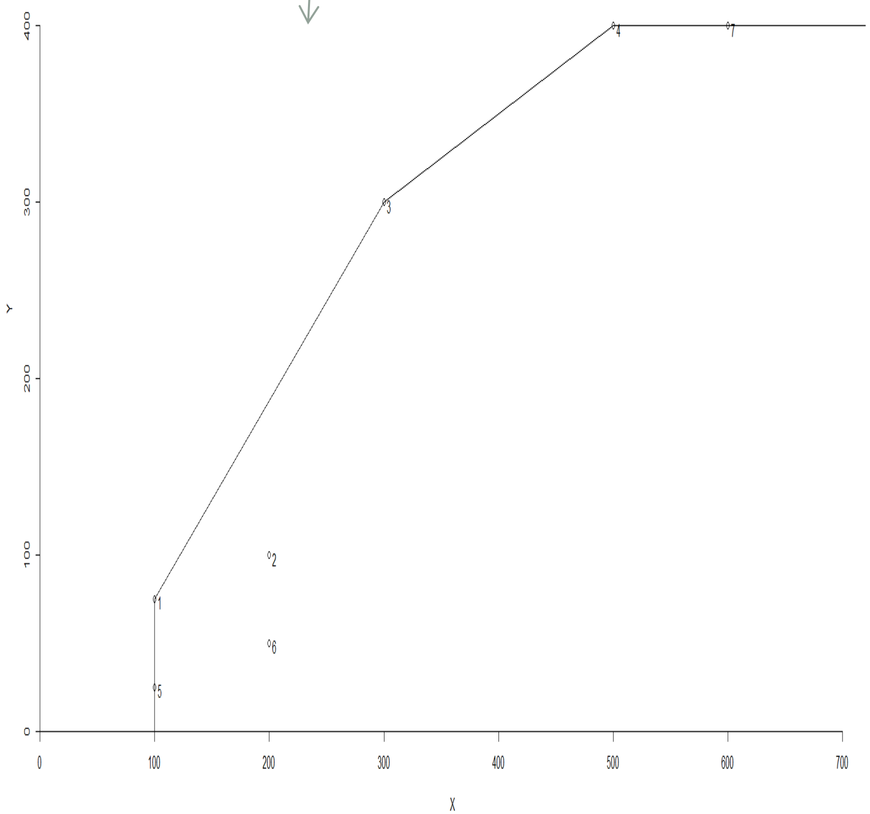
```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c(75,100,300,400,25,50,400),ncol=1)
 or  data(charnes1981-name of file)
x <- with(charnes1981, cbind(x1,x2,x3,x4,x5))
y <- with(charnes1981, cbind(y1,y2,y3))
```

