Εργαστήριο 5 - Απαντήσεις

In[1]:=
$$f[x\_] := \frac{2}{3} x^{3/2}$$

In[2]:=
$$\int_1^2 \sqrt{1 + D[f[x], x]^2}\, dx$$

Out[2]=
$$-\frac{4\sqrt{2}}{3} + 2\sqrt{3}$$

In[3]:=
$$g[x\_] := Log[Sin[x]]$$

In[4]:=
$$\int_{\pi/4}^{\pi/2} \sqrt{1 + D[g[x], x]^2}\, dx$$

Out[4]=
$$-Log\left[Tan\left[\frac{\pi}{8}\right]\right]$$

In[5]:=
$$Limit\left[\frac{3x}{x}, x \to 0\right]$$

Out[5]=
$$3$$

In[6]:=
$$Limit\left[\frac{3x}{x}, x \to 0, Direction \to 1\right]$$

Out[6]=
$$3$$

In[7]:=
$$Limit\left[\frac{Abs[x]}{x}, x \to 0\right]$$

Out[7]=
$$1$$

In[8]:=
$$Limit\left[\frac{Abs[x]}{x}, x \to 0, Direction \to 1\right]$$

Out[8]=
$$-1$$

In[9]:=
$$Limit\left[\frac{Cos[x]^{Sin[x]} - 1}{x!}, x \to \infty\right]$$

Out[9]=
$$Limit\left[\frac{-1 + Cos[x]^{Sin[x]}}{x!}, x \to \infty\right]$$

In[10]:=
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<< "NumericalCalculus`"
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In[11]:=
$$NLimit\left[\frac{Cos[x]^{Sin[x]} - 1}{x!}, x \to \infty\right]$$

Out[11]=
$$0. + 0.\,i$$

In[12]:=
$$f[x\_] := x^3\, e^{-2x}$$

In[13]:=
$$f'[x]$$

Out[13]=
$$3\, e^{-2x} x^2 - 2\, e^{-2x} x^3$$

In[14]:=
$$g[x\_] := x * ArcTan[x]$$

In[15]:=
$$g'[x]$$

Out[15]=
$$\frac{x}{1 + x^2} + ArcTan[x]$$

In[16]:=
$$h[x\_] := (2x + 1)\left(3x^2 - 4x + 2\right)$$

In[17]:=
$$h''[x]$$

Out[17]=
$$6\,(1 + 2x) + 4\,(-4 + 6x)$$

In[18]:=
$$k[x\_] := \frac{\text{ArcSin}[x]}{x^2 - 1}$$

In[19]:=
$$\partial_{\{x,3\}} k[x]$$

Out[19]=
$$-\frac{6x^2}{(1-x^2)^{3/2}(-1+x^2)^2} + \frac{\frac{3x^2}{(1-x^2)^{5/2}} + \frac{1}{(1-x^2)^{3/2}}}{-1+x^2} + \frac{3\left(\frac{8x^2}{(-1+x^2)^3} - \frac{2}{(-1+x^2)^2}\right)}{\sqrt{1-x^2}} + \left(-\frac{48x^3}{(-1+x^2)^4} + \frac{24x}{(-1+x^2)^3}\right)\text{ArcSin}[x]$$

In[20]:=
$$\int e^{-2x}\,\text{Sin}[3x]\,dx$$

Out[20]=
$$-\frac{1}{13}e^{-2x}(3\,\text{Cos}[3x] + 2\,\text{Sin}[3x])$$

In[21]:=
$$\int y^3\,\text{Log}[y]^2\,dy$$

Out[21]=
$$\frac{y^4}{32} - \frac{1}{8}y^4\,\text{Log}[y] + \frac{1}{4}y^4\,\text{Log}[y]^2$$

In[22]:=
$$\int_0^{\sqrt[3]{\pi}} e^{-x^2}\,\text{Cos}\left[x^3\right]\,dx$$

Out[22]=
$$\int_0^{\pi^{1/3}} e^{-x^2}\,\text{Cos}\left[x^3\right]\,dx$$

In[23]:=
$$N[\%]$$

Out[23]=
$$0.701566$$

In[24]:=
$$s1 = \sqrt{a^2 - \left(x - \frac{a}{2}\right)^2};$$

In[25]:=
$$s2 = \frac{1}{2}\left(2\sqrt{4a^2 - x^2} - a\sqrt{3}\right);$$

In[26]:=
$$4\left(\int_a^{3a/2} s1\,dx + \int_0^a s2\,dx\right)$$

Out[26]=
$$4\left(\frac{1}{24}a\sqrt{a^2}\left(-3\sqrt{3} + 4\pi\right) + \frac{1}{6}a\left(-3\sqrt{3}\,a + \sqrt{a^2}\left(3\sqrt{3} + 2\pi\right)\right)\right)$$

In[27]:=
$$\text{Simplify}[\%]$$

Out[27]=
$$\frac{1}{2}a\left(-4\sqrt{3}\,a + \sqrt{a^2}\left(3\sqrt{3} + 4\pi\right)\right)$$

In[28]:=
$$\text{PowerExpand}[\%]$$

Out[28]=
$$\frac{1}{2}a\left(-4\sqrt{3}\,a + a\left(3\sqrt{3} + 4\pi\right)\right)$$

In[29]:=
$$\text{Simplify}[\%]$$

Out[29]=
$$-\frac{1}{2}a^2\left(\sqrt{3} - 4\pi\right)$$

In[30]:=
$$a = \text{Normal}[\text{Series}[\text{Log}[x], \{x, 1, 8\}]]$$

Out[30]=
$$-1 - \frac{1}{2}(-1+x)^2 + \frac{1}{3}(-1+x)^3 - \frac{1}{4}(-1+x)^4 + \frac{1}{5}(-1+x)^5 - \frac{1}{6}(-1+x)^6 + \frac{1}{7}(-1+x)^7 - \frac{1}{8}(-1+x)^8 + x$$

In[31]:=
$$b = \text{Normal}[\text{Series}[\text{ArcTan}[x], \{x, 0, 5\}]]$$

Out[31]=
$$x - \frac{x^3}{3} + \frac{x^5}{5}$$

In[32]:=
$$\text{Together}[\text{Simplify}[a + b]]$$

Out[32]=
$$\frac{1}{840}\left(-2283 + 7560x - 11760x^2 + 15400x^3 - 14700x^4 + 9576x^5 - 3920x^6 + 960x^7 - 105x^8\right)$$