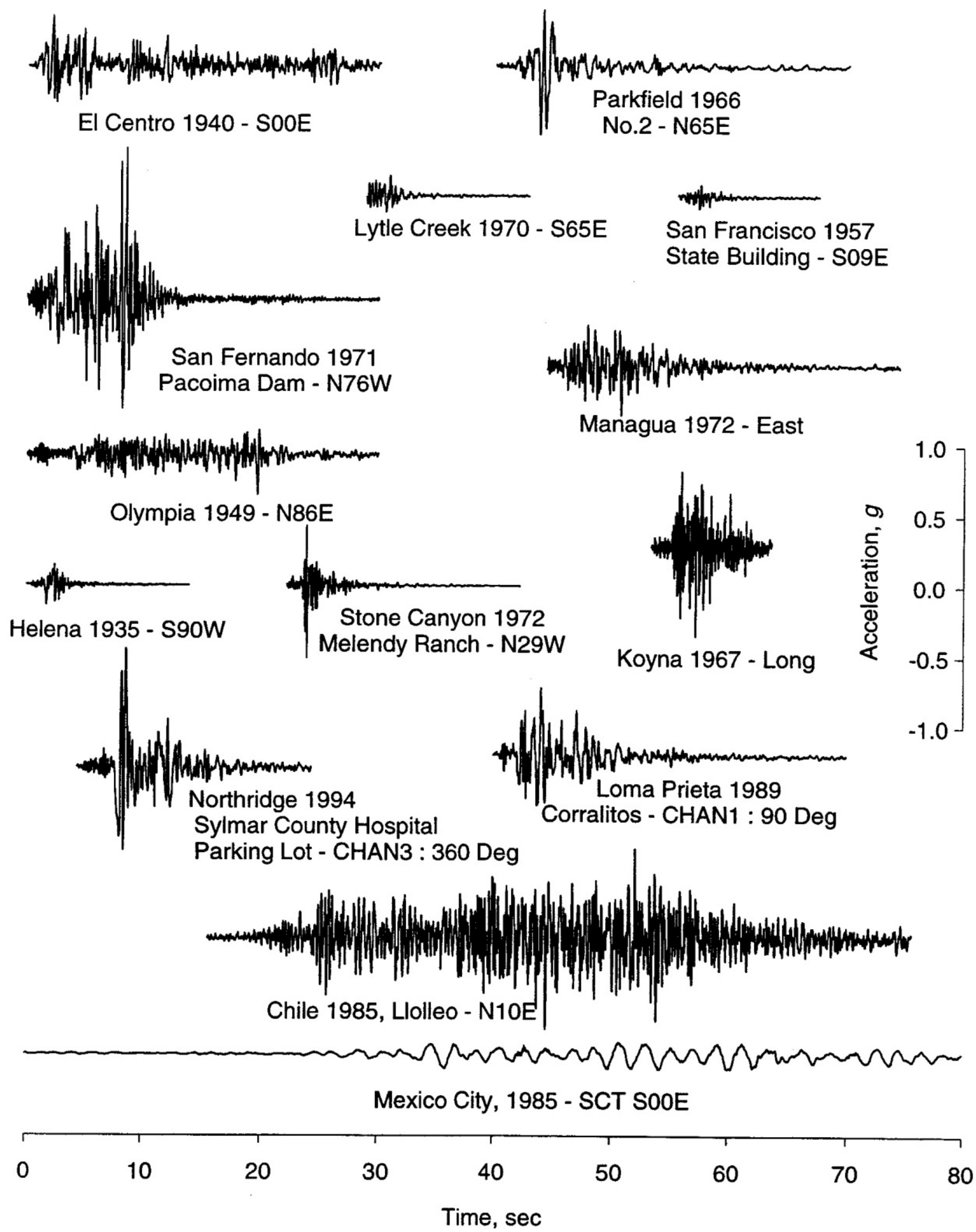
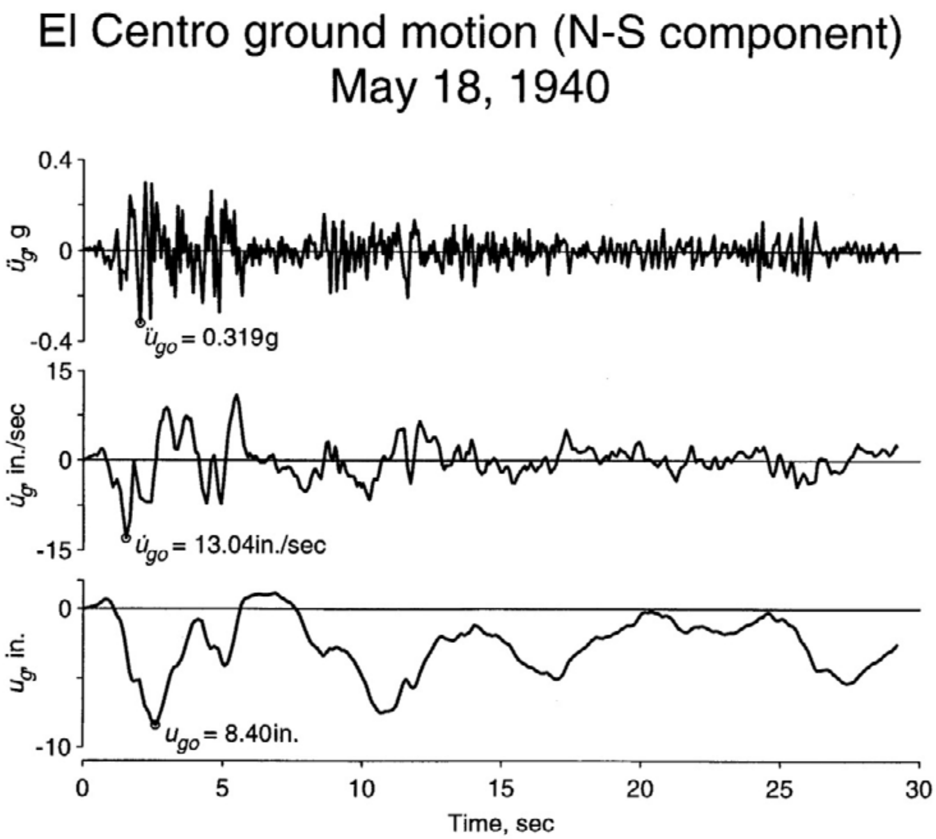
EARTHQUAKE RESPONSE OF LINEAR SYSTEMS

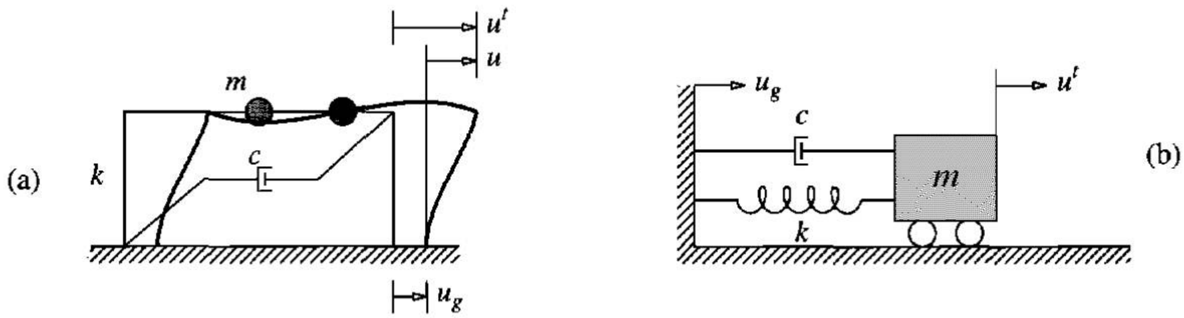
- Characteristics of Earthquake Response Spectra
- Design Spectra (**Ελαστικό Φάσμα Σχεδιασμού**)

Ground motions




El Centro ground motion (N-S component)
May 18, 1940
0.4
0
-0.4
$\ddot{u}_g$, g
$\ddot{u}_{go}$ = 0.319g
15
0
-15
$\dot{u}_g$, in./sec
$\dot{u}_{go}$ = 13.04in./sec
0
-10
u_g, in.
u_{go} = 8.40in.
0
5
10
15
20
25
30
Time, sec

EQUATION OF MOTION:



Single-degree-of-freedom systems

$$\left.\begin{aligned} m\ddot{u}^t + c\dot{u} + ku = 0 \\ u^t = u_g + u \end{aligned}\right\} \Longrightarrow \quad m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$$

$$\Longrightarrow \quad \boxed{\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_g}$$

