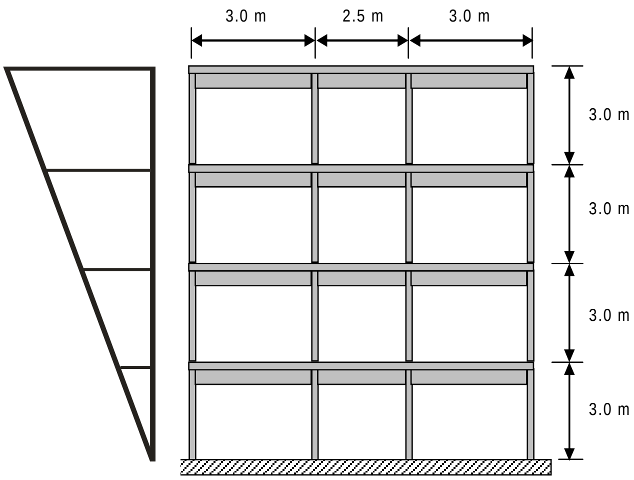


Question 5.2

An old concrete frame (see Fig. 5.18) has all columns the same: 0.25 m wide and 0.4 m deep, with the strong direction in the plane of the frame, with only one 18 mm dia. bar at each corner. All the beams have a depth of 0.5 m and a width of 0.25 m; they have two 14 mm dia. bars at top and bottom, continuous across all spans, plus two additional 14 mm top bars over the interior supports on the columns. The quasi-permanent gravity load, $g+\psi_2 q$, is 8kN/m^2 (all inclusive) and is applied over the 4.0 m wide tributary floor strip of the frame. Gravity loads go to the column which is nearest in plan.

Concrete is C30/37 and steel S500, with a 25 mm concrete cover.

The frame is evaluated under a seismic action represented by a system of horizontal forces on the floors, with inverted triangular heightwise distribution: $f_i = 0.1 i V_b$, where i indexes the storeys (from bottom to top) and V_b is the total seismic base shear. The overturning moment due to these seismic loads induces axial forces only in the two outer columns; seismic axial forces in the interior columns may be neglected.

1) Given that the size and the reinforcement of members is the same in all storeys and that only the column axial force changes from storey to storey, around which beam-column joint is the strong column-weak beam criterion $\sum M_{Rd,c} > \sum M_{Rd,b}$ most likely to be met; around which one is it least likely? Provide separate answers for interior and exterior columns, excluding the top floor and taking into account any effects of the overturning moment. For the two interior and the two exterior joints

expected to be most or least likely to fulfill this criterion, identify where the plastic hinges will form around these joints by checking numerically the criterion $\sum M_{Rd,c} > \sum M_{Rd,b}$. On the basis of the outcome, identify the most likely plastic hinge pattern and plastic mechanism in the frame under lateral seismic loading.

2) On the same basis as in 1) above, identify the beam span and the interior or exterior column in the frame with the largest capacity design shear force according to Eurocode 8. For the beam span and the interior and exterior columns with the expected highest capacity design shear, calculate its value. You may calculate any effects of the overturning moment using a seismic base shear equal to 20% of the weight.

3) Estimate the maximum horizontal force resistance that the frame can develop at its base, from the shear forces that can develop in its four columns when plastic hinges form at the base of these columns and around their top joint. Express it as a fraction of the weight of the frame.

Answer of Question 5.2:

1) Only the values of $M_{Rd,c}$ change from storey to storey in the criterion $\sum M_{Rd,c} > \sum M_{Rd,b}$; the change is due to the axial force alone. In interior columns, the axial force ranges from 8×4×(3+2.5)/2 = 88 kN in the top storey to 4×88= 352 kN in the ground storey. The corresponding dimensionless values are 88/(0.25×0.4×20000) = 0.044 and 4×0.044 = 0.176, well below the limit value v_1 in Eq. (5.37a), which gives the maximum moment resistance for given reinforcement ("balance point"). So, the moment resistance of the column increases with increasing axial force, i.e., from the top to the base of the building. Therefore, as far as the interior joints are concerned, it is around those in the 3rd floor that the criterion $\sum M_{Rd,c} > \sum M_{Rd,b}$ is most likely to be violated, and the ground storey joints where it is least likely.

In exterior columns, the axial force due to gravity ranges from 8×4×3/2 = 48 kN in the top storey to 4×48= 192 kN in the ground storey. The overturning moment at mid-height of the top storey is $f_4H/2$ = $0.1V_b$×4×3/2 = $0.6V_b$, inducing axial forces of $\pm 0.6V_b/(3+2.5+3) = \pm 0.0705V_b$ in the exterior columns; at mid-height of the ground storey it is equal to $f_4(7H/2)+f_3(5H/2)+f_2(3H/2)+f_1(H/2)$ = $0.1V_b\times3/2\times(7\times4+5\times3+3\times2+1\times1)=7.5V_b$, inducing axial forces of $\pm7.5V_b/(3+2.5+3) = \pm0.8825V_b$ in the exterior columns; i.e., the seismic axial force increases in absolute terms from top to bottom much faster than the one due to gravity actions. At mid-height of the 3rd or the 2nd storey it is equal to $f_4(3H/2)+f_3(H/2) = 0.1V_b\times3/2\times(3\times4+1\times3) = 2.25V_b$ or $f_4(5H/2)+f_3(3H/2)+f_2(H/2)$ = $0.1V_b\times3/2\times(5\times4+3\times3+1\times2) = 4.65V_b$, respectively, inducing axial forces of $\pm0.2645V_b$ or $\pm0.547V_b$, respectively, in the exterior columns. What counts is the effect of the axial force variation on the sum of $M_{Rd,c}$ above and below a joint, which roughly depends on the average of the axial forces in the columns above and below. At the 3rd storey joints the average seismic axial force is $\pm0.1675V_b$ and at the 2nd storey ones $\pm0.715V_b$; the difference of $0.547V_b$ in the seismic axial tension of these two joints can overcome the 96 kN increase in axial compression due to gravity if $V_b > 96/0.547 = 177.5$ kN. (Note that we arrive at the same outcome if we consider the difference between the ground and top storey columns). As the total weight is 8×4×4×(3+2.5+3) = 1088 kN, there will be a net reduction of the axial force from top to bottom of the windward column, if the base shear is more than 177.5/1088 = 16.3% of the weight. In that case, the exterior joint most likely to violate criterion $\sum M_{Rd,c} > \sum M_{Rd,b}$ is the ground storey one on the windward side; otherwise, it is the one at the 3rd storey. No matter the magnitude of V_b, the least likely violator is the ground storey joint of the leeward column.

We calculate first the values of $M_{Rd,b}$:

f_{cd}=30/1.5=20 MPa; f_{yd}=500/1.15=434.8 MPa; ε_{yd}=434.8/200000=0.217%;

Distance of centre of longitudinal bars from nearest concrete surface:

$d_1=c+d_{bh}+d_{bL}/2\sim0.025+0.006+0.014/2\sim0.04$ m; d=0.46 m.

Bottom reinforcement: 308 mm^2; top reinforcement: 308 mm^2, except over the two interior supports, where it is 616 mm^2**.**

$M_{Rd,b}^+$=308×0.9×0.46×0.4348 =55.5 kNm.

$M_{Rd,b}^{-}$ = 55.5 kNm at the outer supports, $M_{Rd,b}^{-}$ = 616×0.9×0.46×0.4348 = 111 kNm at the interior ones.

So: $\sum M_{Rd,b}$ = 55.5 kNm at exterior joints, $\sum M_{Rd,b}$ = 111+55.5 = 166.5 kNm at interior ones.

The value of $M_{Rd,c}$ depends on the axial load:

d_1= 40 mm, d= 360 mm, $\delta_1 = d_1/d$= 40/360= 0.111, $\omega_{1d}=\omega_{2d}$= 509/(250×360)×434.8/20 = 0.123, ω_{vd}=0.

The limits of Eq. (5.37a) are:

ν_1 = (0.0035-0.002/3)/(0.0035+0.00217) = 0.5.

ν_2 = 0.111×(0.0035-0.002/3)/(0.0035-0.00217) = 0.237.

A ground storey interior column has: ν_d = 352/(0.25×0.36×20000) = 0.1955 < ν_2 = 0.237. Hence Eqs. (5.38b), (5.39b) apply to all columns. Eq. (5.38b) gives:

$$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi[0.40476 - 0.33676\xi] + 0.05467\left(1 + 1.613\frac{\xi - 0.111}{\xi}\right)$$

with the value of ξ from Eq. (5.39b), which takes the form:

[1-0.002/(3×0.0035)]ξ^2-[ν_d+0.123×(1-0.0035/0.00217)]ξ-0.123×0.111×0.0035/0.00217=0 →

0.8095ξ^2-[ν_d-0.0754]ξ-0.022=0

Above the 3rd storey interior joint (the most likely violator of the criterion): ν_d = 88/(0.25×0.36×20000) = 0.049, and below it: ν_d =0.098, giving: $M_{Rd,c}$ = 51.3 and 66.9 kNm, respectively, i.e., $\sum M_{Rd,c}$ = 118.2 kNm < $\sum M_{Rd,b}$ = 166.5 kNm. (Note that, if we had used in the calculation the average of the axial forces above and below the joint, the outcome would have been $\sum M_{Rd,c}$ = 118.6 kNm).

Above the 1st storey interior joint (the least likely violator of the criterion): ν_d = 0.147 and below it: ν_d = 0.196, giving: $M_{Rd,c}$ = 81 and 93.6 kNm, respectively, i.e., $\sum M_{Rd,c}$ = 174.6 kNm > $\sum M_{Rd,b}$ = 166.5 kNm. (If we had based the calculation on the average of the axial forces above and below the joint, the outcome would have been $\sum M_{Rd,c}$ = 175 kNm).

The conclusion is that at the interior joints of the 3rd floor plastic hinges will form in the columns, but at those of the 1st storey in the beams. To see where the mechanism changes, we repeat the excercise at the 2nd storey joints, this time with the average axial force above and below the joint: $\nu_d = 0.122$. The outcome is $\sum M_{Rd,c} = 148$ kNm; hence, also at the second storey interior joints, plastic hinges develop in the columns, and not in the beams.

Concerning the exterior joints: even for zero average axial force in the columns, $\sum M_{Rd,c} = 68.8$ kNm $> \sum M_{Rd,b} = 55.5$ kNm. So, the plastic hinge forms in the beam at all joints of both outer columns.

So, the frame will develop a beam sway mechanism in the lower storey and a mixed one in the other three, with plastic hinges forming in the interior columns from the 2nd floor up.

2) The maximum capacity design shear in a beam takes place when the beam itself forms plastic hinges at both ends, not the columns it is connected to. Moreover, the central beam span is more critical than the two outer ones: first, because its hogging moment resistance is high at both ends, while that of the outer spans is lower; second, because it has a shorter clear span. This points to the central span of the first storey, where the capacity design shear, without a γ_{Rd} factor, is:

$V_{CD,1,int,b} = (M_{Rd,b}^{+}+M_{Rd,b}^{-})/L_{cl} = (111+55.5)/2.1 = 79.3$ kN.

The shear due to the transverse loading of 32kN/m should be added to V_{CD}.

In principle, the maximum capacity design shear in a column takes place when the column itself forms plastic hinges at both ends, not the beams it is connected to. This is the case of the 3rd storey interior column, where plastic hinges form at both ends, with $M_{Rd,c} = 66.9$ kNm. There, without a γ_{Rd} factor:

$V_{CD,3,int,c} = (M_{Rd,c}^{+}+M_{Rd,c}^{-})/H_{cl} = (66.9+66.9)/2.5 = 53.5$ kN.

However, there is a possibilty that the 1st storey interior column may develop even higher capacity design shear, despite the fact that it will not form a plastic hinge at the top (the 1st storey beams will, instead); in fact, that column has $M_{Rd,c} = 93.6$ kNm at both ends and $\sum M_{Rd,c} = 174.6$ kNm $> \sum M_{Rd,b} = 166.5$ kNm at its top joint. Therefore, it can develop a much higher capacity design shear:

$V_{CD,1,int,c} = (93.6+93.6\times166.5/174.6)/2.5 = 73.1$ kN.

By the same token, the maximum capacity design shear in an outer column takes place at the location with the maximum possible compression. This is obviously the 1st storey leeward column; its maximum compressive axial force is the sum of the maximum possible beam shears applied on that column over all four storeys. These are indeed the capacity design shears at the outer end of the side span, for hogging bending there and sagging bending at the end connected to the interior column, plus the shear due to the transverse load in a simply supported beam:

max$N$ = 4×[(55.5+55.5)/2.6+8×4×3/2]=170.8+192=362.8 kN

(This level of force is reached about when the base shear reaches the value V_b = 170.8/0.8825 = 193.5 kN, i.e., 17.8% of the weight).

