Exercise WS11.1

1. Draw sketch graphs of $y = 10^x$ and $x = \log_{10} y$ for values of x from -3 to $+3$. Show algebraically how one graph may be derived from the other, using the definition of a logarithm. Use the graphs to explain why (i) only positive numbers can have logarithms, and (ii) why the logarithms of numbers between 0 and 1 are negative.

   (Reminder: By convention, "$\log y$" means $\log_{10} y$.)

2. (a) Sketch the graphs of each of the following functions.

   (b) In each case, write down the inverse function.

   (i) $y = (25)10^x$

   (ii) $y = 10^{-0.5x}$

   (iii) $y = -\log 0.5x$

3. Using your graphs drawn in questions 1 and 2 above, estimate the values of the following. Then check your estimates using a calculator.

   (a) $10^{2.5}$

   (b) $10^{0.5}$

   (c) $\log 300$

   (d) $\log 15$

Exercise WS11.2

1. Attempt the following without using a calculator. Then use a calculator to check your answers.

   (a) Given $\log 100 = 2$ and $\log 3 = 0.4771$, what is $\log 300$?

   (b) Given $\log\left(\frac{100}{x}\right) = -1$, what is x?

(c) Given $\log 3 = 0.4771$, what is (i) $\log 0.3$, and (ii) $\log 30$?

2. Solve the equations:

(a) $105^{0.5x} = 200$

(b) $200 = 100(1+r)^{20}$

(c) $50^2 = 25^x$

3. The levels of traffic of various types on UK roads (defined as total distance travelled, in billions of kilometres, by all vehicles in the chosen type) was as follows:

Vehicle type	1980	2003
All vehicles	277	495
Light vans	26	58
Heavy goods vehicles	20	28
Buses and coaches	3.5	5.4
Source: www.dft.gov.uk		

(a) Calculate the average annual growth rate for each type of vehicle.

(b) Between 1990 and 2003 the increase in traffic for all vehicles was 19%. If this growth rate continues, (i) calculate what the level of traffic will be in 2015; (ii) after how many years will traffic have increased to 50% above its 2003 level?