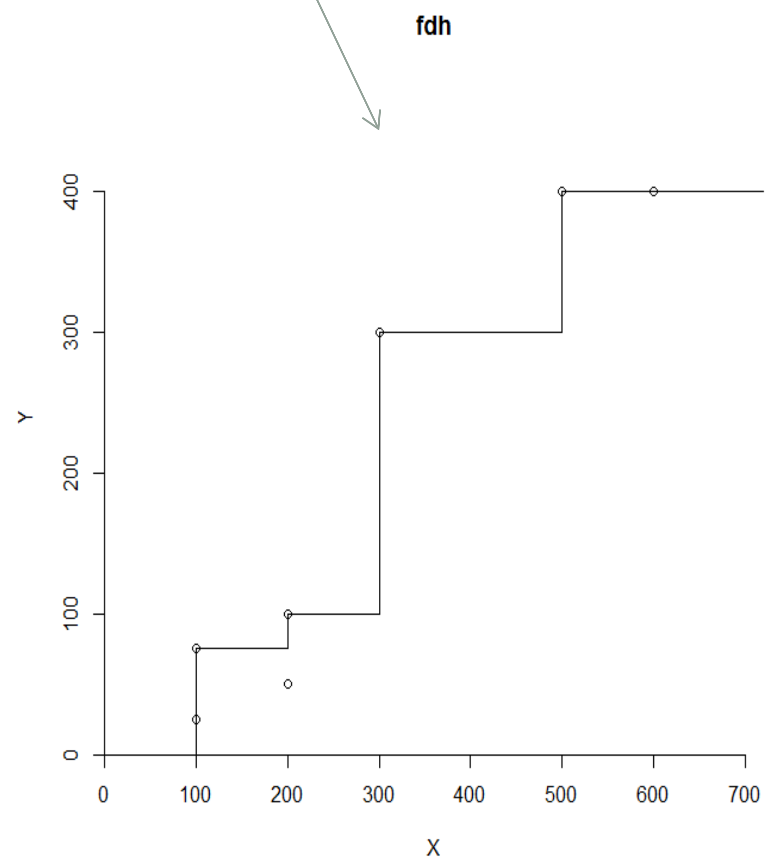
- **Plot**

```
dea.plot.frontier(x,y,txt=TRUE)
dea.plot(x,y,RTS="vrs",ORIENTATION="in-out",txt=rownames(x))
dea.plot(x,y,RTS="drs",ORIENTATION="in-out",add=TRUE,lty="dashed",lwd=2)
dea.plot(x,y,RTS="crs",ORIENTATION="in-out",add=TRUE,lty="dotted")
dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=rownames(x),main="fdh")
dea.plot(x,y,RTS="irs",ORIENTATION="in-out",txt=TRUE,main="irs")