For N = 362.8 kN, ν_d = 362.8/(0.25×0.36×20000) = 0.20155, for which Eqs. (5.38b), (5.39b) give $M_{Rd,c}$ = 95 kNm, while in the storey above N = 3×362.8/4 = 272.1 kN, ν_d = 272.1/(0.25×0.36×20000) = 0.15117 and $M_{Rd,c}$ = 82.1 kNm. Then:

$V_{CD,1,ext,c}$ = (95+95×55.5/(82.1+95))/2.5 = 49.9 kN

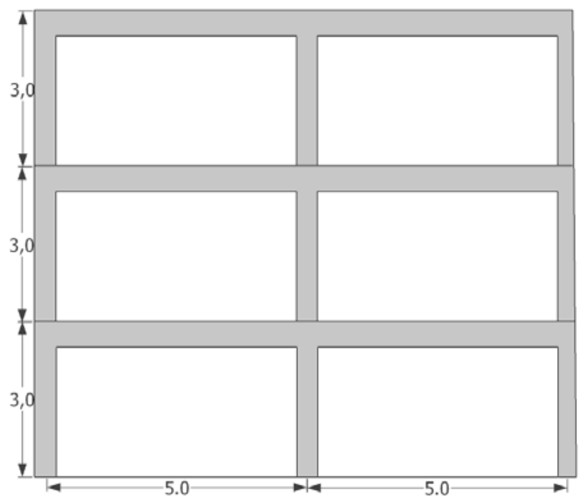
which is still less than the value applying to the interior column of the storey.

3) The maximum shear that can develop at the ground storey of each interior colum has been computed in 2) above as: $V_{CD,3,int,c}$ = 53.5 kN. If we want to add the value $V_{CD,1,ext,c}$ = 49.9 kN determined above for the leeward column under its maximum possible compression, we should also add the value of V_{CD} in the opposite (windward) colum under an axial force of min$N$ = -170.8+192=21.2 kN, i.e., for ν_d = 21.2/(0.25×0.36×20000) = 0.0118, which gives $M_{Rd,c}$ = 38.6 kNm. At the storey above, ν_d = 0.75×0.012 = 0.009 and $M_{Rd,c}$ = 37.6 kNm. Then that column develops a shear force of (38.6+38.6×55.5/(37.6+38.6))/2.5 = 26.7 kN. Therefore, the maximum horizontal force resistance that the frame can develop at the base is: maxV_b = 26.7+2×53.5+49.9 = 183.6 kN, i.e., 16.9% of the weight.

Note that if we had used the axial load of the two outer columns due to gravity actions alone, i.e., N = 192 kN, ν_d = 0.107 in the ground storey and 75% of these values in the storey above, giving $M_{Rd,c}$ =

69.5 kNm and $M_{Rd,c}$ = 61.4 kNm in these two floors, the capacity design shears of these columns would have been (69.5+69.5×55.5/(69.5+61.4))/2.5 = 39.6 kN, giving a maximum horizontal force resistance of the frame: maxV_b = 2×(53.5+39.6) = 186.2 kN, i.e., 17.1% of the weight. This value, though, is not realistic, because it corresponds to a frame staying still, not swaying. The smaller value of 183.6 kN estimated above is more representative.

At any rate, even if there is no shear failure anywhere in the frame (which can be spotted by comparing the capacity design shears computed above to the shear resistance of the member), it cannot develop a base shear equal to 20% of the weight: a flexural plastic mechanism will form before that.

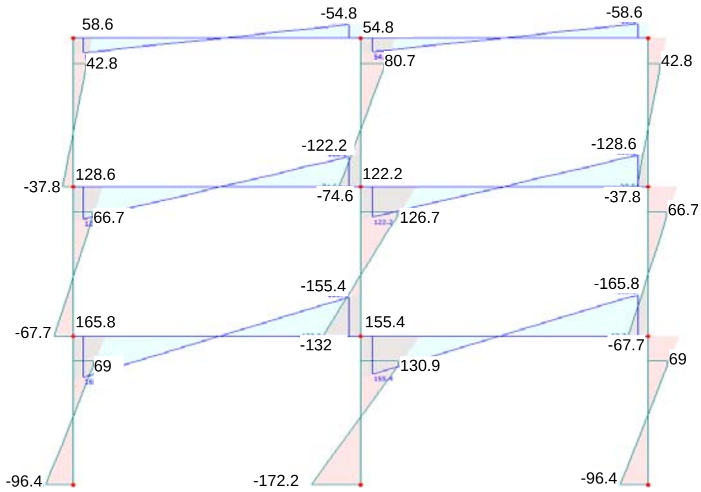


Question 5.3

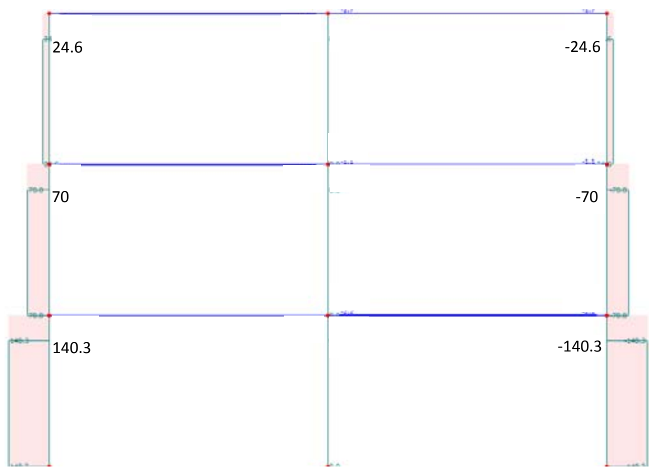
A 3-storey RC frame, with storey height H = 3 m, has two bays, each one with span length L = 5 m (Fig. 5.19). The central column is 0.4 m square; the outer ones 0.35 m square. The beams have width b_w = 0.3 m and depth h_b= 0.5 m and are connected on both sides to a 150 mm thick slab. Design is for a ground motion with design peak ground acceleration (on rock) of 0.30g and the type 1 spectrum per Eurocode 8 on ground type C. Ductility Class (DC) is Medium (M).

The moment, M, and the axial force, N, diagrams shown in Figure 5.20 over the clear member length (joints are considered rigid) are obtained from linear analysis for the quasi-permanent gravity loads, $G+\psi_2 Q$, with ψ_2 = 0.3, and for the design seismic action. For the latter, the full quasi-permanent gravity loads are taken to produce inertia forces (without reduction for the calculation of masses).

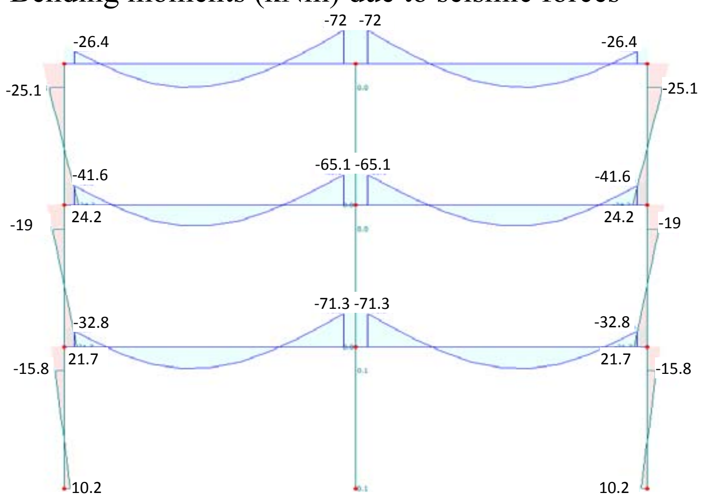
The lateral force method is used, but, since the fundamental period, T_1, is not known yet, the *M*- and *N*-diagrams have been constructed assuming that T_1 is in the constant-acceleration range, i.e., shorter than the corner period T_C = 0.6 s on type C ground. The column axial forces at the base give the total weight, and hence the mass of the frame, which corresponds to the quasi-permanent gravity loads; its distribution to the floors is obtained from the storey shears. Columns are taken as fixed at the top of the footing.



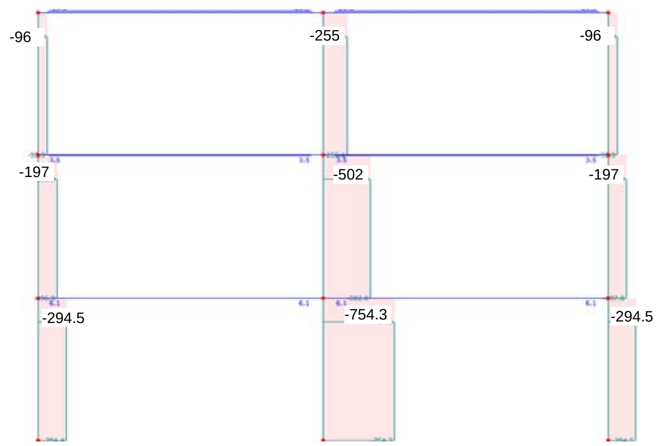
Bending moments (kNm) due to seismic forces



Axial forces (kN) due to seismic loading



Bending moments (kNm) due to quasi-permanent loads



Axial forces (kN) due to quasi-permanent loads

Concrete grade is C30/37 and steel is of Class C with 500 MPa nominal yield stress; the concrete cover to reinforcement is *c* = 25 mm. Importance Class is II (ordinary).

1) Calculate from the moment diagram the lateral forces, f_j, and the resulting floor displacements, u_j; use these values to calculate the fundamental period of the frame through the Rayleigh quotient, Eq.

(3.109); correct the moment and axial force diagrams in Figure 5.21 to be consistent with the computed value of T_1.

2) Calculate the interstorey drifts under the damage limitation seismic action and the sensitivity coefficient for second-order effects.

3) Dimension the longitudinal bars of the beams in floors 1 and 2, taking into account the "persistent and transient design situation" for the combinations of Eqs. (6.10a), (6.10b) of EN 1990 (the most unfavourable of $(1.35\xi)G_k$ "+" $1.5Q_k$ or $1.35G_k$ "+" $(1.5\psi_0)Q_k$, with $\xi = 0.85$ and $\psi_0 = 0.7$); to this end, you may assume that the ratio of permanent-to-imposed nominal loads, G_k-to-Q_k, is 3.

4) Dimension the vertical reinforcement of the central and outer columns in storeys 1 and 2 to meet the strong-column/weak beam capacity design rule, Eq. (5.31).

5) Calculate the capacity design shears at the end sections of the first and second storey beams and columns from Eqs. (5.42), (5.44).

6) Dimension the transverse reinforcement of the first storey beams.

7) Dimension and detail the transverse reinforcement of the first storey columns, including confinement at the base.

Answer of Question 5.3:

1) Total weight = Sum of axial forces at the base: 1343.3 kN.

Storey shears from the lateral force method moments:

Storey 3: $V_3 = (80.7-(-74.6))/2.5+2\times(42.8-(-37.8))/2.5 = 126.5$ kN

Storey 2: $V_2 = (126.7-(-132))/2.5+2\times(66.7-(-67.7))/2.5 = 211$ kN

Storey 1: $V_1 = (130.9-(-172.2))/2.5+2\times(69-(-96.4))/2.5 = 253$ kN

Storey lateral forces as difference of shears across floors:

Storey 3: $f_3 = V_3 = 126.5$ kN

Storey 2: $f_2 = V_2 - V_3 = 84.5$ kN

Storey 1: $f_1 = V_1 - V_2 = 42$ kN

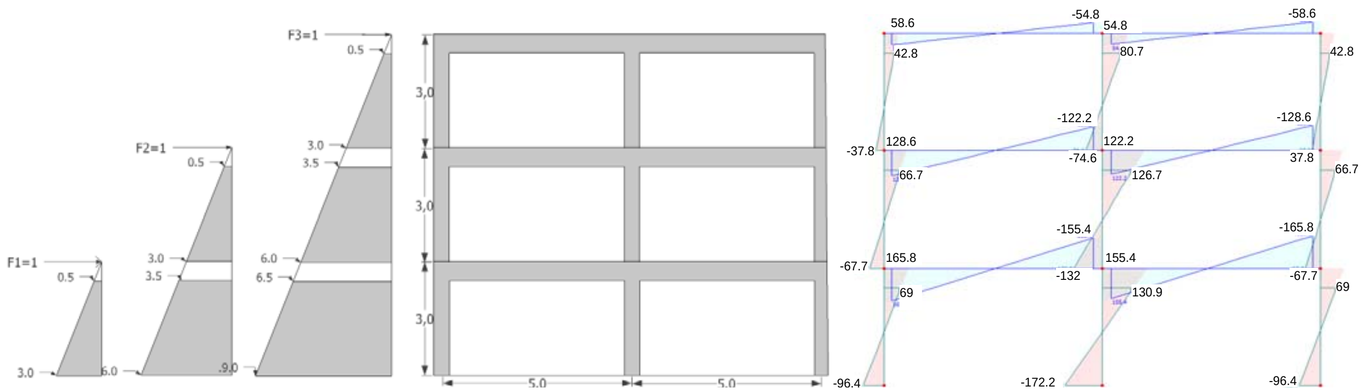
Storey weights and masses:

Storey 3: W_3 = (126.6/1.5)×1343.3/(126.6/1.5+84.4+42.56/0.5)= 446.5 kN, m_3 = 446.5/9.81 = 45.5 tn

Storey 2: W_2 = (84.4/1.0)×1343.3/(126.6/1.5+84.4+42.56/0.5)= 446.5 kN, m_2 = 446.5/9.81 = 45.5 tn

Storey 1: W_1 = (42.56/0.5)×1343.3/(126.6/1.5+84.4+42.56/0.5)= 450.5 kN, m_3 = 450.5/9.81 = 45.9 tn

The floor displacements, u_j, due to the above seismic lateral forces, f_j, are calculated from the moment diagram, by applying unit forces at floor levels and using the Virtual Work principle: the single cantilever moment diagrams for f_j = 1 shown in the figure below on the left are multiplied with those due to the full suite of lateral forces on the right; the product is integrated over the height of the columns (excluding the 0.5 m lengths within beam-column joints, which are taken as rigid). The final outcome is essentially the same, no matter whether the integration takes place along an exterior (I_{ext} = $0.35^4/12$ = 0.00125 m^4) or the central column (I_{int} = $0.4^4/12$ = 0.002133 m^4).



