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In[1]:= DuffingODE = x''[t] + x[t] + ε x[t]^3 == 0
Out[1]= x[t] + ε x[t]^3 + x''[t] == 0

In[2]:= DuffingIC = {x[0] == a, x'[0] == b}
Out[2]= {x[0] == a, x'[0] == b}

In[3]:= xse[t_] = x0[t] + x1[t] ε + x2[t] ε^2
Out[3]= x0[t] + ε x1[t] + ε^2 x2[t]

In[4]:= DuffingODEN = Collect[DuffingODE /. x -> xse // Expand, ε]
Out[4]= x0[t] + 3 ε^6 x1[t] x2[t]^2 + ε^7 x2[t]^3 + ε^3 (3 x0[t] x1[t]^2 + 3 x0[t]^2 x2[t]) +
ε^4 (x1[t]^3 + 6 x0[t] x1[t] x2[t]) + ε^5 (3 x1[t]^2 x2[t] + 3 x0[t] x2[t]^2) +
x0''[t] + ε (x0[t]^3 + x1[t] + x1''[t]) + ε^2 (3 x0[t]^2 x1[t] + x2[t] + x2''[t]) == 0

In[5]:= DuffingICN = Collect[DuffingIC /. x -> xse // Expand, ε]
Out[5]= {x0[0] + ε x1[0] + ε^2 x2[0] == a, x0'[0] + ε x1'[0] + ε^2 x2'[0] == b}

In[6]:= IVP0 = {Coefficient[DuffingODEN[[1]], ε, 0] == 0,
Coefficient[DuffingICN[[1, 1]], ε, 0] == a, Coefficient[DuffingICN[[2, 1]], ε, 0] == b}
Out[6]= {x0[t] + x0''[t] == 0, x0[0] == a, x0'[0] == b}

In[7]:= IVP1 = {Coefficient[DuffingODEN[[1]], ε, 1] == 0,
Coefficient[DuffingICN[[1, 1]], ε, 1] == 0, Coefficient[DuffingICN[[2, 1]], ε, 1] == 0}
Out[7]= {x0[t]^3 + x1[t] + x1''[t] == 0, x1[0] == 0, x1'[0] == 0}

In[8]:= IVP2 = {Coefficient[DuffingODEN[[1]], ε, 2] == 0,
Coefficient[DuffingICN[[1, 1]], ε, 2] == 0, Coefficient[DuffingICN[[2, 1]], ε, 2] == 0}
Out[8]= {3 x0[t]^2 x1[t] + x2[t] + x2''[t] == 0, x2[0] == 0, x2'[0] == 0}

In[9]:= sol0 = DSolve[IVP0, x0[t], t]
Out[9]= {{x0[t] -> a Cos[t] + b Sin[t]}}

In[10]:= x0p[t_] = A Cos[t + B]
Out[10]= A Cos[B + t]

In[11]:= sol1 = DSolve[IVP1 /. x0 -> x0p, x1[t], t] // TrigReduce
Out[11]= {{x1[t] -> 1/32 (6 A^3 Cos[B - t] + A^3 Cos[3 B - t] -
6 A^3 Cos[B + t] - 2 A^3 Cos[3 B + t] + A^3 Cos[3 B + 3 t] - 12 A^3 t Sin[B + t])}}

In[12]:= x1p[t_] = sol1[[1, 1, 2]]
Out[12]= 1/32 (6 A^3 Cos[B - t] + A^3 Cos[3 B - t] -
6 A^3 Cos[B + t] - 2 A^3 Cos[3 B + t] + A^3 Cos[3 B + 3 t] - 12 A^3 t Sin[B + t])

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In[13]= sol2 = DSolve[IVP2 /. {x0 → x0p, x1 → x1p}, x2[t], t] // TrigReduce

$$\text{Out[13]= } \left\{ \left\{ x2[t] \rightarrow \frac{1}{1024} \left(3 A^5 \cos[B - 3 t] - 186 A^5 \cos[B - t] - 21 A^5 \cos[3 B - t] + \right. \right. \right.$$

$$2 A^5 \cos[5 B - t] + 165 A^5 \cos[B + t] - 72 A^5 t^2 \cos[B + t] + 60 A^5 \cos[3 B + t] +$$

$$3 A^5 \cos[5 B + t] + 18 A^5 \cos[B + 3 t] - 39 A^5 \cos[3 B + 3 t] - 6 A^5 \cos[5 B + 3 t] +$$

$$A^5 \cos[5 B + 5 t] + 120 A^5 t \sin[B - t] + 24 A^5 t \sin[3 B - t] + 276 A^5 t \sin[B + t] -$$

$$\left. \left. 24 A^5 t \sin[3 B + t] - 12 A^5 t \sin[5 B + t] - 36 A^5 t \sin[3 B + 3 t] \right) \right\} \right\}$$

In[14]= xstr[t_] = xse[t] /. {x0 → x0p, x1 → x1p, x2 → sol2[[1, 1, 2]]}

$$\text{Out[14]= } A \cos[B + t] + \frac{1}{32} \varepsilon \left(6 A^3 \cos[B - t] + A^3 \cos[3 B - t] - \right.$$

$$6 A^3 \cos[B + t] - 2 A^3 \cos[3 B + t] + A^3 \cos[3 B + 3 t] - 12 A^3 t \sin[B + t] \left. \right) +$$

$$\varepsilon^2 \frac{1}{1024} \left(3 A^5 \cos[B - 3 t] - 186 A^5 \cos[B - t] - 21 A^5 \cos[3 B - t] + 2 A^5 \cos[5 B - t] + \right.$$

$$165 A^5 \cos[B + t] - 72 A^5 t^2 \cos[B + t] + 60 A^5 \cos[3 B + t] + 3 A^5 \cos[5 B + t] +$$

$$18 A^5 \cos[B + 3 t] - 39 A^5 \cos[3 B + 3 t] - 6 A^5 \cos[5 B + 3 t] + A^5 \cos[5 B + 5 t] +$$

$$120 A^5 t \sin[B - t] + 24 A^5 t \sin[3 B - t] + 276 A^5 t \sin[B + t] -$$

$$\left. 24 A^5 t \sin[3 B + t] - 12 A^5 t \sin[5 B + t] - 36 A^5 t \sin[3 B + 3 t] \right) [t]$$