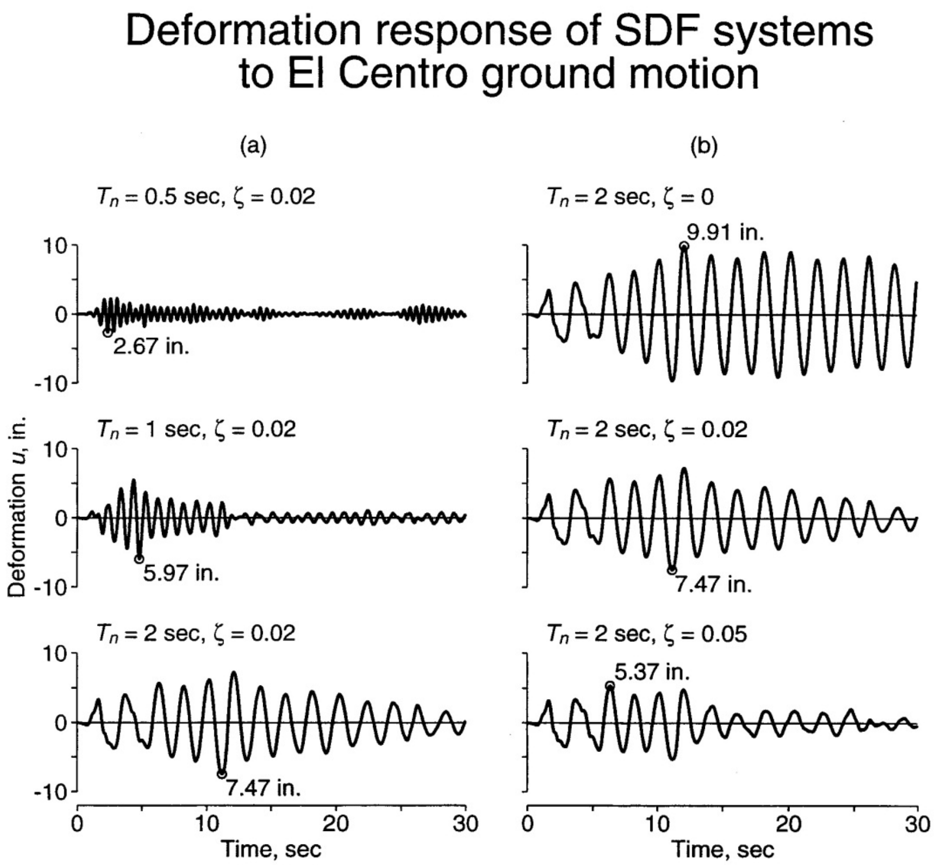
For a given $\ddot{u}_g(t)$, the **deformation** $u(t)$ **of the system depends only on the natural frequency** ω or **natural period** T of the system and **its damping ration** ξ, writing formally $u \overset{\text{def}}{=} u(t, T, \xi)$.

RESPONSE QUANTITIES:

$u(t)$: **deformation of the system, to which the internal forces are linearly related**

$u^t(t)$: **total displacement of mass**; useful in providing enough separation between adjacent buildings **to avoid pounding**

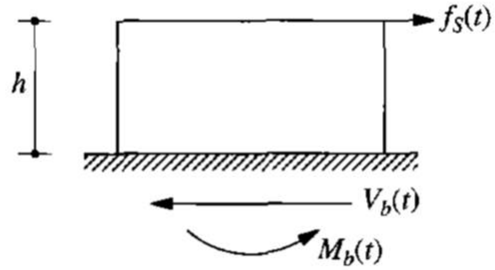
$\ddot{u}^t(t)$: **total acceleration**; useful if structure is supporting sensitive equipment



It is seen that **the time required for an SDOF system to complete a cycle of vibration** when subjected to earthquake ground motion **is very close to the natural period of the system**.

This interesting result, valid for 'typical' ground motions containing a wide range of frequencies (*i.e.*, having a ***broad-band*** **spectrum**), can be proven using ***Random Vibration Theory***.

RESPONSE HISTORY:



Equivalent Static Force $f_S(t)$

The internal forces can be determined by static analysis of the structure at each time instant.

Equivalent Static Force Method:

$$\begin{aligned} f_S(t) &= ku(t) \\ &= m\underbrace{\omega^2 u(t)}_{A(t):} = mA(t) \end{aligned}$$

$A(t)$: ***pseudo – acceleration***

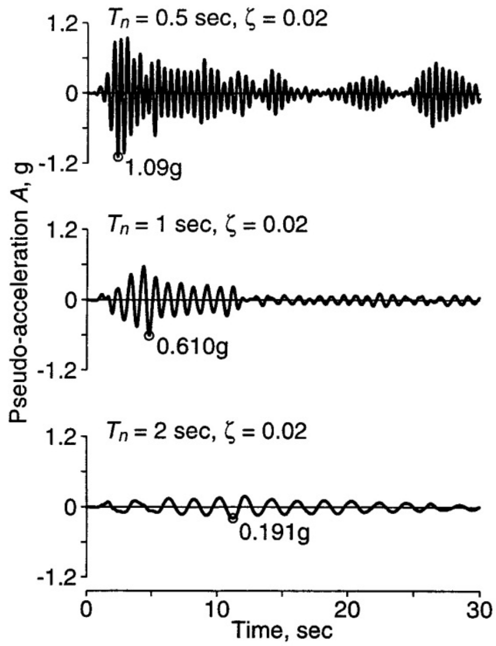
Base Shear $V_b(t)$:

$$\begin{aligned} V_b(t) &= f_S(t) \\ &= mA(t) \end{aligned}$$

Overturning Moment $M_b(t)$:

$$\begin{aligned} M_b(t) &= hf_S(t) \\ &= hV_b(t) \end{aligned}$$

Pseudo-acceleration response of SDF systems to El Centro ground motion



RESPONSE SPECTRUM CONCEPT:

A plot of the peak value of a response quantity as a function of the natural vibration period T of the system, or a related parameter such as circular frequency ω or cyclic frequency f, **is called the response spectrum of the quantity**.

A variety of response spectra can be defined depending on the response quantity that is plotted:

The Deformation Response Spectrum:
(**Φάσμα Απόκρισης Μετατόπισης**)

$$u_o(T,\xi) \quad \stackrel{\text{def}}{=} \quad \max_t |u(t,T,\xi)|$$

The Relative Velocity Response Spectrum:
(**Φάσμα Απόκρισης Σχετικής Ταχύτητας**)

$$\dot{u}_o(T,\xi) \quad \stackrel{\text{def}}{=} \quad \max_t |\dot{u}(t,T,\xi)|$$

The Acceleration Response Spectrum:
(**Φάσμα Απόκρισης Απολύτου Επιταχύνσεως**)

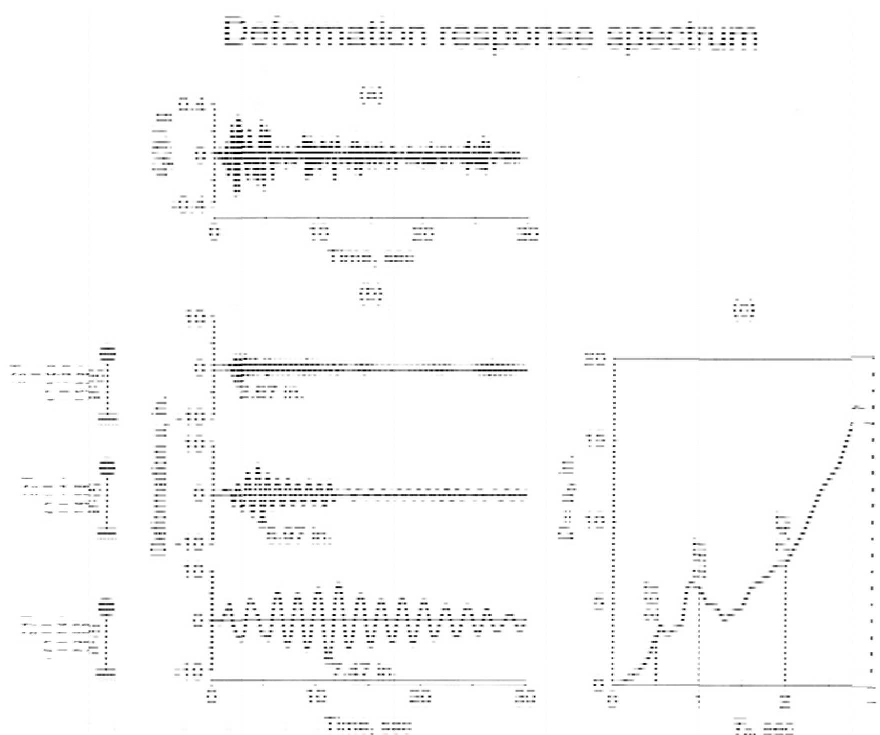
$$\ddot{u}_o^t(T,\xi) \quad \stackrel{\text{def}}{=} \quad \max_t |\ddot{u}^t(t,T,\xi)|$$

DEFORMATION, PSEUDO-VELOCITY, AND PSEUDO-ACCELERATION RESPONSE SPECTRA
(Φάσματα Απόκρισης Μετατοπίσεως, Ψευτοταχύτητος & Ψευτοεπιταχύνσεως)

The *Deformation Spectrum* provides all the information necessary to compute the **peak values of deformation** $D \stackrel{\text{def}}{=} u_o$, and **internal forces**.