For example, for an exterior column and 50% of the uncracked section stiffness:

u_1 =-(2×0.5×69-2×3.0×96.4-0.5×96.4+3.0×69)×2.5/(6×0.5×33000000×0.00125) = 0.0071 m

u_2 =-(2×0.5×66.7-2×3.0×67.7-0.5×67.7+3.0×66.7+2×3.5×69-2×6.0×96.4-3.5×96.4+6.0×69)

×2.5/(6×0.5×33000000×0.00125) = 0.0156 m

u_3 =-(2×0.5×42.8-2×3.0×37.8-0.5×37.8+3.0×42.8+2×3.5×66.7-2×6.0×67.7-3.5×67.7+

6.0×66.7+2×6.5×69-2×9×96.4-6.5×96.4+9×69)×2.5/(6×0.5×33000000×0.00125) = 0.0223 m

With the above values of u_j, f_j, m_j, Eq. (3.109) gives:

T_1 = 0.565 s,

which is less than T_C = 0.6 s; therefore the internal force diagrams do not need to be corrected for the value of the period. As $T_1 < 2T_C$=1.2 sec, we may reduce the base shear by 15% in the lateral force method of analysis (Section 3.1.6).

The behaviour factor is equal to $q=q_o$=3x1.3=3.9.

Then, at T_1 = 0.565 s the base shear is:

$V_b = 0.85\times(0.3\times1.15)\times2.5/3.9\times1343.3 = 253$ kN.

2) The floor deflections calculated above should be multiplied by q = 3.9 and by the reduction factor of 0.5 for the damage limitation seismic action. Then the interstorey drifts are:

$\Delta u_3/h_{st} = 0.5\times(0.0225\text{-}0.0156)\times3.9/3.0 = 0.45\%$;

$\Delta u_2/h_{st} = 0.5\times(0.0156\text{-}0.0071)\times3.9/3.0 = 0.55\%$;

$\Delta u_1/h_{st} = 0.5\times0.0071\times3.9/3.0 = 0.46\%$.

The interstorey drift limit of 0.5% for brittle partitions is slightly violated at storey 2.

The sensitivity coefficient for second-order effects is:

$\theta_3 = 3.9\times(0.0225\text{-}0.0156)\times446.5/(126.5\times3.0) = 0.032 < 0.10$;

$\theta_2 = 3.9\times(0.0156\text{-}0.0071)\times2\times446.5/(211\times3.0) = 0.047 < 0.10$;

$\theta_1 = 3.9\times0.0071\times1343.3/(253\times3.0) = 0.049 < 0.10$.

3) Dimensioning of beam longitudinal reinforcement

Parameters:

f_{cd}=30/1.5=20 MPa; f_{ctm}=2.9 MPa, E_c=33000000 kPa;

f_{yd}=500/1.15=434.8 MPa; ε_{yd}=434.8/200000=0.217%;

Distance of centre of longitudinal bars from nearest concrete surface:

$d_1=c+d_{bh}+d_{bL}/2$~0.025+0.008+0.018/2 ~0.04m.

Curvature ductility demand for detailing:

As $T < T_C$, Eq. (5.64b) applies: $\mu_\varphi = 2(q_o - 1)T_C/T + 1 = 2\times(3.9\text{-}1)\times0.6/0.565+1 = 7.16$.

$\rho_{min} = 0.5f_{ctm}/f_{yk} = 0.5\times2.9/500 = 0.0029$

d=0.46m, z=0.46-0.04=0.42m, $A_{s,min}$=0.0029×300×460=400mm^2 (2Φ16 - 402mm^2).

If $G/Q = 3$, then:

For Eq. (6.10a) in EN1990: $(1.35\xi G+1.5Q)/(G+\psi_2 Q)=(1.35\times0.85\times3+1.5\times1)/(3+0.3\times1)=1.498$,

For Eq. (6.10b): $(1.35G+1.5\psi_0 Q)/(G+\psi_2 Q)=(1.35\times3+1.5\times0.7\times1)/(3+0.3\times1)=1.545$ (i.e., more unfavourable).

The beam moments in the persistent and transient design situation are equal to those due to the quasi-permanent gravity loads, $G+\psi_2 Q$ times 1.545.

Design moments of 1st floor beams B1:

At the support on the central column:

– Hogging design moment for the top reinforcement:

$M_{d1} = \max[M_{EN1990}; M_{g+\psi 2q}+|\max M_E|] = \max[1.545\times71.3=110.2; 71.3+155.4=226.7] = 226.7$ kNm

– Sagging design moment for the bottom reinforcement:

$M_{d2} = -M_{g+\psi 2q}+|\max M_E|= 155.4-71.3=84.1$ kNm

At the support on the exterior column:

– Hogging design moment for the top reinforcement:

$M_{d1} = \max[M_{EN1990}; M_{g+\psi 2q}+|\max M_E|] = \max[1.545\times32.8=50.7; 32.8+165.8=198.6] = 198.6$ kNm.

– Sagging design moment for the bottom reinforcement:

$M_{d2} = -M_{g+\psi 2q}+|\max M_E|= 165.8-32.8=133$ kNm

Beam longitudinal reinforcement (computed as $A_s=M_d/zf_{yd}$, with $z=d-d_1$):

Maximum diameter of bars, d_{bL}, at the support on the exterior column, per Eq. (5.2b):

$\min N=\min(\min N_{1st\ storey}, \min N_{2nd\ storey})=\min(294.5-140.3=154.2; 197-70=127)= 127$ kN

For $v_d=\min N_d/(0.35^2\times20000)=0.052$: $d_{bL}/h_{c,ex}\leq7.5\times1.042\times2.9/(1\times434.8)=0.0521$, $d_{bL}\leq18$ mm

At the support on the central column, per Eq. (5.2a):

$v_d=\min(N_{1st\ story,g+\psi 2q}, N_{2nd\ storey,g+\psi 2q})/(0.4^2\times20000)=0.157$.

If $\rho'=0.5\rho_{max}$: $d_{bL}/h_{c,int}\leq 7.5\times1.1255\times2.9/(434.8\times1.25)=0.045$, $d_{bL}\leq 18$mm

Longitudinal reinforcement at the central support:

– Top: A_{s1} =226.7×10^3/(0.42×434.8)=1241 mm^2: 5Φ16+1Φ18 (1260mm^2), ρ = 1257/(460×300)=0.0091.

– Bottom: A_{s2} = 84.1×1241/226.7 = 460 mm^2: 4Φ16 (804 mm^2> 1260/2 = 630 mm^2), ρ'= 804/(460×300) = 0.0058,

From Eq. (5.4b): $\rho_{max}=\rho'+0.0018 f_{cd}/(\mu_{\varphi}\varepsilon_{yd}f_{yd})$=0.0058+0.0018×20/(7.16×0.00217×434.8) = 0.0111, $\rho_{max} > \rho$=0.0091

$M_{Rd,b}^{-}$ = 1260×0.42×434.8/10^3 = 230 kNm, $M_{Rd,b}^{+}$ = 804×0.42×434.8/10^3 = 146.8 kNm, Σ$M_{Rd,b}$=376.8 kNm.

Longitudinal reinforcement at the support on the exterior column:

– Top reinforcement: A_{s1} = 198.6×1241/226.7 = 1087 mm^2: 5Φ16 (1005 mm^2) > $A_{s,min}$ = 400 mm^2

ρ=1005/(460×300)=0.00728 < ρ_{max}=0.0111.

– Bottom reinforcement: A_{s2} = 133×1241/226.7 = 728 mm^2: 4Φ16 (804mm^2> 1260/2=630mm^2), ρ'=804/(460×300)=0.0058

$M_{Rd,b}^{-}$ =1005×0.42×434.8/10^3 = 183.5 kNm, $M_{Rd,b}^{+}$ = 804×0.42×434.8/10^3=146.8 kNm

Design moments of 2nd floor beams B1:

At the support on the central column:

– Hogging design moment for the top reinforcement:

M_{d1} = max[M_{EN1990}; $M_{g+\psi2q}$+|maxM_E|] = max[1.545×65.1=100.6; 65.1+122.2=187.3] = 187.3 kNm

– Sagging design moment for the bottom reinforcement:

M_{d2} = -$M_{g+\psi2q}$+|maxM_E|= 122.2-65.1 = 57.1 kNm

At the support on the exterior column:

– Hogging design moment for the top reinforcement:

M_{d1} = max[M_{EN1990}; $M_{g+\psi2q}$+|maxM_E|] = max[1.545×41.6=64.3; 41.6+128.6=170.2]= 170.2 kNm

– Sagging design moment for the bottom reinforcement:

M_{d2} = -$M_{g+\psi 2q}$+|maxM_E|= 128.6-41.6=87 kNm

Beam longitudinal reinforcement (as $A_s = M_d/zf_{yd}$, with $z = d$-d_1):

Maximum diameter of bars, d_{bL}, at the support on the exterior column, per Eq. (5.2b):

min$N$ = min(min$N_{3rd\ storey}$, min$N_{2nd\ storey}$) = min(197-70=127; 96-24.6=71.4)= 71.4 kN

For ν_d = minN_d/(0.35^2×20000)=0.029: $d_{bL}/h_{c,ext} \leq$ 7.5×1.023×2.9/434.8 = 0.0512, $d_{bL} \leq$ 18mm

At the support on the central column, per Eq. (5.2a):

ν_d = min($N_{3rd\ story,g+\psi 2q}$, $N_{2nd\ storey,g+\psi 2q}$)/(0.4^2×20000) = 255/(0.4^2×20000) =0.08.

If ρ'=0.5ρ_{max}: $d_{bL}/h_{c,int} \leq$7.5×1.064×2.9/(434.8×1.25)=0.0426, $d_{bL} \leq$ 17 mm

Longitudinal reinforcement at the central support:

– Top: A_{s1} = 187.3×10^3/(0.42×434.8) = 1026 mm^2: 5Φ16 (1005 mm^2), ρ = 1005/(460×300) = 0.0073

– Bottom: A_{s2} = 57.1×1026/187.3 = 313 mm^2: 2Φ16+1Φ14 (555 mm^2> 1005/2 = 502 mm^2), ρ'= 555/(460×300) = 0.004

– ρ_{max} = 0.004+0.0018×20/(7.16×0.00217×434.8) = 0.0093>ρ=0.0073.

$M_{Rd,b}^-$ = 1005×0.42×434.8/10^3 = 183.5 kNm, $M_{Rd,b}^+$= 555×0.42×434.8/10^3 = 101.4 kNm,

Σ$M_{Rd,b}$ = 284.9 kNm

Longitudinal reinforcement at the support on the exterior column:

– Top: A_{s1} = 170.2×10^3/(0.42×434.8) = 932 mm^2: 4Φ16+1Φ14 (958 mm^2) > $A_{s,min}$ = 400 mm^2 , ρ = 958/(460×300) = 0.0069

– Bottom: A_{s2} = 87×932/170.2 = 476 mm^2: 2Φ16+1Φ14 (555 mm^2> 958/2=479 mm^2), ρ' = 555/(460×300) = 0.004

ρ_{max} = 0.004+0.0018×20/(7.16×0.00217×434.8) = 0.0093 > ρ = 0.0069.

$M_{Rd,b}^-$ =958×0.42×434.8/10^3=175 kNm, $M_{Rd,b}^+$=555×0.42×434.8/10^3= 101.4 kNm

4) Dimensioning of column vertical reinforcement, including capacity design. Calculation of column moment resistance.