dea.plot(x,y,RTS="irs2",ORIENTATION="in-out",txt=rownames(x),main="irs2")
dea.plot(x,y,RTS="add",ORIENTATION="in-out",txt=rownames(x),main="add")
```

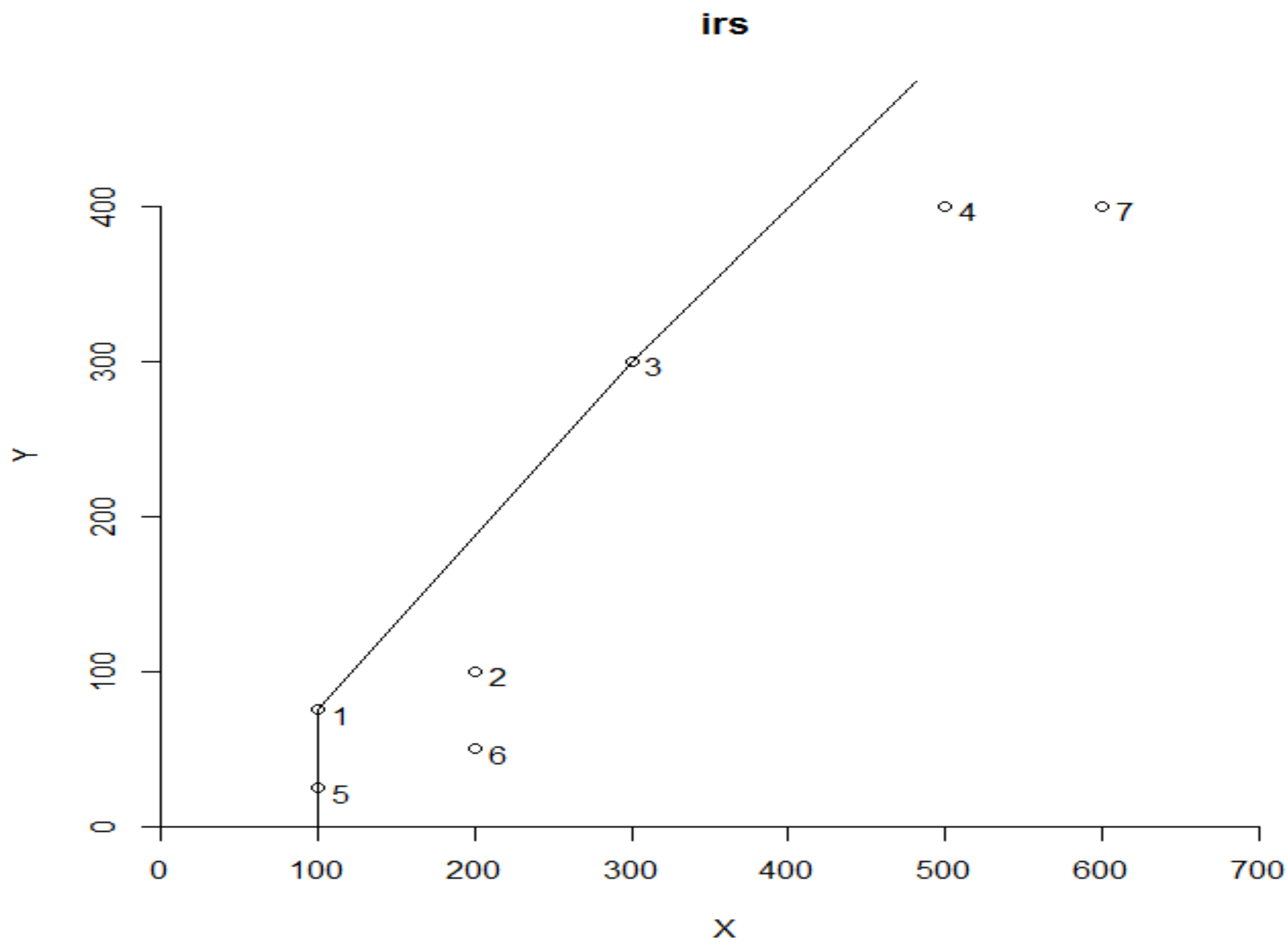
```
dea.plot.frontier(x,y,txt=TRUE)
```

```
dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=rownames(x),main="fdh")
```





```
dea.plot(x,y,RTS="irs",ORIENTATION="in-out",txt=TRUE,main="irs")
```