The ***Pseudo-Velocity*** and ***Pseudo-Acceleration Response Spectra*** are included, however, because they are useful in:

- **studying characteristics of response spectra**
- **constructing design spectra**
- **relating structural dynamics results to building codes**



Usually, the peak occurs **during ground shaking**; however, for **lightly damped systems** with **very long periods** the peak response may occur **during the free vibration phase after the ground shaking has stopped**.

Pseudo-Velocity Response Spectrum:

$$\boxed{V = \omega D = \frac{2\pi}{T} D}$$

- V has the units of '***velocity***'
- V is **related to the <u>peak value of strain energy</u>** E_{S_o} **<u>stored in the system</u>** during the earthquake:

$$\boxed{E_{S_o} = \frac{1}{2} k u_o^2 = \frac{1}{2} k D^2 = \frac{1}{2} k \left(\frac{V}{\omega}\right)^2 = \frac{1}{2} m V^2}$$

Pseudo-Acceleration Response Spectrum:

$$\boxed{A = \omega^2 D = \left(\frac{2\pi}{T}\right)^2 D}$$

- A has the units of '***acceleration***'
- A is **related to the <u>peak value of base shear</u>** V_{bo} [or **<u>the peak value of the equivalent static force</u>** f_{So}]

$$\boxed{V_{bo} = f_{So} = mA \qquad [A \overset{\text{def}}{=} \max_t |A(t)|]}$$

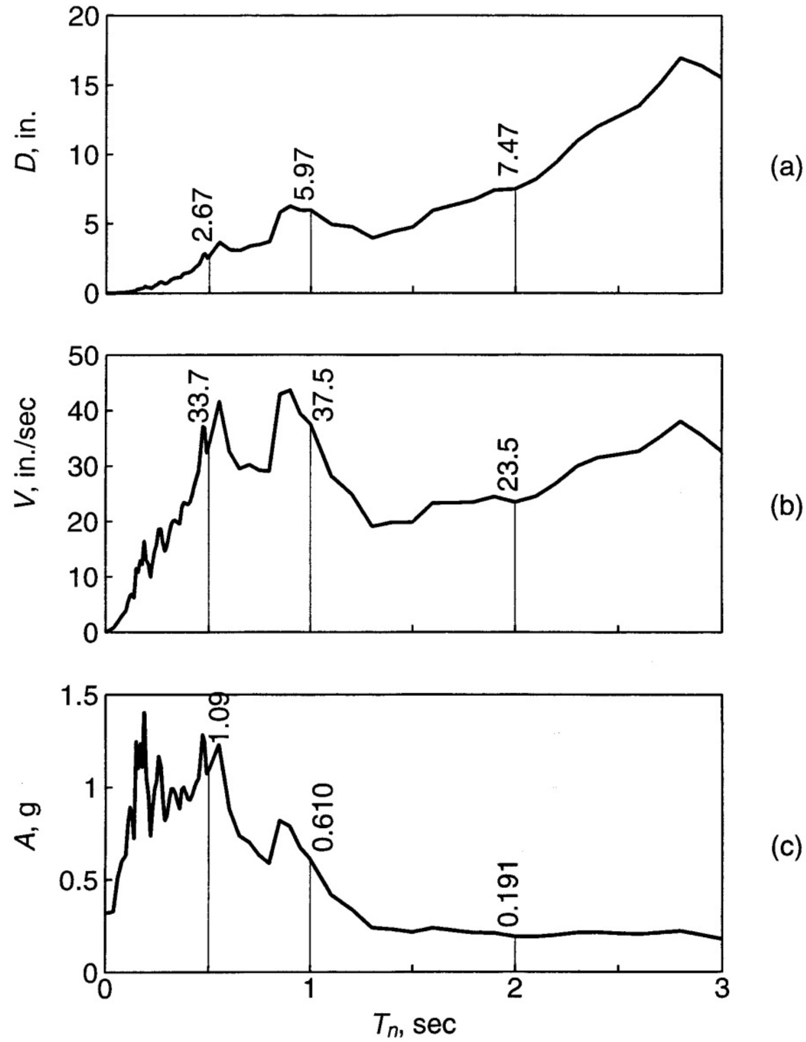
Also:

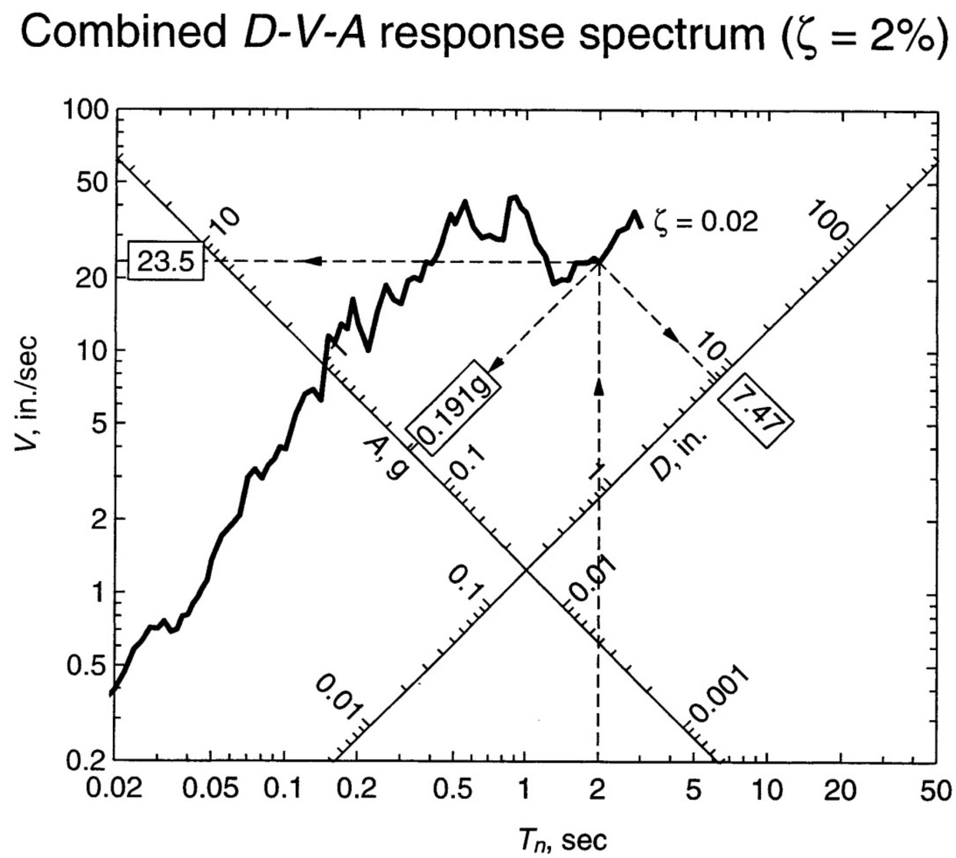
$$\boxed{V_{bo} = \frac{A}{g} W \qquad [W = weight\ of\ the\ structure]}$$

$$\boxed{\frac{A}{g} = \begin{cases} base - shear\ coefficient \\ or \\ lateral\ force\ coefficient \end{cases}}$$

[The ***base-shear coefficient*** (**συντελεστής τέμνουσας βάσης** ή **συντελεστής πλευρικής δύναμης**) is **used in building codes to represent <u>the coefficient by which the structural weight is multiplied to obtain the base shear</u>**.]