The peak values of the column bending moments from the analysis are listed in the first half of the table below, along with the concurrent axial force. The second half of the table gives the moment resistance of the columns required in order to meet Eq. (5.31), with the value of 1.3ΣM_{Rb} equally split between the column sections above and below the joint; in that case the mean axial load above and below the joint applies.

Moments and axial forces at column ends for dimensioning of the vertical reinforcement

Column			Exterior					Central				
Storey			1st		2nd		3rd	1st		2nd		3rd
End section			base	top	base	top	base	base	Top	base	top	base
From the analysis	+E	M (kNm)	106.6	84.8	89.4	85.7	62	172.2	130.9	132	126.7	74.6
		N (kN)	434.8	434.8	267	267	120.6	754.3	754.3	502	502	255
	-E	M (kNm)	86.2	53.2	46	47.7	13.6	-	-	-	-	-
		N (kN)	154.2	154.2	127	127	71.4	-	-	-	-	-
(1.3Σ$M_{Rd,b}$)/2 at mean value of N above & below joint	+E	M_{Rc} (kNm)	-	119.3		113.8		-	244.9		185.2	
		N (kN)	-	350.9		193.8		-	628.2		378.5	
	-E	M (kNm)	-	95.4		65.9		-				
		N (kN)	-	140.6		99.2		-				

The moments from the analysis are much lower than the demands of capacity design; so, they are not considered. The starter bars at the connection to the foundation should be dimensioned for the internal forces at the base section from the analysis; however, this is omitted, because the internal forces for the column design at the top of the 1st storey are more critical: the vertical bars extending all along a column's 1st storey and into the base of the 2nd, extend also downwards into the foundation.

a) Central column.

$\delta_1 = d_1/d$=40/360=0.111, ρ_{min}=0.01, $A_{s,min}$=0.01×400×400 = 1600 mm^2; minimum number of bars: 8 (3 per side).

At the 1st storey and the base of the 2nd storey:

μ_d = 244.9/(0.4×0.36^2×20000) = 0.236, ν_d= 628.2/(0.4×0.36×20000)=0.218.

At the 2nd storey and the base of the 3rd storey:

μ_d = 185.2/(0.4×0.36^2×20000) = 0.179, ν_d = 378.5/(0.4×0.36×20000) = 0.1315.

The limit per Eqs. (5.33a), (5.33b) is ν_2 = 0.111×(0.0035-0.002/3)/(0.0035-0.00217)=0.2365, and exceeds ν_d at both storeys.

Eqs. (5.34b), (5.35b) apply, as follows:

$$\omega_{1d}\,\frac{4}{9}\left(1+1.613\,\frac{\xi-0.111}{\xi}\right)=\mu_d-\xi\left[0.4048-0.3367\,\xi\right],$$

$$0.8095\,\xi^2-\left[v_d-0.613\,\omega_{1d}\right]\xi-0.1792\,\omega_{1d}=0$$

They are solved iteratively, giving:

- At the 1st storey and the base of the 2nd storey: ω_{1d}=0.175; A_{s1} = 0.175×(400×360)×20/434.8 = 1160 mm^2

We place 5Φ16 per side (with the mid-side bars restrained at the corner of a diamond-shaped tie), i.e. 16Φ16 total (3216mm^2), giving: ρ=3216/(400×400)=0.0201>ρ_{min}=0.01.

We calculate next the moment resistance above and below the joint resulting from this reinforcement; to this end, half of the corner bars count in ω_{vd}, in order to have the "web" reinforcement uniformly spread between ω_{1d} and ω_{2d}: ω_{1d}=ω_{2d}=0.25×3216/(400×360)×434.8/20 = 0.1214, ω_{vd}=2ω_{1d}=0.2428.

The limits of Eq. (5.37a) are:

ν_1 = 0.2428×[(0.0035-0.00217)/(0.0035+0.00217)-0.111]/(1-0.111)+(0.0035-0.002/3)/(0.0035+0.00217) = 0.53.

ν_2 = 0.2428×[0.111×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.111)+0.111×(0.0035-

0.002/3)/(0.0035-0.00217) = 0.093.

The limits are met both above and below the joint:

- Above the joint: v_d= 502/(0.4×0.36×20000)=0.174 > v_2.
- Below the joint: v_d= 754.3/(0.4×0.36×20000)=0.262 > v_2.

Eqs. (5.38a), (5.39a) apply, giving ξ = (0.889v_d+1.111×0.2428)/(0.889×0.8095+2×0.2428):

- Above the joint: ξ = 0.352.
- Below the joint: ξ = 0.417.

The moment resistance provided by this reinforcement per Eq. (5.38a):

$$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi\left[0.40476 - 0.33676\xi\right] + 0.889\cdot 0.1214 + 0.2428\left[(\xi - 0.111)(1-\xi) - 0.1281\xi^2\right]/0.889$$, giving:

- Above the joint: $M_{Rd,c}$= 256.1 kNm.
- Below the joint: $M_{Rd,c}$= 270.4 kNm.

$\sum M_{Rd,c}$ = 526.5 kNm > 1.3$\sum M_{Rd,b}$ = 489.8 kNm.

(Note that, if we had placed 3Φ20 per side, with the mid-side bar restrained at the corner of a cross-tie or hoop, the resulting moment resistances would give $\sum M_{Rd,c}$ = 466.7 kNm < 1.3$\sum M_{Rd,b}$ = 489.8 kNm. The alternative option of 4Φ18 per side satisfies $\sum M_{Rd,c}$ > 1.3$\sum M_{Rd,b}$, but requires restraining both intermediate bars at a hoop corner).

- At the 2nd storey and the base of the 3rd storey: ω_{1d}=0.144; A_{s1} = 0.144×(400×360)×20/434.8= 954 mm^2. We place 3Φ20 per side (with the mid-side bar restrained at the corner of a cross-tie or hoop), i.e. 8Φ20 total (2513 mm^2), giving:

ρ=2513/(400×400)=0.0157>ρ_{min}=0.01.

ω_{1d}=ω_{2d}=0.25×2513/(400×360)×434.8/20 = 0.0948, ω_{vd}=2ω_{1d}=0.1896.

The limits of Eq. (5.37a) are:

v_1 = 0.1896×[(0.0035-0.00217)/(0.0035+0.00217)-0.111]/(1-0.111)+(0.0035-0.002/3)/(0.0035+0.00217) = 0.525.

$\nu_2 = 0.1896\times[0.111\times(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.111)+0.111\times(0.0035-0.002/3)/(0.0035-0.00217) = 0.124$.

- Above the joint: $\nu_d = 255/(0.4\times0.36\times20000)=0.0885$.
- Below the joint: $\nu_d = 502/(0.4\times0.36\times20000)=0.174$.

Below the joint, Eqs. (5.38a), (5.39a) apply, giving:

$\xi = (0.889\times0.174+1.111\times0.1896)/(0.889\times0.8095+2\times0.1896) = 0.3325$.

$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi[0.40476-0.33676\xi]+0.889\cdot0.0948+0.1896[(\xi-0.111)(1-\xi)-0.1281\xi^2]/0.889$, $M_{Rd,c}$= 217.9 kNm.

Above the joint, Eqs. (5.38b), (5.39b) apply:

$[1-0.002/(3\times0.0035)+0.1853\times(0.0035+0.00217)^2/(2\times0.0035\times0.00217\times(1-0.111)]\xi^2$

$-[0.0885+0.0927\times(1-0.0035/0.00217)+0.1896\times(1+0.111\times0.0035/0.00217)/(1-0.111)]\xi-$

$[0.0948-0.5\times0.1896\times0.111/(1-0.111)]\times0.111\times0.0035/0.00217=0 \rightarrow 1.2609\xi^2-0.28185\xi-0.01485=0$

$\rightarrow \xi = 0.2675$

Eq. (5.38b) gives: $\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi[0.40476-0.33676\xi]+0.0421\left(1+1.613\frac{\xi-0.111}{\xi}\right)+ 0.05332[1.62\xi-0.111]\left[1+1.613\left(\frac{\xi-0.111}{\xi}\right)\right][0.963-1.08\xi]$ $M_{Rd,c}$= 195.5 kNm

$\sum M_{Rd,c} = 413.4$ kNm > $1.3\sum M_{Rd,b} = 370.4$ kNm.

b) Exterior column

$\delta_1 = d_1/d = 40/310 = 0.129$

At the 1st storey and the base of the 2nd storey.

For max$N$: $\nu_d = 350.9/(0.35\times0.31\times20000)=0.1617$; $\mu_d = 119.3/(0.35\times0.31^2\times20000) = 0.1773$.

For min$N$: $\nu_d = 140.6/(0.35\times0.31\times20000)=0.0648$; $\mu_d = 95.4/(0.35\times0.31^2\times20000) = 0.1418$.

At the 2nd storey and the base of the 3rd storey.

For max$N$: $\nu_d = 193.8/(0.35\times0.31\times20000)=0.0893$; $\mu_d = 113.8/(0.35\times0.31^2\times20000) = 0.1692$.

For min$N$: $\nu_d = 99.2/(0.35\times0.31\times20000)=0.0457$; $\mu_d = 65.9/(0.35\times0.31^2\times20000) = 0.098$.

The limit per Eqs. (5.33a), (5.33b) is $\nu_2 = 0.129\times(0.0035-0.002/3)/(0.0035-0.00217) = 0.2748$, and exceeds all values of v_d above; therefore, Eqs. (5.34b), (5.35b) apply; they take the form:

$$0.4355\,\omega_{1d}\left(1+1.613\,\frac{\xi-0.129}{\xi}\right)=\mu_d-\xi\left[0.4048-0.3367\,\xi\right],$$

$$0.8095\xi^2-\left[v_d-0.613\,\omega_{1d}\right]\xi-0.208\omega_{1d}=0$$

They are solved iteratively, giving:

- At the 1st storey and the base of the 2nd storey:

For max*N*: ω_{1d}=0.123; for min*N*: ω_{1d}=0.121; A_{s1}=0.123×(310×350)×20/434.8=614 mm^2. We place 3Φ16 per side (with the mid-side bar restrained at the corner of a cross-tie or hoop), i.e. 8Φ16 total (1608 mm^2), giving: ρ=1608/(350×350)=0.0131>ρ_{min}=0.01. This reinforcement gives:

$\omega_{1d}=\omega_{2d}$=0.25×1608/(350×310)×434.8/20 = 0.0805, $\omega_{vd}=2\omega_{1d}$=0.1611.

For max*N*:

- Above the joint: v_d= 267/(0.35×0.31×20000)=0.123.
- Below the joint: v_d= 434.8/(0.35×0.31×20000)=0.20.

For min*N*:

- Above the joint: v_d= 127/(0.35×0.31×20000)=0.0585.
- Below the joint: v_d= 154.2/(0.35×0.31×20000)=0.071.

The values of v_d for max*N* and min*N* are less than the limit of Eq. (5.37b):

v_2 = 0.1611×[0.129×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.129)+0.129×(0.0035-0.002/3)/(0.0035-0.00217) = 0.1916.

Eq. (5.38b) applies and takes the form:

[1-0.002/(3×0.0035)+0.1611×(0.0035+0.00217)^2/(2×0.0035×0.00217×(1-0.129)]ξ^2 -[v_d+0.0805×(1-0.0035/0.00217)+0.1611×(1+0.129×0.0035/0.00217)/(1-0.129)]ξ-[0.0805-0.5×0.1611×0.129/(1-0.129)]×0.129×0.0035/0.00217=0 → 1.201ξ^2-[v_d+0.1741]ξ-0.01427=0

Eq. (5.38b) gives:

$$\frac{M_{Rd,c}}{bd^2f_{cd}}=\xi\left[0.40476-0.33676\xi\right]+0.03506\left(1+1.613\frac{\xi-0.129}{\xi}\right)+0.04624\left[1.62\xi-0.129\right]\left[1+1.613\left(\frac{\xi-0.129}{\xi}\right)\right]\left[0.957-1.08\xi\right]$$

For max$N$:

- Above the joint: v_d= 0.123, ξ= 0.2885, $M_{Rd,c}$= 117.2 kNm.
- Below the joint: v_d= 0.20, ξ= 0.346, $M_{Rd,c}$= 130.2 kNm

$\sum M_{Rd,c}$ = 247.4 kNm > 1.3$\sum M_{Rd,b}$ = 238.6 kNm

For min$N$:

- Above the joint: v_d= 0.0585, ξ= 0.2425, $M_{Rd,c}$= 104.1 kNm
- Below the joint: v_d= 0.071, ξ= 0.2515, $M_{Rd,c}$= 106.8 kNm

$\sum M_{Rd,c}$ = 210.9 kNm > 1.3$\sum M_{Rd,b}$ = 190.8 kNm

- At the 2nd storey and the base of the 3rd storey:

For max$N$: ω_{1d}=0.1417; for min$N$: ω_{1d}=0.0807; A_{s1}=0.1417×(310×350)×20/434.8=707 mm^2.