Calculate efficiency

- dea(x,y, RTS="vrs", ORIENTATION="in")
- e <- dea(x,y)
- eff(e)
- print(e)
- summary(e)
- lambda(e)
- peers(e)
- **Input savings potential for each firm**
- (1-eff(e)) * x
- (1-e$eff) * x

Slacks and Super Efficiency

- calculate slacks

```
el <- dea(x,y,SLACK=TRUE)
data.frame(e$eff,el$eff,el$slack,el$sx,el$sy)
```

- Fully efficient units, eff==1 and no slack

```
which(eff(e) == 1 & !el$slack)
```

- Calculating super efficiency

```
esuper <- sdea(x,y, RTS="vrs", ORIENTATION="in")
esuper
print(peers(esuper,NAMES=TRUE),quote=FALSE)
```

Program DEAP

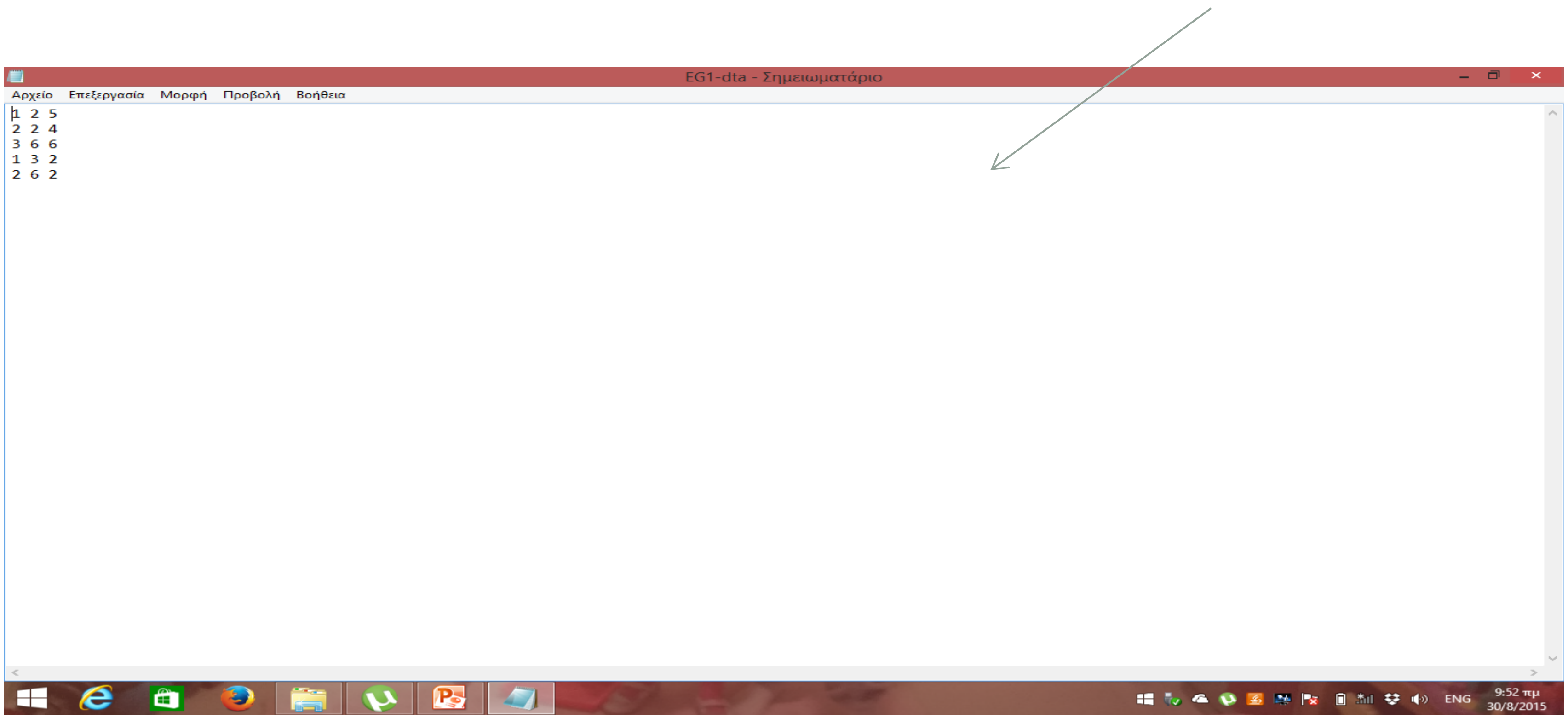
- Download and install the DEAP program from the abovementioned site.
- Please also read the instructions from the pdf file.
- Follow the instructions presented in the presentation in order to have your first results.

DEAP Program

- A computer program which has been written to conduct Data Envelopment Analysis for calculating efficiencies.
- It's free and open source.
- Provide a variety of DEA specifications (CRS,VRS e.t.c)
- It has been used in order to calculate malmquist productivity index.
- It is easy to implement.
- http://www.uq.edu.au/economics/cepa/deap.php

