```
In[1]:= DuffingODE = x''[t] + x[t] + ε x[t]^3 == 0
Out[1]= x[t] + ε x[t]^3 + x''[t] == 0

In[2]:= DuffingIC = {x[0] == a, x'[0] == b}
Out[2]= {x[0] == a, x'[0] == b}

In[3]:= subs = {x[t] → x[τ], x''[t] → w^2 x''[τ]}
Out[3]= {x[t] → x[τ], x''[t] → w^2 x''[τ]}

In[4]:= DuffingNIC = {x[0] == a, w x'[0] == b}
Out[4]= {x[0] == a, w x'[0] == b}

In[5]:= DuffingNODE = DuffingODE /. subs
Out[5]= x[τ] + ε x[τ]^3 + w^2 x''[τ] == 0

In[6]:= xse[τ_] = x0[τ] + x1[τ] ε
Out[6]= x0[τ] + ε x1[τ]

In[7]:= DuffingNODEN = Collect[DuffingNODE /. {x → xse, w → 1 + w1 ε} // Expand, ε]
Out[7]= x0[τ] + ε^4 x1[τ]^3 + x0''[τ] + ε (x0[τ]^3 + x1[τ] + 2 w1 x0''[τ] + x1''[τ]) +
          ε^2 (3 x0[τ]^2 x1[τ] + w1^2 x0''[τ] + 2 w1 x1''[τ]) + ε^3 (3 x0[τ] x1[τ]^2 + w1^2 x1''[τ]) == 0

In[8]:= DuffingNICN = Collect[DuffingNIC /. {x → xse, w → 1 + w1 ε} // Expand, ε]
Out[8]= {x0[0] + ε x1[0] == a, x0'[0] + w1 ε^2 x1'[0] + ε (w1 x0'[0] + x1'[0]) == b}

In[9]:= IVP0 = {Coefficient[DuffingNODEN[[1]], ε, 0] == 0,
          Coefficient[DuffingNICN[[1, 1]], ε, 0] == a, Coefficient[DuffingNICN[[2, 1]], ε, 0] == b}
Out[9]= {x0[τ] + x0''[τ] == 0, x0[0] == a, x0'[0] == b}

In[10]:= IVP1 = {Coefficient[DuffingNODEN[[1]], ε, 1] == 0,
           Coefficient[DuffingNICN[[1, 1]], ε, 1] == 0, Coefficient[DuffingNICN[[2, 1]], ε, 1] == 0}
Out[10]= {x0[τ]^3 + x1[τ] + 2 w1 x0''[τ] + x1''[τ] == 0, x1[0] == 0, w1 x0'[0] + x1'[0] == 0}

In[11]:= sol0 = DSolve[IVP0, x0[τ], τ]
Out[11]= {{x0[τ] → a Cos[τ] + b Sin[τ]}}

In[12]:= x0p[τ_] = A Cos[τ + B]
Out[12]= A Cos[B + τ]

In[13]:= sol1 = DSolve[IVP1 /. x0 → x0p, x1[τ], τ] // TrigReduce
Out[13]= {{x1[τ] → 1/32 (6 A^3 Cos[B - τ] + A^3 Cos[3 B - τ] - 6 A^3 Cos[B + τ] -
            2 A^3 Cos[3 B + τ] + A^3 Cos[3 B + 3 τ] - 12 A^3 τ Sin[B + τ] + 32 A w1 τ Sin[B + τ])}}

In[14]:= x1p[τ_] = sol1[[1, 1, 2]]
Out[14]= 1/32 (6 A^3 Cos[B - τ] + A^3 Cos[3 B - τ] - 6 A^3 Cos[B + τ] -
           2 A^3 Cos[3 B + τ] + A^3 Cos[3 B + 3 τ] - 12 A^3 τ Sin[B + τ] + 32 A w1 τ Sin[B + τ])
```

In[15]:= **solw = Solve[-12 A^3 + 32 A w1 == 0, w1]**

Out[15]= $\left\{\left\{w1 \to \frac{3 A^2}{8}\right\}\right\}$

In[16]:= **x1pp[τ_] = x1p[τ] /. solw**

Out[16]= $\left\{\frac{1}{32}\left(6 A^3 \text{Cos}[B-\tau] + A^3 \text{Cos}[3 B-\tau] - 6 A^3 \text{Cos}[B+\tau] - 2 A^3 \text{Cos}[3 B+\tau] + A^3 \text{Cos}[3 B+3 \tau]\right)\right\}$

In[17]:= **xl[t_] = xse[τ] /. {x0 → x0p, x1 → x1pp, τ → (1 + 3 A^2/8 ε) t}**

$$\left\{\mathbf{x0} \to \mathbf{x0p},\ \mathbf{x1} \to \mathbf{x1pp},\ \tau \to \left(1 + \frac{3 A^2}{8}\varepsilon\right) t\right\}$$

Out[17]= $$\left\{A \text{Cos}\left[B + t\left(1 + \frac{3 A^2 \varepsilon}{8}\right)\right] + \frac{1}{32}\varepsilon\left(6 A^3 \text{Cos}\left[B - t\left(1 + \frac{3 A^2 \varepsilon}{8}\right)\right] + A^3 \text{Cos}\left[3 B - t\left(1 + \frac{3 A^2 \varepsilon}{8}\right)\right] - 6 A^3 \text{Cos}\left[B + t\left(1 + \frac{3 A^2 \varepsilon}{8}\right)\right] - 2 A^3 \text{Cos}\left[3 B + t\left(1 + \frac{3 A^2 \varepsilon}{8}\right)\right] + A^3 \text{Cos}\left[3 B + 3 t\left(1 + \frac{3 A^2 \varepsilon}{8}\right)\right]\right)\right\}$$