We keep the same reinforcement as in the 1st storey: 8Φ16: 1608 mm^2 (ω_{vd} = 0.1611, ω_{1d} = ω_{2d} = 0.0805). So, the only additional information to be computed on the basis of the above is the moment resistance for the axial load values above the joint. Whatever has been computed for the section above the 1st storey joint applies here at the section below the 2nd storey joint:

For max$N$:

- Above the joint: v_d= 120.6/(0.35×0.31×20000)=0.056, ξ= 0.241, $M_{Rd,c}$= 103.6 kNm
- Below the joint: v_d= 267/(0.35×0.31×20000)=0.123, ξ= 0.2885, $M_{Rd,c}$= 117.2 kNm

$\sum M_{Rd,c}$ = 220.8 kNm ~ 1.3$\sum M_{Rd,b}$ = 227.6 kNm

(The minor shortfall is ignored, to avoid increasing the reinforcement over what is placed in the 1st storey).

For min$N$:

- Above the joint: v_d= 71.4/(0.35×0.31×20000)=0.033, ξ= 0.224, $M_{Rd,c}$= 98.5 kNm
- Below the joint: v_d= 127/(0.35×0.31×20000)=0.0585, ξ= 0.2425, $M_{Rd,c}$= 104.2 kNm

$\sum M_{Rd,c}$ = 212.7 kNm > 1.3$\sum M_{Rd,b}$ = 131.8 kNm

5) Capacity design shears in the beams and the columns of the 1st and 2nd storey

The moment resistance of beams and columns around joints are summarised in the tables below. Note that the direction of the seismic action which causes compression and maximum axial force in an exterior column is associated with tension at the top flange of the beam end connected to that column ($M_{Rd,b}^-$) and at the bottom flange at the opposite end ($M_{Rd,b}^+$).

Moment resistance at beam end sections

Storey	1st				2nd			
Beam end at:	Outer column		Central column		Outer column		Central column	
Earthquake causes in column:	max$N$	min$N$	max$N$	min$N$	max$N$	min$N$	max$N$	min$N$
Moment resistance, kNm	$M_{Rd,b}^-$	$M_{Rd,b}^+$	$M_{Rd,b}^+$	$M_{Rd,b}^-$	$M_{Rd,b}^-$	$M_{Rd,b}^+$	$M_{Rd,b}^+$	$M_{Rd,b}^-$
	183.5	146.8	146.8	230	175	101.4	101.4	183.5

Column moment resistance $M_{Rd,c}$ (kNm) around joints

	1st storey joints			2nd storey joints		
Column	Exterior		Central	Exterior		Central
Seismic action direction causes:	max$N$	min$N$		max$N$	min$N$	
Above joint	117.2	104.1	256.1	103.6	98.5	195.5
Below joint	130.2	106.8	270.4	117.2	104.2	217.9

Assuming one-way frame action, the column axial forces balance the shear forces of the 2-bay frame alone. Then, the shears at beam ends due to the quasi-permanent gravity loads may be calculated from the column axial forces. By subtracting from these shears the part due to the moment difference between the two beam ends, we can estimate the shear force induced by the gravity loads on the beam, $V_{g+\psi q,o}$, with the beam considered as simply supported. The calculation is carried out in the first part of the table below; in the second part, the maximum capacity design shear at a beam end is estimated from Eq. (5.42), considering that a hogging plastic hinge forms at the beam end in question and a negative one at the opposite end. Recall that plastic hinges form at beam ends (even in the case of the 2nd storey exterior joints under the action which causes max$N$ in the columns).

Beam capacity design shears (kN)

Storey	1st		2nd	
Beam end	Exterior column	Central column	Exterior column	Central column
Shear, $V_{g+\psi q}$	294.5-197=97.5	(754.3-502)/2=126.2	197-96=101	(502-255)/2=123.5
V due to end moments	(32.8-71.3)/4.625= -8.3	(71.3-32.8)/4.625= 8.3	(46.1-65.1)/4.625= -4.1	(65.1-46.1)/4.625= 4.1
$V_{g+\psi q,o}$ in simply supported beam	97.5-(-8.3) =105.8	126.2-8.3=117.9	101-(-4.1) =105.1	123.5-4.1=119.4
Shear due to plastic hinging	(183.5+146.8)/ 4.625 = 71.4	(230+146.8)/4.625 = 81.5	(175+101.4)/4.625 = 59.8	(183.5+101.4)/ 4.625 = 61.6
Capacity design shear	105.8+71.4=177.2	117.9+81.5=199.4	105.1+59.8=164.9	119.4+61.6=181

The column capacity design shears are estimated in the table below from Eq. (5.44).

Column capacity design shears

Storey		1st			2nd		
Column		Exterior		Central	Exterior		Central
Seismic action direction causing:		max$N$	min$N$		max$N$	min$N$	
Column top	$M_{Rd,c}$	127.7	103.9	270.4	117.2	104.2	217.9
	$\sum M_{Rd,b}$	183.5	146.8	376.8	175	101.4	284.9
	$\sum M_{Rd,c}$	247.4	211.0	526.5	220.8	202.7	413.4
Column base	$M_{Rd,c}$	127.7	103.9	270.4	117.2	104.2	256.1
	$\sum M_{Rd,b}$	-	-	-	183.5	146.8	376.8

	$\sum M_{Rd,c}$	-	-	-	244.9	208.1	526.5
Capacity design shear		97.9	77.5	204.1	79.5	55.2	146.7

6) Dimensioning of transverse reinforcement in the first storey beams.

In Part 3 of this Answer, it was estimated that the action effects in the persistent and transient design situation are equal to those due to the quasi-permanent gravity loads, $G+\psi_2 Q$, times 1.545. So, the shear forces at the "persistent and transient design situation" vary (presumably linearly) from 1.545×97.5= 150.6 kN at the face of the exterior support to 1.545×126.2=195 kN at the face of the central support.

The shear resistance at the face of the support for shear compression in the web is according to Eurocode 2:

$V_{Rd,max} = 0.3(1\text{-}f_{ck}(\text{MPa})/250)b_w z f_{cd}\sin 2\theta = 0.3\times(1\text{-}30/250)\times0.3\times0.9\times0.46\times20000\sin 2\theta = 655.8\sin 2\theta$

with $1 \leq \cot\theta \leq 2.5$.

Acting shears:

At the face of the exterior support: $V_{Ed}(0) = \max(150.6, 177.2) = 177.2$ kN < $V_{Rd,max} \rightarrow \theta > 8^o$

At the face of the central support: $V_{Ed}(0) = \max(195, 199.4) = 199.4$ kN < $V_{Rd,max} \rightarrow \theta > 9^o$

So, $\cot\theta = 2.5$, i.e. $\theta = 21.8^o$ over the full length of the beam.

– Shear reinforcement in the "critical regions" of the beam next to the column faces:

Design shear force at a distance h=0.5m from the support on the exterior column.

$V_{Ed}(0.5m)=71.4+105.8\times(1-0.5/(4.625/2)) = 154.3$ kN

Design shear force at a distance h=0.5 m from the support on the central column.

$V_{Ed}(0.5m)=81.5+117.9\times(1-0.5/(4.625/2)) = 173.9$ kN

For $V_{Rd,s}=b_w z \rho_w f_{ywd}\cot\theta = 0.3\times0.9\times0.46\times434800\times2.5\rho_w = 135000\rho_w > 173.9$ kN $\rightarrow \rho_w > 0.00129$, larger than the minimum stirrup ratio required by Eurocode 2: $\min\rho_w = 0.08\sqrt{f_{ck}(\text{MPa})}/f_{yk}(\text{MPa}) = 0.000876$.

mind_{bL} =16 mm, mind_{bw} = 6 mm and maxs_w=8d_{bL}=128 mm: 6 mm-dia. stirrups at 125 mm centres, in the “critical region” next to the exterior column, giving ρ_w=2×28.3/(125×300)= 0.00151 > 0.00129. According to Eurocode 8, the 1st stirrup is not further than 50 mm from the face of the column; so five 6 mm-dia. stirrups @ 125 mm centres extend up to 500 mm from the face of the exterior column. Similar stirrup reinforcement (6 mm stirrups @ 125 mm centres) is placed in the “critical region” next to the exterior column.

– Shear reinforcement between the “critical regions” at the two ends of the beam:

The length of the beam outside the “critical regions” is: 4.625-2×0.50=3.625m.

Design shear force at a distance $z\cot\theta$ = 0.9×0.46×2.5=1.035 m from last stirrup of the "critical region", i.e., 1.535 m from the face of the support:

V_{Ed}(1.535 m)=81.5+117.9×(1-1.535/(4.625/2)) = 121 kN

There, the minimum stirrup spacing according to Eurocode 2 is 0.75d=0.75×460=345 mm. 18 6 mm-dia. stirrups at 200 mm centres cover that length, giving ρ_w= 2×28.3/(200×300) =0.000943 > minρ_w=0.000876 and providing shear resistance: $V_{Rd,s}$= 135000ρ_w = 127.5 kN which exceeds the maximum design shear V_{Ed}(1.535 m)=121 kN outside the “critical regions”.

7) Dimensioning and detailing of transverse reinforcement of the first storey columns, including confinement at the base.

a) Central column:

Outside the critical regions, the maximum stirrup spacing that allows lap-splicing is ≤ 12d_{bL}=12×16 = 192 mm, 0.6h_c = 0.6×400 = 240 mm, 240 mm, i.e., the minimum is Φ6/190 (149 mm^2/m per stirrup leg). In addition to the perimeter tie, a diamond-shaped one is placed, engaging the four mid-side bars. The shear resistance provided by these stirrups is:

$V_{Rd,s}$ = 754.3×0.9×0.36/2.5+0.9×0.36×149×0.4348×(2+√2)cotθ= 97.8+71.7cotθ (where the first term is the contribution of the axial force to shear resistance and the parenthesis at the end is the effective no. of stirrup legs).

$V_{\text{Rd,max}} = 0.3\times(1\text{-}30/250)\times0.4\times0.9\times0.36\times(1+754.3/(0.4^2\times20000))\times20000\sin2\theta = 845\sin2\theta$;

For $\cot\theta = 2.5$: $V_{\text{Rd,max}} = 583$ kN, $V_{\text{Rd,s}} = 277$ kN, both above the capacity design shear of $V_{\text{Ed}} = 204.1$ kN.

The critical region length should be $\geq h_c = 0.4$ m, ≥ 0.45 m and $\geq H_{cl}/6 = 0.417$ m, i.e., 0.45 m. For a distance from the end sections equal to the critical region length:

- stirrup diameter, $d_{bw} \geq 6$ mm, $0.25d_{bL} = 0.25\times16 = 4$ mm;
- spacing $\leq 8d_{bL} = 8\times16 = 128$ mm; $b_o/2 = (400\text{-}2\times25\text{-}8)/2 = 171$ mm.

Stirrups chosen are Φ6/125 (226 mm^2/m per stirrup leg). There is no need to check the shear resistance provided by these ties, because the lighter arrangement of Φ6/190 chosen outside the critical regions is sufficient for the same design shears along the column.

In the critical region at the base the transverse reinforcement should also provide confinement through an effective mechanical ratio satisfying the conditions:

$\omega_{wd} > 0.08$, and

$a\omega_{wd} > 30\mu_\phi \nu_d \varepsilon_{yd} b_c/b_o - 0.035$,

where $\varepsilon_{yd} = 0.00217$, $\mu_\phi = 2(q_o\text{-}1)T_C/T+1 = 2\times(3.9\text{-}1)\times0.6/0.565+1 = 7.16$, $b_c = 0.4$ m, $b_o = 0.4\text{-}2\times0.025\text{-}0.008 = 0.342$ m, $\nu_d = 754.3/(0.4^2\times20000) = 0.236$,

i.e., $a\omega_{wd} > 30\times7.16\times0.236\times0.00217\times0.4/0.342 - 0.035 = 0.0937$.

The pattern of Φ6/125 stirrups adopted for the critical regions above the base gives:

$a = (1\text{-}0.5\times125/342)^2[1\text{-}8/(4\times6)] = 0.445$

$\omega_{wd} = 2\times(2+\sqrt{2})\times226/(342\times1000)\times(434.8/20) = 0.0981 > 0.08$ and

$a\omega_{wd} = 0.445\times0.0981 = 0.0437 < 0.0937$. So, specifically in the critical region at the base, the stirrups increase to Φ8/100 (503 mm^2/m per stirrup leg), giving:

$a = (1\text{-}0.5\times100/342)^2[1\text{-}8/(4\times6)] = 0.486$;

$\omega_{wd} = 2\times(2+\sqrt{2})\times503/(342\times1000)\times(434.8/20) = 0.218$, and

$a\omega_{wd} = 0.486\times0.218 = 0.1059 > 0.0937$.