Deformation, pseudo-velocity, and pseudo-acceleration response spectra ($\zeta = 2\%$)



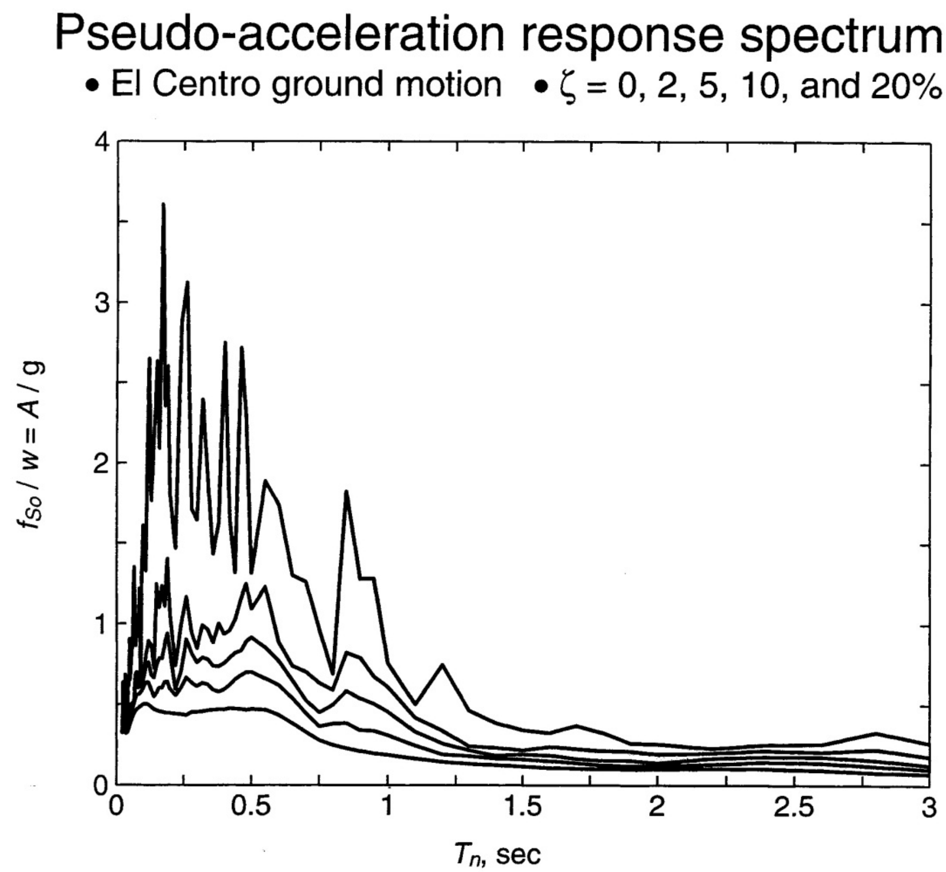


The integrated representation is possible because the three spectral quantities, $D - V - A$, are interrelated as follows:

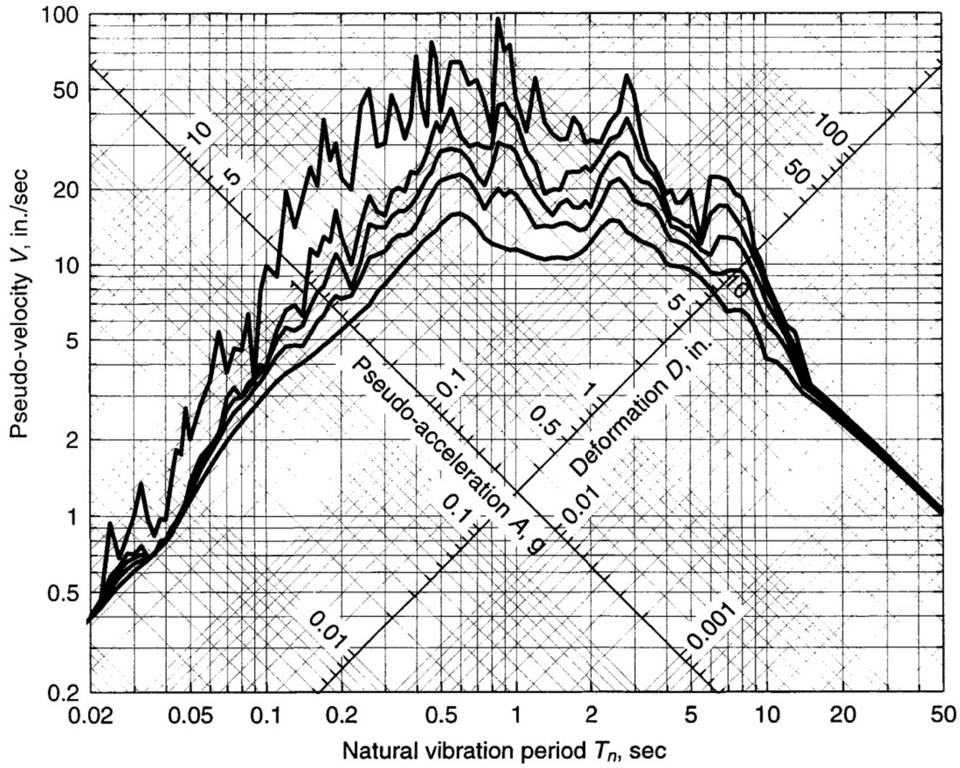
$$\frac{A}{\omega} = V = \omega D \quad or \quad \frac{T}{2\pi} A = V = \frac{2\pi}{T} D$$

This type of plot was **used** for **earthquake response spectra** for the first time **by A.S. Veletsos & N.M. Newmark in 1960**.

NOTE: The '***tri-partite log form***' ***graph paper*** was invented in the **late 1950s** by **Edward FISHER** of the **Westinghouse Research Laboratory** in **Pittsburgh** and published in the ***Shock and Vibration Handbook***.

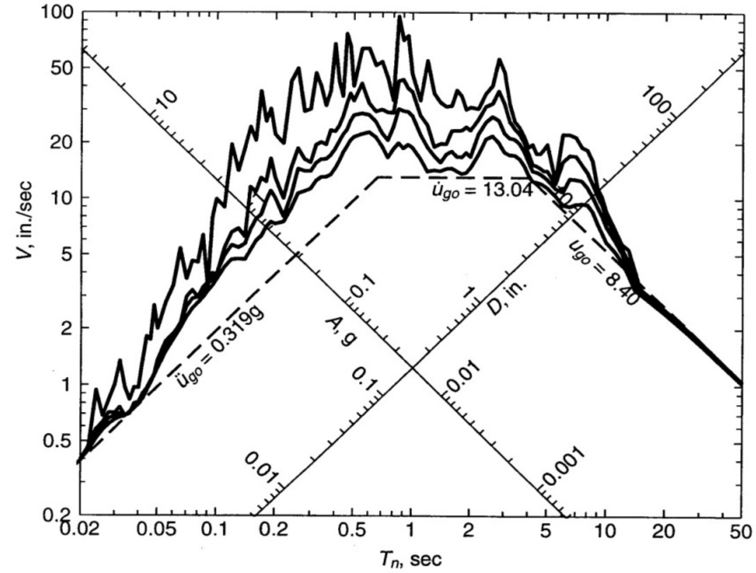

Pseudo-acceleration response spectrum
• El Centro ground motion • ζ = 0, 2, 5, 10, and 20%
fSo / w = A / g
Tn, sec
0
0.5
1
1.5
2
2.5
3
1
2
3
4

Response spectrum for El Centro ground motion
ζ = 0, 2, 5, 10, and 20%.

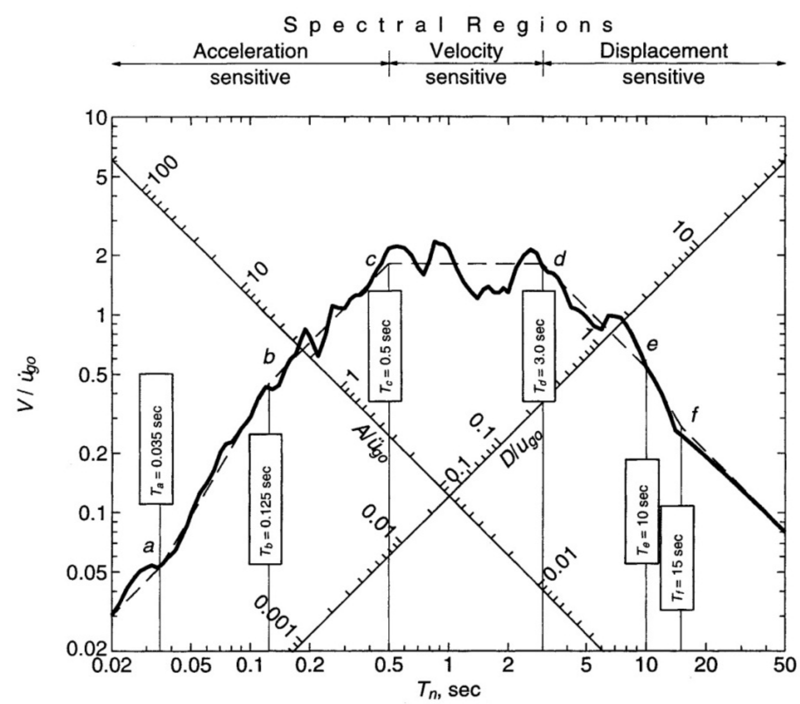


(**Συνδιασμένο Φάσμα *D-V-A***)