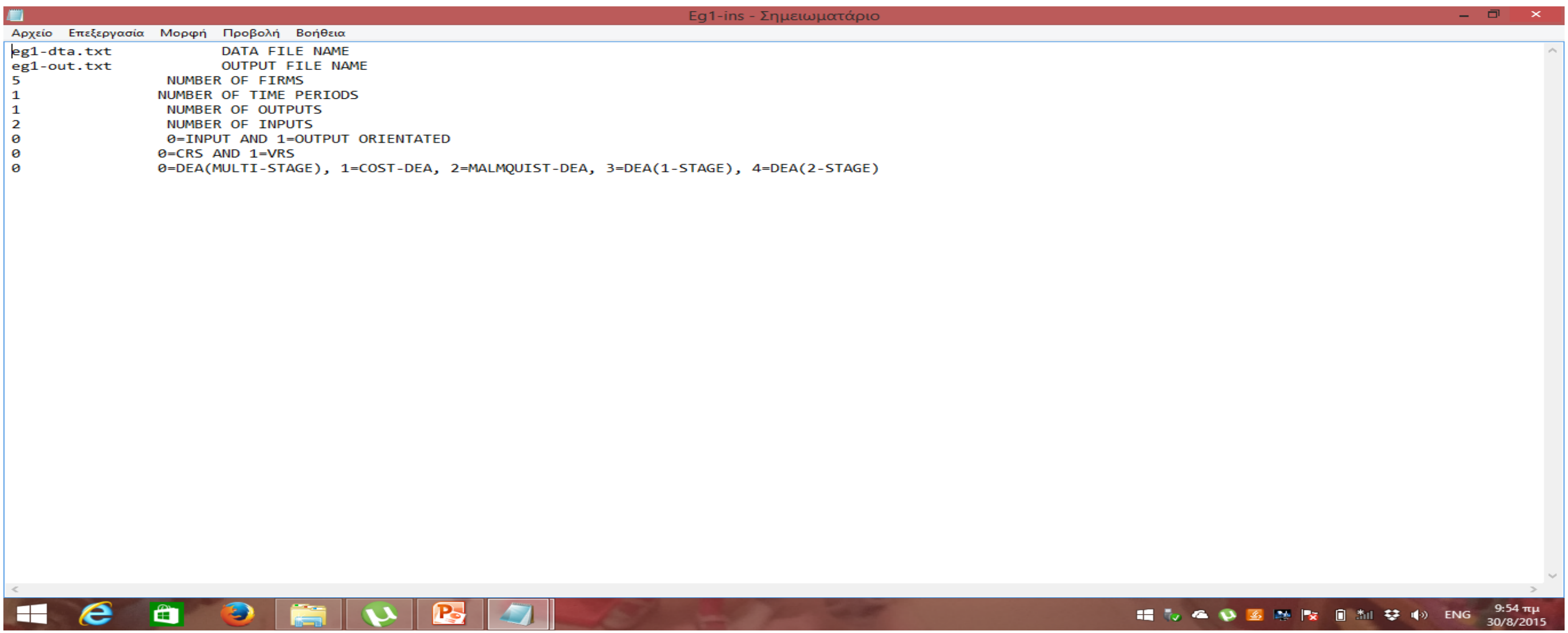
DEAP I

- Calculation of your results.
- First you have to create your file with data from the excel and you definitely have to save it as *.dta (Tab Delimited) in the DEAP program file. Must have the following structure.



DEAP II

- The second step demand the creation of the *ins file.
- In this *.ins file you must define the following:




```
eg1-dta.txt            DATA FILE NAME
eg1-out.txt            OUTPUT FILE NAME
5                 NUMBER OF FIRMS
1                NUMBER OF TIME PERIODS
1                 NUMBER OF OUTPUTS
2                 NUMBER OF INPUTS
0                 0=INPUT AND 1=OUTPUT ORIENTATED
0                0=CRS AND 1=VRS
0                0=DEA(MULTI-STAGE), 1=COST-DEA, 2=MALMQUIST-DEA, 3=DEA(1-STAGE), 4=DEA(2-STAGE)
```

DEAP III

You have to specify

1. The number of participated firms
2. The number of time periods
3. The number of inputs
4. The number of outputs
5. The CRS,VRS
6. The type of orientation
7. Which DEA model you want to be estimated

DEAP IV

- In order to have your first results type in the DOS prompt "DEA" and then your instruction file name.
- The program will take few minutes for its calculations to run the corresponding LP problem.
- A new file with the name *.OUT is going to be produced having the appropriate results.

Case study

The file countries2009.xlsx contains productive characteristics for 104 countries for 2009. More specifically, Labor Capital (estimated using the PIM) and Gross Domestic Product (as an output) has be represented. Please using the two open source software estimate the corresponding efficiencies and write a short report presenting your results. Also provide different estimations regarding the CO2 emissions participation as input and output.

Please report any differences.

Deadline 4/11/2020.

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