So, the stirrups of the 1st storey central column are:

- $\Phi 6/190$ between the two 0.45 m long end regions,
- $\Phi 6/125$ in the end region at the top of the column and
- $\Phi 8/100$ in the end region at the base.

b) Exterior column:

The same minimum reinforcement applies and the same stirrups are chosen as in the central column: $\Phi 6/190$ (149 mm^2/m per stirrup leg), in the form of a perimeter tie and a diamond-shaped one, engaging the four mid-side bars. The shear resistance provided by these stirrups is:

- For max$N$ = 434.8 kN:

  $V_{Rd,s}$ = 434.8×0.9×0.31/2.5+0.9×0.31×149×0.4348×(2+√2)cotθ= 48.5+61.7cotθ;

  $V_{Rd,max}$ = 0.3×(1-30/250)×0.35×0.9×0.31×20000×(1+434.8/(0.35^2×20000))×sin2θ = 608sin2θ;

  for cotθ = 2.5: $V_{Rd,s}$ = 202.8 kN, $V_{Rd,max}$ = 419.3 kN, both well above the capacity design shear of V_{Ed} = 97.9 kN.

- For min$N$ = 154.2 kN:

  $V_{Rd,s}$ = 154.2×0.9×0.31/2.5+0.9×0.31×149×0.4348×(2+√2)cotθ= 17.2+61.7cotθ,

  $V_{Rd,max}$ = 516×(1+154.2/(0.35^2×20000))sin2θ=548sin2θ

  for cotθ = 2.5: $V_{Rd,s}$ = 171.5 kN, $V_{Rd,max}$ = 378 kN, both well above the capacity design shear of V_{Ed} = 77.5 kN.

The critical region length should be ≥ h_c= 0.35 m, ≥ 0.45 m and ≥ $H_{cl}/6$ = 0.417 m, i.e., 0.45 m. For a distance from the end sections equal to the critical region length:

- stirrup diameter, $d_{bw} \geq$ 6 mm, 0.25d_{bL}= 0.25×16 = 4 mm;
- spacing ≤ 6d_{bL}=8×16 = 128 mm; b_o/2= (350-2×25-8)/2= 146 mm.

The stirrups chosen are again $\Phi 6/125$ (226 mm^2/m per stirrup leg), with the same pattern of a perimeter tie and a diamond-shaped one engaging the four mid-side bars. The shear verification is ommited, because the lighter arrangement of $\Phi 6/190$ chosen outside the critical regions suffices for the same design shears along the column.

The confinement requirements are larger for max$N$ = 434.8 kN, i.e., v_d =434.8/(0.35^2×20000) = 0.1775:

i.e., $a\omega_{wd}$ > 30×7.16×0.1775×0.00217×0.35/0.292-0.035= 0.0642.

The pattern of Φ6/125 stirrups adopted for the critical regions above the base is not sufficient. It is changed in the critical region to Φ8/125 (402 mm^2/m per stirrup leg), giving:

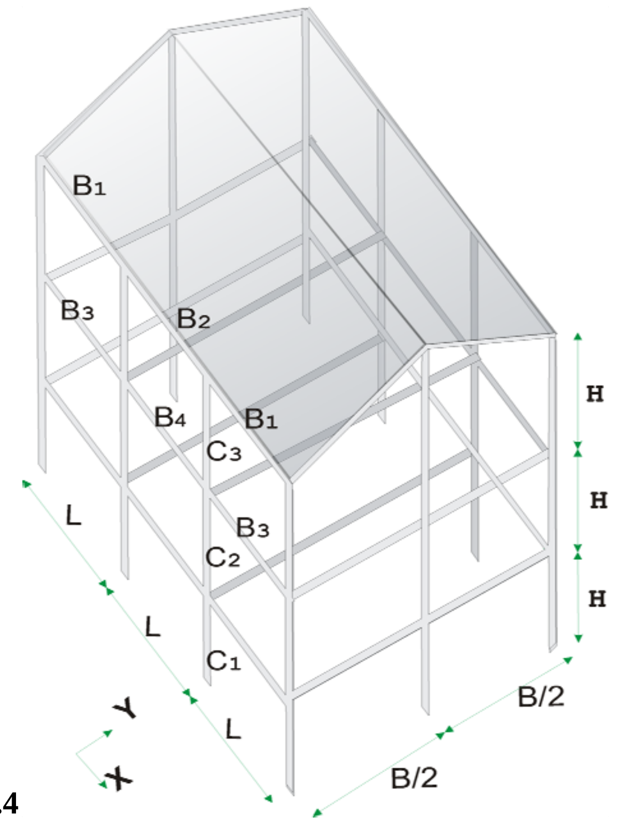
a = (1-0.5×125/292)^2[1-8/(4×6)] = 0.412

ω_{wd} = 2×(2+√2)×402/(292×1000)×(434.8/20)=0.2044 > 0.08

$a\omega_{wd}$= 0.412×0.2044 = 0.0842 > 0.0642.

The stirrups of the 1st storey exterior column are:

- Φ6/190 between the two 0.45 m long end regions,
- Φ6/125 in the end region at the top of the column and
- Φ8/125 in the end region at the base.



Question 5.4

For the building shown in Figure 5.21:

- The seismic action in direction X is considered to be resisted by the two exterior 3-bay frames alone. Seismic forces are applied at floor levels and at the lowest level of the roof; they are derived from the masses and a presumed inverted triangular pattern of horizontal displacements.The two interior columns of these frames have twice the moment of inertia of the corner ones and take twice as large seismic shears as the corner columns; hence the seismic moments at the two ends of the beams of that frame are numerically equal across all three spans of a floor. The columns of the two X-direction frames may be considered to develop zero seismic moment (inflection point) at storey mid-height. At the top they are fixed against rotation within the X-direction vertical plane, because the sloping roof works with the type B1, B2 perimeter beams as a very wide, inclined flange, imparting to these beams very high stiffness and flexural resistance for bending in the plane of the X-direction frames; for that reason, column C3 and the like cannot escape from plastic hinging at the top under strong seismic action in direction X.

- The pitched roof is supported by beams only along the perimeter. Its ridge is a non-deflecting support of the two roof slabs on either side. These slabs are one-way and, by in-plane action, transfer to the perimeter beams which are parallel to the ridge the vertical reaction to gravity loads which would normally go to the support along the ridge. So, the full gravity load of the roof goes to beams B1 and B2. Under a uniform load p (kN/m), these beams develop bending moments at the interior supports equal to $0.1pL^2$, and zero at the end supports.
- Gravity loads go to the closest column in plan, but may be taken to induce no bending moments to columns. Floor slabs are one-way and can be taken to be supported only on their long sides in plan: floor beams B3, B4 and the like are considered as unloaded by gravity loads.

1) Estimate the seismic moments and axial forces in the members, due to the seismic action in direction X.

2) Dimension the longitudinal reinforcement at the end sections of the second floor beams B3 and B4 and of the roof beams B1 and B2.

3) Dimension the vertical reinforcement of the third and second floor columns, C3 and C2, to meet the strong column/weak beam capacity design rule around its joint with beams B3, B4 and resist at the top the seismic moments from the analysis for earthquake in direction X.

4) Calculate the capacity design shears of the second storey beam B4 and of the third storey column C3 in the plane of the exterior X-direction frame.

- Ductility Class H (High).
- Base shear in direction X: 25% of the weight of the building.
- Bay lengths: $L = 5.0$ m, $B = 11$ m. Storey height $H = 3.6$ m. Roof slope to the horizontal: 12°.
- Concrete C25/30, S500 steel. Cover of reinforcement 30 mm.
- Permanent loads (all inclusive): for the roof 8 kN/m^2 per m^2 of horizontal projection; for the floors 9 kN/m^2.
- Live loads: 2 kN/m^2 on the floors; zero on the roof.

- ψ_2=0.30
- Beam width 0.3 m and depth 0.5 m. Slab thickness 0.16 m.
- Interior columns: 0.6 m square; corner ones: 0.5 m square.
- Curvature ductility demand for detailing: $\mu_\varphi = 2q_o - 1$, where q_o is the behaviour factor appropriate for the building.

Answer of Question 5.4:

1) Effects of the seismic action in direction X:

The total quasi-permanent loads are:

On the roof: W_3 = 8×11×15= 1320 kN; on each floor: $W_1 = W_2$ = (9+0.3×2)×11×15= 1584 kN.

Total: W = 1320+2×1584=4488 kN

Total base shear in X: 0.25×4488 = 1122 kN.

Seismic forces at the three levels:

f_3= 1122×3×1320/(3×1320+2×1584+1×1584) = 510 kN;

f_2= 1122×2×1584/(3×1320+2×1584+1×1584)= 408 kN;

f_1= 1122×1584 /(3×1320+2×1584+1×1584) = 204 kN.

Storey seismic shears:

V_3= 510 kN;

V_2= 510+408 = 918 kN;

V_1= 918+204 = 1122 kN.

Seismic shears in an interior column, presuming that the share of a corner column to the storey shear is half that of an interior column and taking into account that the two exterior frames share the total seismic shear:

$V_{Ec,3}$= 0.5×510/3 = 85 kN;

$V_{Ec,2}$= 0.5×918/3 = 153 kN;

$V_{Ec,1}$= 0.5×1122/3 = 187 kN.

Bending moments at interior column ends, for point of inflection presumed to be at storey mid-height:

$M_{Ec,3}$= $V_{Ec,3}H/2$ = 85×3.6/2 = 153 kNm;

$M_{Ec,2}$= $V_{Ec,2}H/2$ = 153×3.6/2 = 275.4 kNm;

$M_{Ec,1}$= $V_{Ec,1}H/2$ = 187×3.6/2 = 336.6 kNm.

Corner columns take just half of the above seismic shears and moments.

Seismic moments at beam ends:

$M_{Eb,2}$= ($M_{Ec,3}$+$M_{Ec,2}$)/2 = 214.2 kNm;

$M_{Eb,1}$= ($M_{Ec,2}$+$M_{Ec,1}$)/2 = 306 kNm.

Beam seismic moments at the face of the supporting column (neglecting the different size of corner columns):

$M_{Eb,2d}$= (1-h_c/L_{cl})$M_{Eb,2}$ = (1-0.6/5)×214.2 = 188.5 kNm;

$M_{Eb,1d}$= (1-h_c/L_{cl})$M_{Eb,1}$ = (1-0.6/5)×306 = 269.3 kNm.

For earthquake in the X-direction, the 3rd-storey beams B1, B2 work as integral with the roof, with the roof playing the role of a deep, inclined flange. The column moments $M_{Ec,3}$ computed above are taken by the inclined roof slab, working together with beams B1, B2.

The columns at mid-length of the Y-direction sides take a negligible share of the X-direction seismic shear, because they are not connected to any beams in that direction. If their tops are considered as fixed against rotation by the large rigidity which the sloping roof presents to bending in a vertical plane parallel to X, these columns may at most be considered as free-standing over a height of 3×3.6+5.5tan(12^o) = 12 m, with double fixity at top and bottom. For such a height, their stiffness – and uptake of seismic forces – in direction X may be neglected. By the same token, their contribution to the uptake of the overturning moment is neglected.

The corner columns develop axial forces resisting the seismic overturning moment; these forces are computed by considering equilibrium of moments at a horizontal section through storey mid-height, where the column moments are assumed to be zero:

Storey 3: Overturning moment: $f_3H/2$ = 510×3.6/2 = 918 kNm, corner column axial forces: $N_{Ec,3}$ = 0.5×918/15=30.6 kN;

Storey 2: Overturning moment: $f_3(3H/2)+f_2H/2$ = 510×3×3.6/2 + 408×3.6/2 = 3488.4 kNm, corner column axial forces: $N_{Ec,2}$ = 0.5×3488.4/15=116.3 kN;

Storey 1: Overturning moment: $f_3(5H/2)+f_2(3H/2)+f_1H/2$ = 510×5×3.6/2 + 408×3×3.6/2 +204×3.6/2 = 7160.4 kNm, corner column axial forces: $N_{Ec,1}$ = 0.5×7160.4/15= 238.7 kN.

2) Dimensioning of the longitudinal reinforcement of the X-direction beams:

Parameters:

f_{cd}=25/1.5=16.67 MPa; f_{ctm}=2.6 MPa, E_c=31000000 kPa;

f_{yd}=500/1.15=434.8 MPa; ε_{yd}=434.8/200000=0.217%;

q_o = 4.5×1.3= 5.85, $\mu_\varphi = 2q_o$ -1= 10.7

Distance of centre of longitudinal bars from nearest concrete surface:

$d_1=c+d_{bh}+d_{bL}/2$~0.030+0.006+0.014/2 ~0.045m.

$\rho_{min}=0.5f_{ctm}/f_{yk}$=0.5×2.6/500=0.0026

a) Beams B1, B2:

Maximum diameter of bars, d_{bL}, at the support by a corner column, Eq. (5.2b):

min$N$=8×2.5×5.5-30.6 = 79.4 kN, ν_d = 79.4/(0.5^2×16667)=0.019: $d_{bL}/h_{c,ext} \leq$ 7.5×1.015×2.6/(1.2×434.8) = 0.038, $d_{bL} \leq$ 0.038×500 = 19 mm

Maximum diameter of bars, d_{bL}, at the support on an intermediate exterior column, Eq. (5.2a):

min$N$=8×5×5.5 = 220 kN.