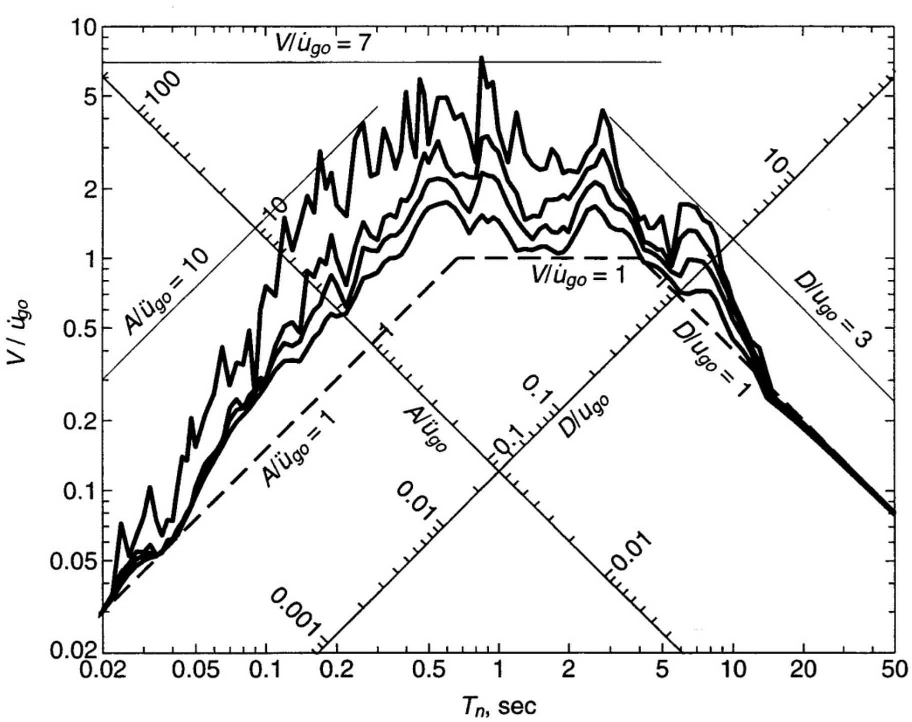
Response spectrum
u_{go}, $\dot{u}_{go}$, and $\ddot{u}_{go}$



Response spectrum and spectral regions



Response spectrum plotted on normalized scales



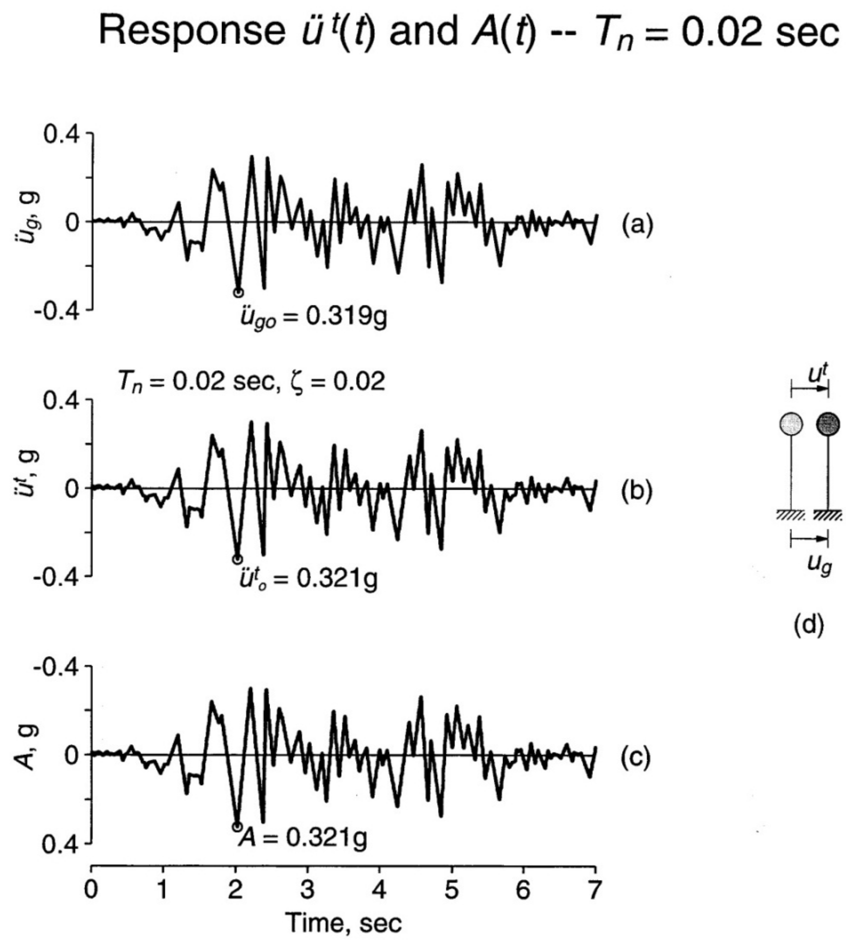
- For systems with **very short period** (*i.e.*, **very stiff systems**), $A \longrightarrow \ddot{u}_{go}$ and D = very small.

- For systems with **very long period** (*i.e.*, **very flexible systems**), $D \longrightarrow u_{go}$ and A = very small.

- For $T_b < T < T_c$, $\boldsymbol{A}$ may be idealized as constant at a value **equal to $\ddot{u}_{go}$ amplified by a factor depending on** ξ.

- For $T_d < T < T_e$, $\boldsymbol{D}$ may be idealized as constant at a value **equal to u_{go} amplified by a factor depending on** ξ.

- For $T_c < T < T_d$, $\boldsymbol{V}$ may be idealized as constant at a value **equal to $\dot{u}_{go}$ amplified by a factor depending on** ξ.



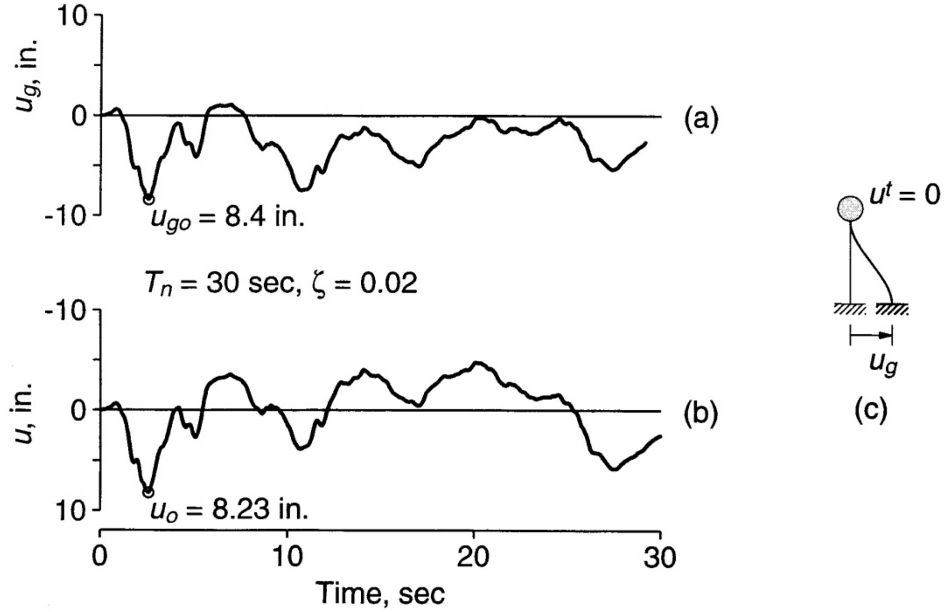
For systems with **very short period**, $\boldsymbol{A \longrightarrow \ddot{u}_{go}}$ and $\boldsymbol{D}$ = **very small**.

For a fixed mass, a **very short period system** is **extremely stiff** or essentially **rigid**. Such a system would be **expected to undergo very little deformation and its mass would move more rigidly with the ground**; its peak acceleration should be approximately equal to $\ddot{u}_{go}$.

Observe that: $\ddot{u}^t(t) \cong \ddot{u}_g(t)$ and $\ddot{u}^t_o \cong \ddot{u}_{go}$

For **lightly damped systems** ($i.e.$, $\xi \rightarrow 0$) $\ddot{u}^t(t) \cong -A(t)$ and $\ddot{u}^t_o \cong A$.
Therefore: $A \cong \ddot{u}_{go}$

Response $u(t)$ -- $T_n = 30$ sec, $\zeta = 2\%$



For systems with **very long period**, $D \longrightarrow u_{go}$ and A = **very small**.

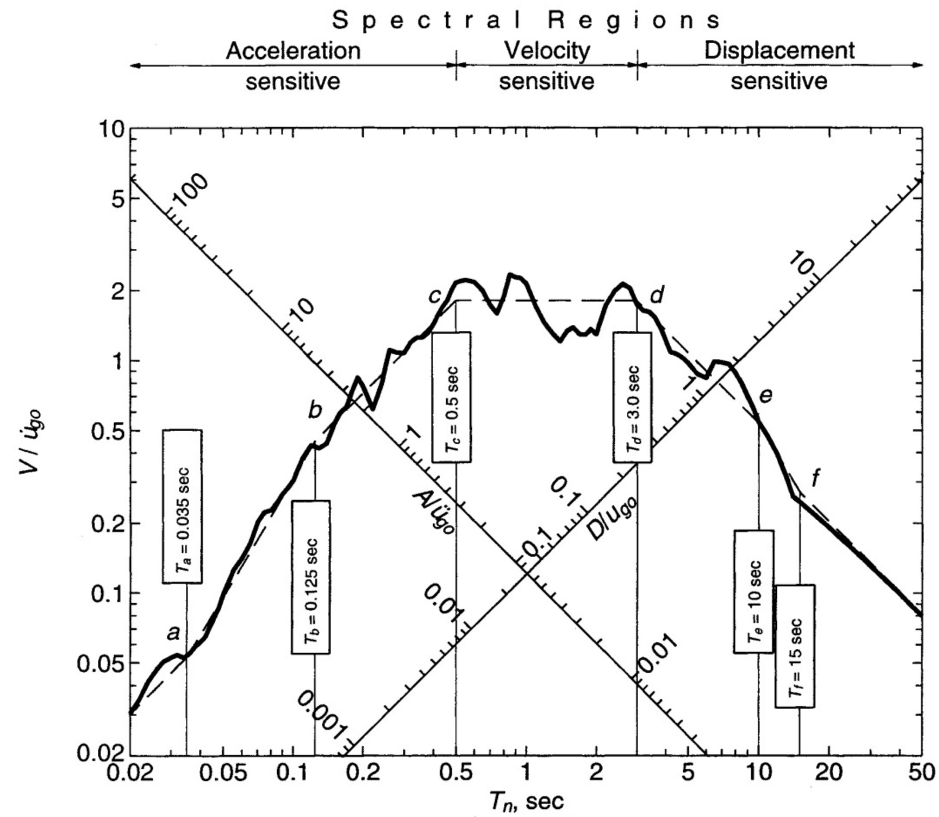
For a fixed mass, a **very long period system** is **extremely flexible**. **The mass would be expected to remain essentially stationary while the ground below moves.**

Thus $\ddot{u}^t(t) \cong 0$, implying that $A(t) \cong 0$ and $u(t) \cong -u_g(t)$, implying that $D \cong u_{go}$.

Observe that: $u_o \cong u_{go}$ and $u(t) \cong -u_g(t)$ but for rotation of the baseline.

[**NOTE**: The **discrepancy** between $u(t)$ & $u_g(t)$ arises, in part, from the **loss of the initial portion of the recorded motion prior to triggering** of the recording (**analog**) accelerograph.]

Response spectrum and spectral regions



Divide the spectrum into **three period ranges**:

$T_d < T$: ***displacement-sensitive region***

$T < T_c$: ***acceleration-sensitive region***

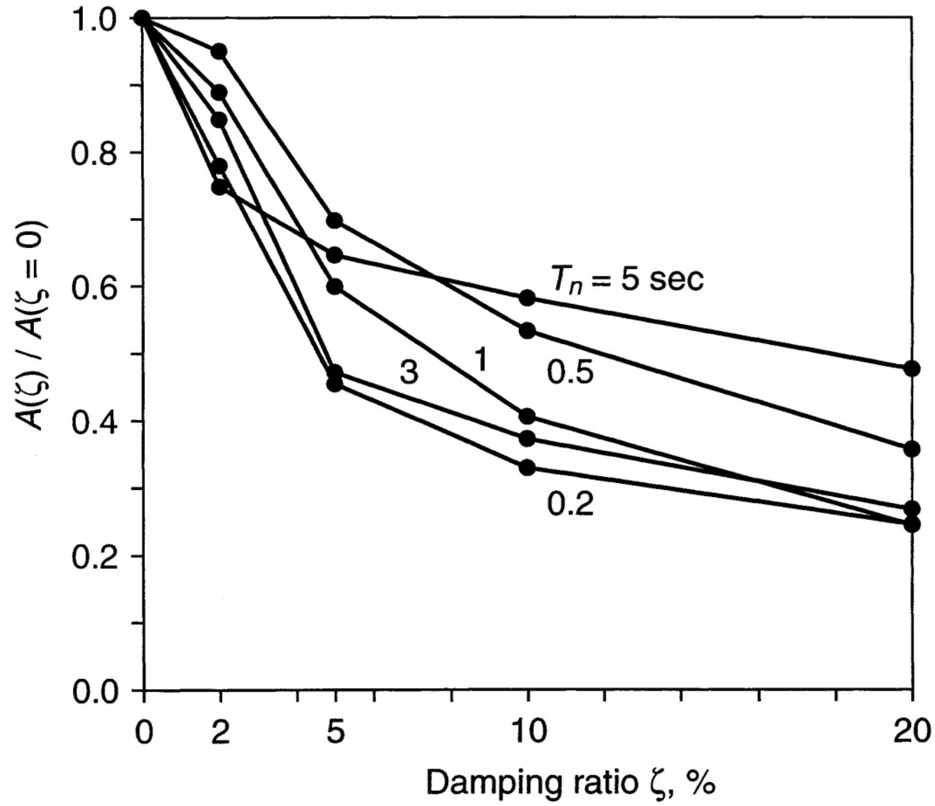
$T_c < T < T_d$: ***velocity-sensitive region***

T_a, T_b, T_e, T_f on idealized spectrum are **independent of damping**

T_c, T_d on idealized spectrum **vary with damping**

NOTE: **Idealizing the spectrum** by a series of straight lines $a - b - c - d - e - f$ in the four-way log plot is obviously **not a precise process**.

Variation of pseudo-acceleration with damping



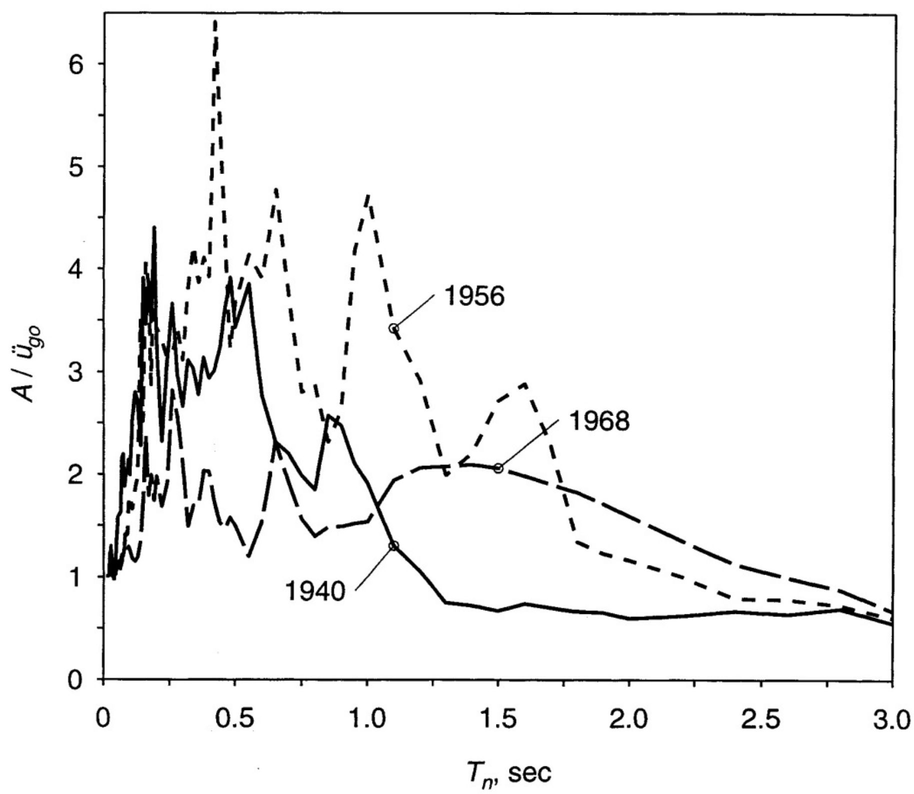
Damping reduces the response of a structure, as expected.