For ν_d = 220/(0.6^2×16667)=0.0367 and $\rho'=0.5\rho_{max}$: $d_{bL}/h_{c,ext} \leq$ 7.5×1.029×2.6/(1.2×1.375×434.8) = 0.028, $d_{bL} \leq$ 0.028×600 = 17 mm.

The roof works as a folded plate, supported in an one-way fashion by the ridge and by beams B1, B2. These beams take the ridge's share of the vertical reactions of the roof slabs and are subjected to a uniform line load due to the permanent loads equal to $p = 5.5\times8 = 44$ kN/m. It makes little sense to design these beams for seismic moments due to earthquake in direction X, as though the roof slab were horizontal. These moments are taken by the roof slab, which works as an inclined deep beam. Therefore, beams B1, B2 are designed for gravity loads alone, and indeed for the factored gravity load, 1.35G= $1.35p = 59.4$ kN/m. Beams B1, B2 may be dimensioned for the resulting moments as T-beams. Controlling, in this respect, is the direction of the flange; so the vertical load of 59.4 kN/m is analysed into two components: $59.4\cos(12^o)=58.1$ kN/m, at right angles to the roof slab, and $59.4\sin(12^o) = 12.4$ kN/m, within the plane of the roof slab. The second component is taken by in-plane action in the slab and is of no interest. The first one causes bending of the beams about an axis parallel to the roof slab; it is for this bending action that these beams are dimensioned in flexure. To this end, we should take as depth of the beam the projection of its real depth onto the normal to the roof slab: $0.5\cos(12^o) = 0.49$ m, the effective depth is computed similarly: $(500-30-6-16/2)\cos(12^o) = 445$ mm; the effective width of its web is the width parallel to the roof slab: $0.3/\cos(12^o) = 0.307$ m. $A_{s,min} = 0.0026\times307\times445 = 355$ mm^2; bottom: 3Φ14 (462 mm^2), top: 2Φ16 (402 mm^2).

The hogging moment at interior supports, with vector parallel to the roof slab, is: $0.1\times58.1\times5^2 = 145$ kNm; it requires 860 mm^2 of top reinforcement: 2Φ16 ($A_{s,min}$)+3Φ14 (additional), in total 864 mm^2, giving: $\rho = 864/(307\times445) = 0.00632$, $\rho' = 462/(307\times445) = 0.00338$, $\rho_{max}=\rho'+0.0018 f_{cd}/(\mu_\varphi \varepsilon_{yd} f_{yd})=$ $0.00338 + 0.0018\times16.667/(10.7\times0.00217\times434.8)= 0.00635 > \rho = 0.00632$.

The detailing rule: $A_{s,bottom}$ (in this case 462 mm^2) > $0.5A_{s,top}$ (864 mm^2) is satisfied.

b) Beams B3, B4:

Unlike B1 and B2, beams B3, B4 bear little gravity loads (neglected in this case). Their bending is exclusively due to the seismic action. To design them in flexure, we first have to estimate the effects of the seismic action component in direction X.

Maximum diameter of bars, d_{bL}, at the support of B3 on a corner column, Eq. (5.2b):

$\min N=(8+9.6)\times2.5\times5.5 - 116.3 = 125.7$ kN, $\nu_d = 125.7/(0.5^2\times166667)=0.03$: $d_{bL}/h_{c,ext} \leq$

$7.5\times1.024\times2.6/(1.2\times434.8) = 0.0384$, $d_{bL} \leq 0.0384\times500 = 19$ mm

Maximum diameter of bars, d_{bL}, at the support on an exterior column, Eq. (5.2a):

min$N$=(8+9.6)×5×5.5 = 484 kN.

For $\nu_d = 484/(0.6^2\times16667)=0.0807$ and $\rho'=0.5\rho_{max}$: $d_{bL}/h_{c,ext} \leq 7.5\times1.065\times2.6/(1.2\times1.375\times434.8) =$

0.029, $d_{bL} \leq 0.029\times600 = 17.5$ mm.

$A_{s,min} = 0.0026\times300\times455 = 355$ mm^2; bottom: 2Φ16 (402 mm^2).

The design moments for beams B3, B4 are: $\pm M_{Eb,2d} = \pm188.5$ kNm, for which 1124 mm^2 are needed; provided at top and bottom: 2Φ16 ($A_{s,min}$)+5Φ14 (additional), in total 1172 mm^2. There is no need to check ρ against ρ_{max}, because $\rho = \rho'$.

$M_{Rd,b}^{-} = M_{Rd,b}^{+} = 1172\times0.9\times0.455\times434.8/10^3 = 208.7$ kNm

3) Design of 2nd and 3rd storey interior columns in flexure.

Distance of centre of vertical bars from the nearest concrete surface: $d_1 = c+d_{bh}+d_{bL}/2 \sim 0.045$ m.

Minimum number of bars per side, to respect the maximum spacing of 150 mm between bars engaged at a stirrup corner or cross-tie: 5 (total 16); $A_{s,min} = 0.01\times600\times600 = 3600$ mm^2.

Minimum reinforcement: 4Φ20 (at the corners) + 12Φ16 (intermediate bars along the sides): 3668 mm^2; $\rho = 3668/(600\times600) = 0.0102 > 0.01$.

At 3rd storey: $N = 8\times5\times5.5 = 220$ kN.

At 2nd storey: $N = (8+9.6)\times5\times5.5 = 484$ kN.

Design moment at the top of column C3 (3rd storey):

$M_{Ec,3} = V_{Ec,3}H/2 = 85\times3.6/2 = 153$ kNm.

Capacity design moment input at the joint of columns C3 and C2 with beams B3 and B4:

$1.3\sum M_{Rd,b} = 1.3\times2\times208.7$ kNm $= 542.6$ kNm

As the column section is large, its moment resistance with the minimum reinforcement may suffice for the moment demands above. It is computed below for the 2nd and 3rd storeys:

$d = 600-45 = 555$ mm; $d_1 = 45$ mm, $\delta_1 = d_1/d = 45/555 = 0.081$, In order to have the "web" reinforcement uniformly spread between ω_{1d} and ω_{2d}, half the area of 1Φ16 is subtracted from each corner bar and counts in ω_{vd}. So:

$\omega_{1d}=\omega_{2d}$ = (2×314+2×201)/(600×555)×434.8/16.67 = 0.0807,

ω_{vd} = 2×4×201/(600×555)×434.8/16.67 = 0.126.

The limit in Eqs. (5.37a), (5.37b) is:

v_2 = 0.126×[0.081×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.081)+ 0.081×(0.0035-0.002/3)/(0.0035-0.00217) = 0.17256-0.0898=0.0828

Top storey: v_d= 220/(0.6×0.555×16667)=0.0396 < 0.0828 .

Second storey: v_d= 484/(0.6×0.555×16667)=0.0872 > 0.0828.

Eq. (5.39b) applies to the top storey column, in the form:

$[1-0.002/(3\times0.0035)+ 0.126\times(0.0035+0.00217)^2/(2\times0.0035\times0.00217\times(1-0.081)]\xi^2$

$-[0.0396+0.0807\times(1-0.0035/0.00217)+ 0.126\times(1+0.081\times0.0035/0.00217)/(1-0.081)]\xi-$

$[0.0807-0.5\times0.126\times0.081/(1-0.081)]\times0.081\times0.0035/0.00217=0 \rightarrow 1.1\xi^2-0.1452\xi-0.00982=0 \rightarrow$

$\xi=0.1812$

Eq. (5.38b) gives: $\frac{M_{Rd,c}}{bd^2 f_{cd}}=\xi[0.40476-0.33676\xi]+0.5\cdot(1-0.081)\cdot0.0807\left(1+1.613\frac{\xi-0.081}{\xi}\right)+0.25\cdot0.126[1.62\xi-0.081]\left[1+1.613\left(\frac{\xi-0.081}{\xi}\right)\right][1-0.081/3-1.08\xi]/(1-0.081)$ = 0.143, from which $M_{Rd,c}$ = 441 kNm.

For v_d= 0.0872, Eq. (5.39a) applies to the top storey column:

ξ = (0.919v_d+1.081×0.126)/(0.919×0.8095+2×0.126)=0.217, and

$\frac{M_{Rd,c}}{bd^2 f_{cd}}=\xi[0.40476-0.33676\xi]+0.0807\cdot(1-0.081)+0.126[(\xi-0.081)(1-\xi)-0.1281\xi^2]/(1-0.081)$ = 0.16, $M_{Rd,c}$ = 492.6 kNm

$\sum M_{Rd,c}$ =492.6+441=933.6 > 1.3$\sum M_{Rd,b}$=542.6 kNm

The minimum reinforcement suffices in both storeys.

4) Capacity design shears:

Beam B4:

As $\sum M_{\mathrm{Rd,c}}$ = 933.6 kNm > $\sum M_{\mathrm{Rd,b}}$ = 417.4 kNm at both ends of the beam, the capacity design shear is:

max$V_{\mathrm{CD,B4}}$ = 1.2×2×208.7/4.4 = 113.8 kN, min$V_{\mathrm{CD,B2}}$ = -113.8 kN, ζ = -1 (full reversal of the shear).

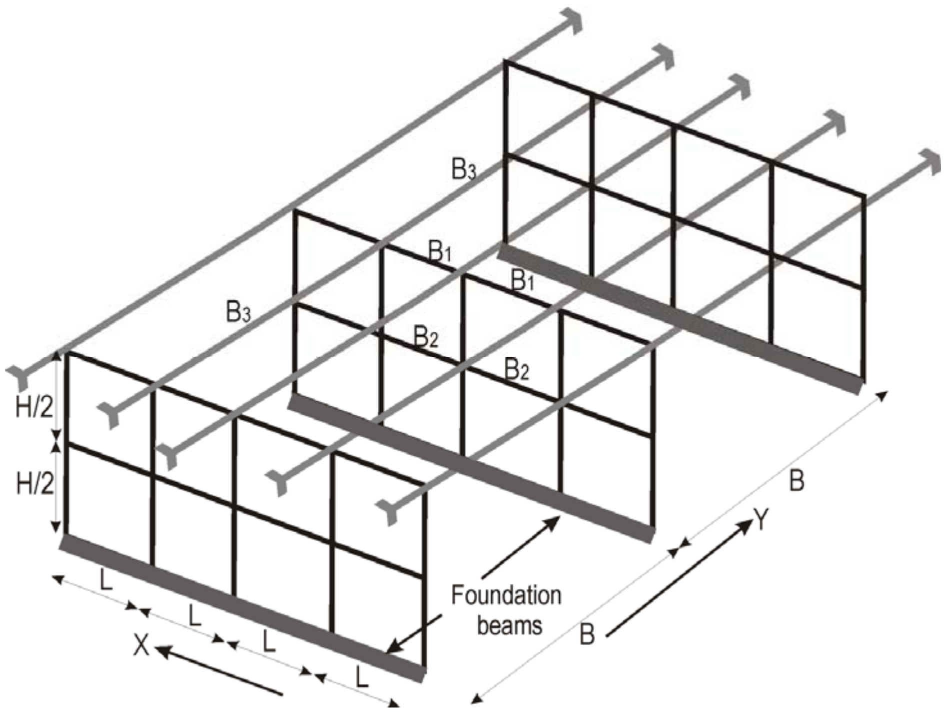
However, maxV_{CD} = 113.8 kN < $(2+\zeta)f_{\mathrm{ctd}}b_{\mathrm{w}}d$ =(2-1)×(0.7×2600/1.5)×0.3×0.455 = 165.6 kN → no need for diagonal or inclined shear reinforcement.

Column C3:

The column is fixed at the top by the sloping roof; so:

As $\sum M_{\mathrm{Rd,c}} > \sum M_{\mathrm{Rd,b}}$ at both ends of the beam, the capacity design shear is:

$V_{\mathrm{CD,C3}}$ = 1.3×(441+441×417.4/933.6)/3.1 = 267.6 kN.



Question 5.5

The building in Fig. 5.22 has many similar 4-bay, 2-storey frames in direction X. Column tops are connected in direction Y through beams of type B3, into five parallel Y-direction frames, each one with practically infinite, similar bays. There is a diaphragm only at roof level.