Among the three period regions defined earlier, the **effect of damping** tends to be **greatest in the velocity-sensitive region** of the spectrum.

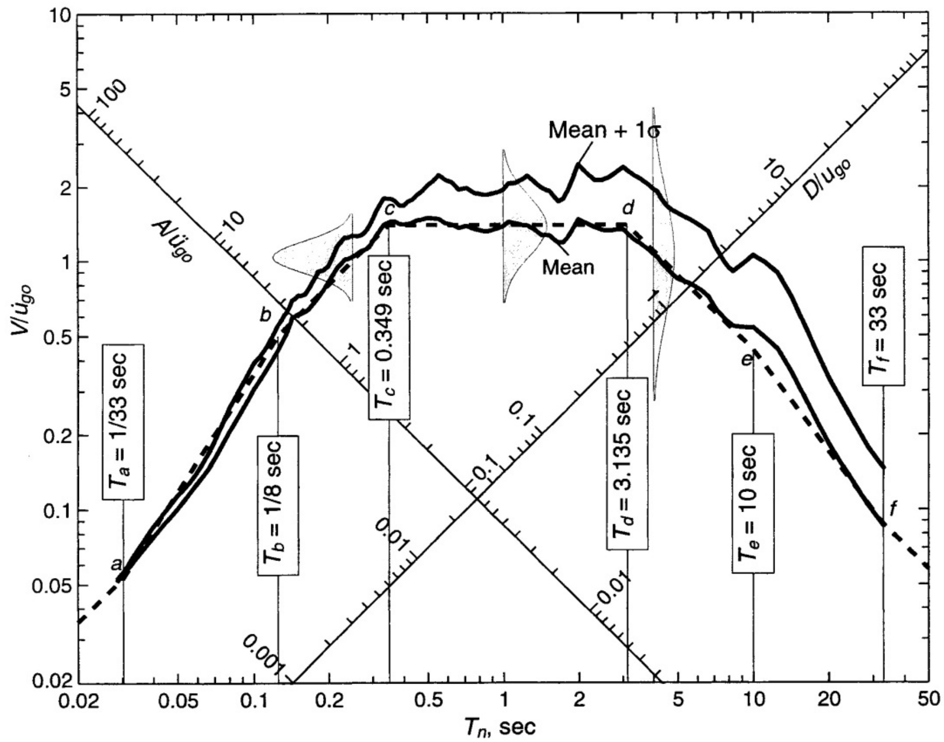
Observe that the **effect of damping** is **stronger for smaller damping values**.

The **effect of damping** in reducing the response **depends on** T of the system, but there is **no clear trend**. This is yet another indication of the complexity of structural response to earthquakes.

Response spectra for three ground motions at El Centro site



- Mean and mean + 1σ spectra
- Design spectrum
- Probability distributions for V



The **design spectrum** is **based on statistical analysis** of the response spectra for an '***appropriately selected***' **ensemble of ground motions**.

Each ground motion is **normalized** (*i.e.*, scaled up or down) so that all ground motions have the same peak ground acceleration, say $\ddot{u}_{go}$; other bases for normalization can be chosen.

[**NOTE**: The above normalization procedure is a highly controversial and, as recently demonstrated by **GRIGORIU (2011)**, wrong procedure if one is interested in assessing the reliability of a structure.]

GRIGORIU, M. (2011). 'To Scale or Not to Scale Seismic Ground-Acceleration Records', *Journal of Engineering Mechanics*, ASCE, Vol. 137, No. 4, 284-293.

The quantities u_{go}, $\dot{u}_{go}$ & $\ddot{u}_{go}$ in the normalized scales (D/u_{go}), $(V/\dot{u}_{go})$, & $(A/\ddot{u}_{go})$, are the **average values** of peak ground displacement, velocity and acceleration – averaged over the ensemble of motions. The **spectral ordinates** are ***lognormally distributed***.

Construction of elastic design spectrum

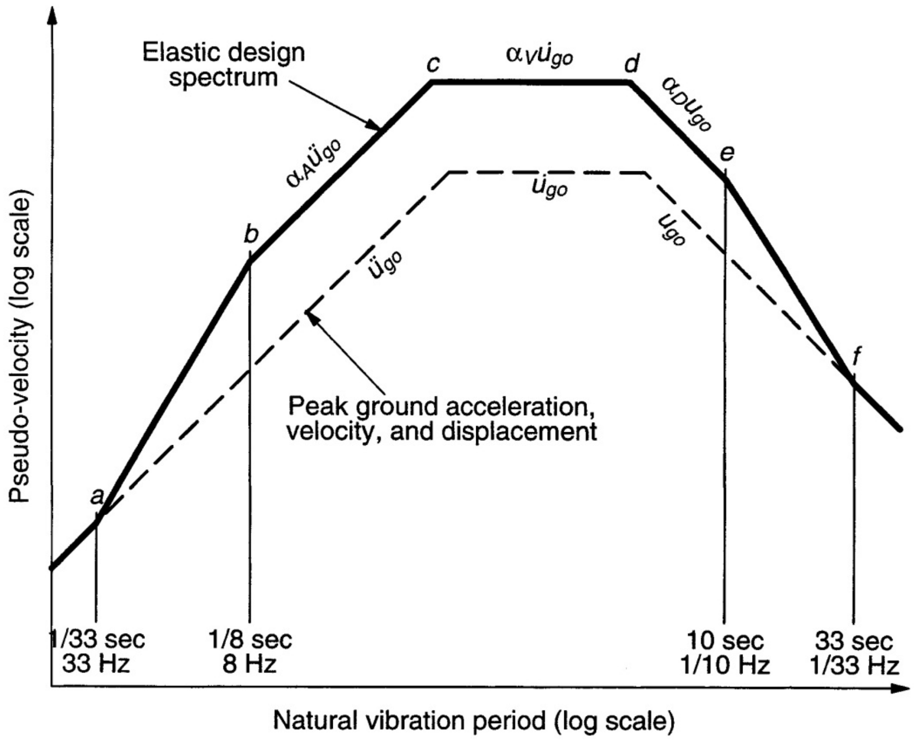


TABLE 6.9.1 AMPLIFICATION FACTORS: ELASTIC DESIGN SPECTRA

Damping, ζ (%)	Median (50 percentile) α_A	α_V	α_D	One Sigma (84.1 percentile) α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.59	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

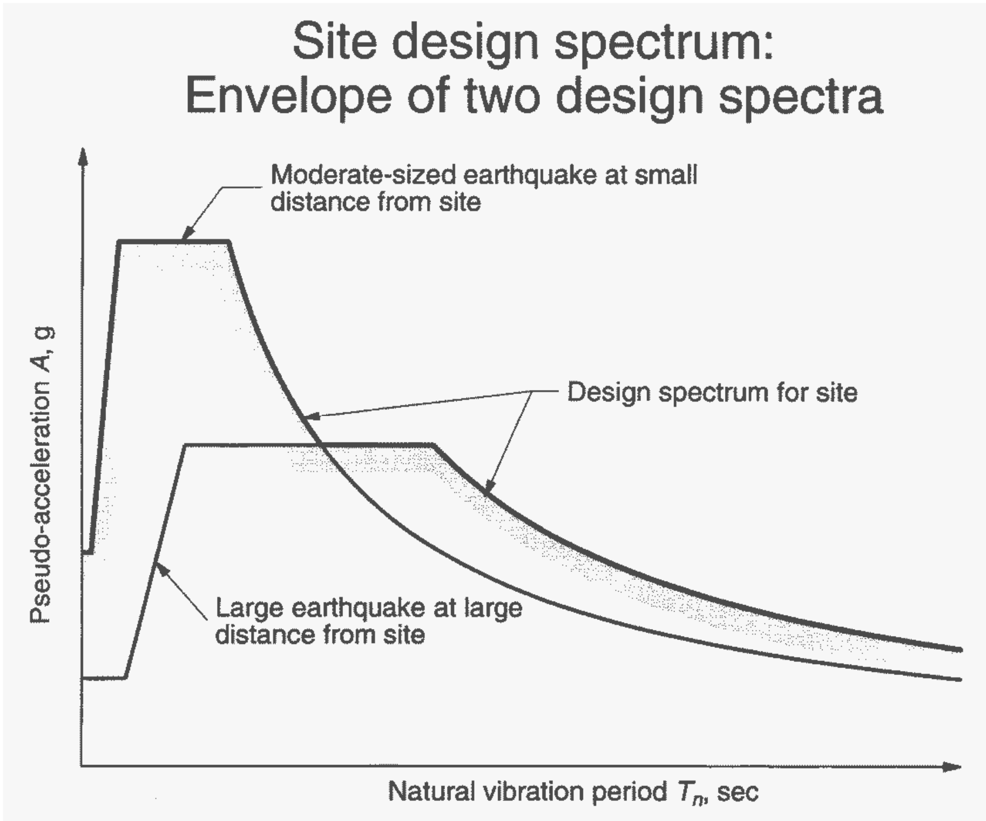
Source: N. M. Newmark and W. J. Hall, *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, Calif., 1982, pp. 35 and 36.

TABLE 6.9.2 AMPLIFICATION FACTORS: ELASTIC DESIGN SPECTRA[a]

	Median (50 percentile)	One Sigma (84.1 percentile)
α_A	$3.21 - 0.68 \ln \zeta$	$4.38 - 1.04 \ln \zeta$
α_V	$2.31 - 0.41 \ln \zeta$	$3.38 - 0.67 \ln \zeta$
α_D	$1.82 - 0.27 \ln \zeta$	$2.73 - 0.45 \ln \zeta$

Source: N. M. Newmark and W. J. Hall, *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, Calif., 1982, pp. 35 and 36.

[a]Damping ratio in percent.


Site design spectrum:
Envelope of two design spectra
Moderate-sized earthquake at small distance from site
Design spectrum for site
Large earthquake at large distance from site
Pseudo-acceleration A, g
Natural vibration period Tn, sec

RELATIVE VELOCITY & ACCELERATION RESPONSE SPECTRA
(not commonly used)

Equation of Motion:

$$m\ddot{u} + c\dot{u} + ku = \underbrace{-m\ddot{u}_g(t)}_{p_{eff}(t)}$$

Response:

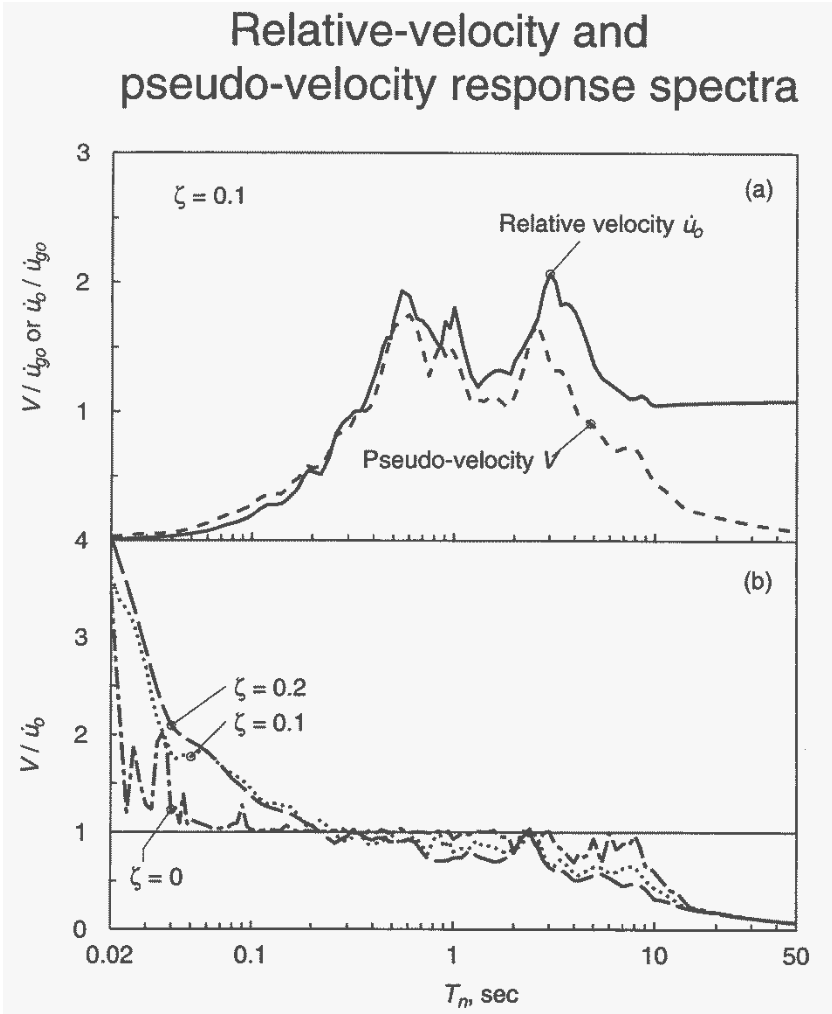
$$u(t) = -\frac{1}{\omega_D}\int_0^t \ddot{u}_g(\tau)e^{-\xi\omega(t-\tau)}\sin[\omega_D(t-\tau)]\ d\tau$$

Applying ***Leibnitz' Rule***, we get:

$$\dot{u}(t) = -\xi\omega u(t) - \frac{1}{\omega_D}\int_0^t \ddot{u}_g(\tau)e^{-\xi\omega(t-\tau)}\cos[\omega_D(t-\tau)]\ d\tau$$

Then:

$$\ddot{u}^t(t) = -\omega^2 u(t) - 2\xi\omega\dot{u}(t)$$

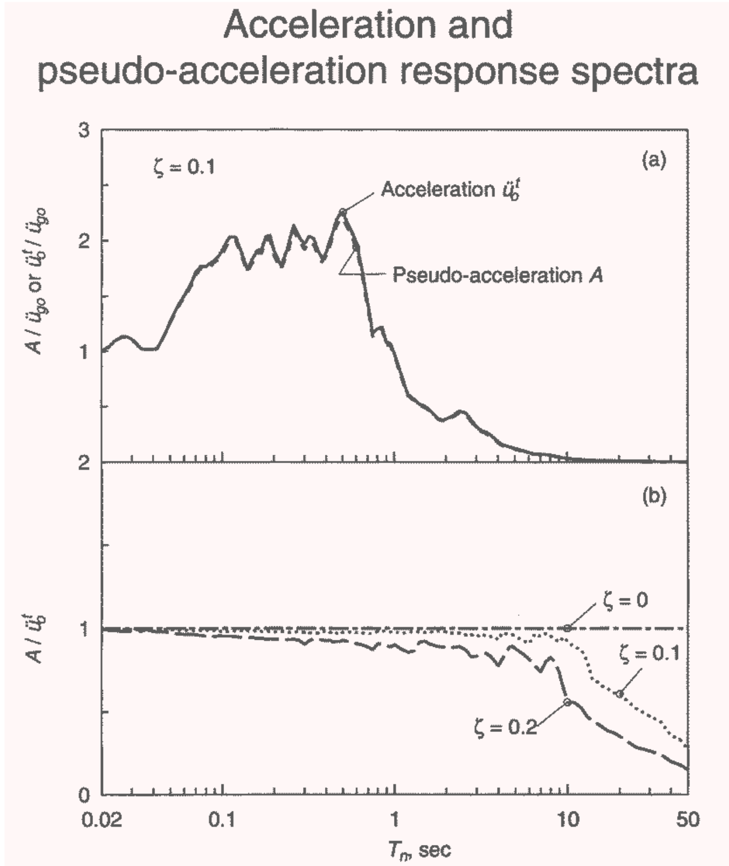


- For **long-period systems**, $V < \dot{u}_o$, and the difference between the two is large.

  As $T \to \infty$, $D \to u_{go}$ and $\dot{u}_o \to \dot{u}_{go}$

  Now, $D \underbrace{\longrightarrow}_{T\to\infty} u_{go} \Longrightarrow V \underbrace{\longrightarrow}_{T\to\infty} 0$ because $V = \frac{2\pi}{T} D$

- For **short-period systems**, $V > \dot{u}_o$, with the difference increasing as $T \to 0$.
- For **medium-period systems**, the differences between V and $\dot{u}_o$ are small over a wide range of T.



The pseudo-acceleration and acceleration spectra are identical for systems without damping (*i.e.*, for $\xi = 0$):

Equation of motion $\Longrightarrow \ddot{u}^t(t) = -\omega^2 u(t) \Longrightarrow \ddot{u}_o^t = \omega^2 u_o = \omega^2 D = A$

The peak values, $\ddot{u}_o^t$ & A, occur at the same time and are equal only for $\xi = 0$.

Differences between $\ddot{u}_o^t$ & A are expected to increase as the damping increases.

Another way of looking at it:

$$f_{So} = mA = \text{peak value of the elastic} - \text{resisting force}$$
$$m\ddot{u}_o^t = \text{peak value of (elastic + damping) forces}$$

As seen in the above Figure, $\ddot{u}_o^t > A$ (*i.e.*, A **resists only elastic forces**).