Simplifying assumptions:

- The self weight of beams and columns is neglected for all purposes.
- The roof comprises one-way slabs, supported only on the Y-direction beams B3; beams of type B1 may be taken as not loaded by the roof slabs.
- Under gravity loads, beams of type B3 are considered as fixed at the end section against rotation.
- Bending of columns due to gravity loads is ignored.
- The seismic action is considered to produce horizontal forces only at the roof level.
- The seismic action components in direction X and Y are taken to act separately, not concurrently.
- Columns take the horizontal seismic forces, as well as the gravity loads acting on the roof, in proportion to their tributary area in plan. Exterior columns have one-half the moment of inertia of interior ones; so, their share of the forces may indeed be assumed to be about half of that of interior columns.
- Under the seismic action, the inflection point (zero moment) of the columns is at the following fraction of the storey height from the base of the column in the storey:
  - In the one-storey frames along the Y-direction: $(6k_Y+1)/(12k_Y+1)$, where $k_Y=(EI)_{B3}/(EI)_{CY}(H/B)$, with $(EI)_{B3}$ denoting the rigidity of beam B3, $(EI)_{CY}$ that of an interior column for bending within a plane parallel to Y (strong axis) and B, H, the length of these elements.
  - In the two-storey frames along the X-direction:
    - at the lower storey: $(3k_{X2}+1)(12k_{X1}+1)/[(6k_{X2}+1)(12k_{X1}+1)-1/2]$,
    - at the upper storey: $[6k_{X2}(6k_{X1}+1)+1/2]/[(6k_{X2}+1)(12k_{X1}+1)-1/2]$,

    where $k_{X1}=(EI)_{B1}/(EI)_{CX}(H/2L)$, $k_{X2}=(EI)_{B2}/(EI)_{CX}(H/2L)$, with $(EI)_{B1}$, $(EI)_{B2}$ denoting the rigidity of beams B1 and B2, $(EI)_{CX}$ that of an interior column for bending within a plane parallel to X (weak axis) and L, $H/2$ the length of these elements.
- The inflection points of the beams under seismic loading are always at mid-span.

- The effective flange width of roof beams B1 and B3 may be taken per Eurocode 2: on each side of the web where there is a slab: 10% of the distance of the beam from the nearest parallel one (but not greater than 7% of the beam span) plus another 7% of the beam span.

1) What is the value of the behaviour factor, q, of the building in directions X and Y according to Eurocode 8 for Ductility Class High (H)?

2) Calculate the fundamental periods of the building in directions X and Y, after establishing the stiffness of the corresponding single-degree-of-freedom (SDOF) system.

3) Using the outcomes of 1) and 2), compute the floor seismic forces for the design of the building in directions X and Y.

4) Calculate the interstorey drifts under the design seismic action in directions X and Y and use them to estimate the sensitivity coefficients to second-order effects and the interstorey drifts under a damage limitation earthquake equal to 50% of the design seismic action.

5) What is the use of X-direction beams of type B2 at building mid-height, since there are no slabs or seismic forces at that level?

6) Dimension the longitudinal reinforcement of interior beams B1, B2 and B3 at the supports.

7) Dimension the vertical reinforcement of an interior column, separately in directions X and Y, on the basis of the analysis results for the seismic forces in 3) above.

8) Calculate the capacity design shears at the ends of interior beams B1, B2, B3 and at both storeys of an interior column, in directions X and Y.

9) Dimension and detail the shear reinforcement of Beams B1, B2 and B3.

10) Dimension and detail the transverse reinforcement of an interior column

- Type 1 spectrum of Eurocode 8 for ground type E and design ground acceleration 0.42g.
- Ductility Class H (High).
- Bay lengths: L = 3.0 m, B = 10 m.
- Height to mid-depth of the roof slab, where the seismic forces are applied: H = 7 m.

- Concrete C35/45, S500 steel. Cover of reinforcement 25 mm.
- The roof slab is 160 mm thick and has only permanent loads: g = 6.5 kN/m^2
- Beams B1, B2: width 0.3 m; depth 0.40 m; beams B3: width 0.3 m; depth 0.50 m.
- Interior column: 0.35 m in direction X, 0.60 m in Y; Exterior column: 0.30 m in X, 0.50 m in Y.

Answer of Question 5.5:

1) The building is regular, in plan and in elevation. The dimensionless axial load at the base of a column is (6.5×3×10)/(0.35×0.6×23333)=195/4900=0.04 < 0.3, hence the building is not considered an inverted pendulum system, despite having all its mass at the top. In direction Y it is a one-storey multi-bay frame; hence α_u/α_1=1.1 and q_Y = 4.5×1.1=4.95. In direction X it is considered as a multi-bay frame with more than one storey, hence α_u/α_1=1.3 and q_X = 4.5×1.3=5.85.

2) Direction Y:

Beam B3: effective flange width on each side of the web: 0.1×3+0.07×10=1 m; total flange width: 2.3 m, I = 0.006759 m^4;

Column in direction Y: I = 0.35×0.6^3/12 = 0.0063 m^4

k_Y= (0.006759/0.0063)×(7/10)=0.751.

Point of inflection: $(6k_Y+1)H/(12k_Y+1)$ = 0.55×7 = 3.85 m from the base, $6k_YH/(12k_Y+1)$ = 3.15 m from the top. Seismic moments: 3.85V at the bottom, 3.15V at the top, where V: seismic shear in the column.

From virtual work:

Top deflection due to horizontal force V at the top: $[2(6k_Y+1)-6k_Y]VH^3/[6EI_{CY}(12k_Y+1)]$ = $(3k_Y+1)VH^3/[3EI_{CY}(12k_Y+1)] = 0.1083VH^3/EI_{CY}$;

Frame stiffness per interior column: K_Y = 9.233×(0.5×34000000×0.0063)/7^3 = 2883 kN/m

Tributary mass of interior column: M=195/9.81=19.88 tn

T=2π√(19.88/2823)=0.527 sec.

Direction X:

Beam B1: effective flange width on each side of the web: 0.07×3+0.07×3=0.42 m; total flange width: 0.3+2×0.42=1.14 m, I = 0.0028 m^4;

Beam B2: I = 0.3×0.4^3/12 = 0.0016 m^4;

Column in X: I = 0.6×0.35^3/12 = 0.002144 m^4

k_{X1}=(0.0028/0.002144)×(3.5/3)=1.524,

k_{X2}=(0.0016/0.002144)×(3.5/3)=0.871.

Points of inflection:

- in the first storey column:
  - from the base (M_{1b}): $(3k_{X2}+1)(12k_{X1}+1)H/[2(6k_{X2}+1)(12k_{X1}+1)-1]$ = 0.5827×3.5=2.04 m,
  - from the top of first storey (M_{1t}): $[3k_{X2}(12k_{X1}+1)-1/2]H/[2(6k_{X2}+1)(12k_{X1}+1)-1]$ = 3.5-2.04 = 1.46 m.

  Seismic moments: 2.04V at the base, 1.46V at the top.

- in the second storey column:
  - from the base of second storey (M_{2b}): $[6k_{X2}(6k_{X1}+1)+1/2]H/[2(6k_{X2}+1)(12k_{X1}+1)-1]$ = 0.4473×3.5 = 1.57 m,
  - from the top (M_{2t}): $12k_{X1}(3k_{X2}+1)H/[2(6k_{X2}+1)(12k_{X1}+1)-1]$ = 3.5-1.57 = 1.93 m.

  Seismic moments: 1.57V at the bottom, 1.93V at the top.

From virtual work:

Top deflection due to horizontal force V at the top: $(5M_{1b}+2M_{2b}-4M_{1t}-M_{2t})H^2/(24EI_{CX})$ = $[24k_{X2}k_{X1}+5k_{X2}+16k_{X1}+3]VH^3/\{8EI_{CX}[2(6k_{X2}+1)(12k_{X1}+1)-1]\}$= 0.03324$VH^3/EI_{CX}$;

Frame stiffness per interior column: K_X = 30.1×(0.5×34000000×0.002144)/7^3 = 3197 kNm^2

We calculate also, for later use, the deflection in the X direction at column mid-height (level of beams B2) due to a horizontal force V at roof level: $(2M_{1b}-M_{1t})H^2/(24EI_{CX})$ =

$[72k_{X2}k_{X1}+6k_{X2}+48k_{X1}+5]VH^3/\{48EI_{CX}[2(6k_{X2}+1)(12k_{X1}+1)-1]\}= 0.01559VH^3/EI_{CX} =$

$0.01559\times 7^3 V/(0.5\times 34000000\times 0.002144) = 0.0001467V$

Tributary mass of an interior column: M=195/9.81=19.88 tn

$T=2\pi\sqrt{(19.88/3197)}=0.495$ sec.

3) Direction Y:

Design spectral acceleration: $S_d(T=0.527)=1.4\times 0.42\times(2.5/4.95)\times(0.5/0.527) = 0.282g$

Seismic force at the level of the roof per span between two X-direction frames:

$V_Y = 0.282\times 12.3\times 10\times 6.5=225$ kN

Direction X:

Design spectral acceleration: $S_d(T=0.587)=1.4\times 0.42\times(2.5/5.85) = 0.251g$

Seismic force at the level of the roof per one X-direction frame:

$V_X = 0.251\times 12.3\times 10\times 6.5=200$ kN

4) Direction Y:

Stiffness of all four interior and two exterior columns: $5K_Y = 5\times 2883$ kN/m.

Roof displacement: $u_Y = 4.95\times 225/(5\times 2883)=0.077$ m

Interstorey drift under 50% of the design seismic action: $0.5u_Y/H= 0.5\times 0.077/7=0.0055 > 0.5\%$.

Sensitivity coefficient for second-order effects: $\theta= 0.077\times(12.3\times 10\times 6.5)/(225\times 7)=0.039 < 0.1$

Direction X:

Stiffness of all four interior and two exterior columns: $5K_X = 5\times 3197$ kN/m.

Roof displacement: $u_Y = 5.85\times 200/(5\times 3197)=0.073$ m

Deflection at column mid-height (level of beams B2) due to a horizontal force $V = 200/5 = 40$ kN acting at roof level: $5.85\times 0.0001467\times 40 = 0.034$ m; deflection from that level to the roof: 0.073-0.034 = 0.039 m.

Interstorey drift under 50% of the design seismic action:

- Lower storey of the frame: $0.5\times 0.034/3.5=0.0049 < 0.5\%$.

- Top storey of the frame: 0.5×0.039/3.5=0.0056 > 0.5%.

The damage limitation limit for brittle infills is not met, but the limit for the frame without interacting infills (1%) is.

Sensitivity coefficient for second-order effects: θ= 0.073×(12.3×10×6.5)/(200×7)=0.042 < 0.1

5) According to 2) above, the maximum column moment due to a horizontal force V is 3.85V for framing action in the Y direction and 2.04V for bending in the X direction. So, beams B2 reduce the moment resistance requirements by almost 50%. Besides, they increase the in-plane stiffness from 9.23EI_C/H^3 in Y to 30.1EI_C/H^3 in X. Thanks to the reduction in the flexural strength demand and the increased stiffness, the columns are slimmer in the X direction than in Y.

6) Dimensioning of the beams in flexure:

Parameters:

f_{cd}=35/1.5=23.33 MPa; f_{ctm}=3.2 MPa, E_c=34000000 kPa;

f_{yd}=500/1.15=434.8 MPa; ε_{yd}=434.8/200000=0.217%;

Distance of centre of longitudinal bars from nearest concrete surface:

d_1=c+d_{bh}+d_{bL}/2~0.025+0.008+0.016/2 ~0.04m.

Curvature ductility demand for detailing:

As $T > T_C$, Eq. (5.64b) applies: $\mu_\varphi = 2q_o - 1$ = 10.7 in X, 8.9 in Y.

Beams B1 and B2 develop only seismic moments, because the elements of the X-direction frame have been taken as weightless and the loads of the roof go to beams B3 alone in one-way action:

d = 0.4-0.04 = 0.36 m; ρ_{min} =0.5f_{ctm}/f_{yk} =0.5×3.2/500=0.0032

$A_{s,min}$ =0.0032×300×360= 346 mm^2.

Maximum diameter of bars, d_{bL}, at the support on the interior column, per Eq. (5.2a):

min$N$= 6.5×3×10=195 kN

For ν_d = min$N$/(0.35×0.6×23333))=0.04 and ρ'=0.5ρ_{max}:

d_{bL}/h_c ≤7.5×1.032×3.2/(1.2×1.375×434.8)=0.0345, d_{bL} ≤ 0.0345×350=12 mm

Beams B1:

The seismic moment is one-half that of the moment at the top of the column for seismic action in direction X: $M_{B1,E} = \pm 0.5 \times 1.93 \times 40 = \pm 38.6$ kNm, $A_s = 267$ mm$^2 < A_{s,min}$. We place 3Φ12 (339 mm$^2 \sim A_{s,min}$) at top and bottom.

$M_{Rd,b}^{-} = M_{Rd,b}^{+} = 339 \times 0.9 \times 0.36 \times 434.8/10^3 = 47.8$ kNm

Beams B2:

The seismic moment is the average moment at the top of the 1st storey column and the bottom of the 2nd storey one: $M_{B2,E} = \pm 0.5 \times (1.46+1.57) \times 40 = 60.6$ kNm, $A_s = 424$ mm$^2 \sim A_{s,min}$. We place 4Φ12 (452 mm^2) at top and bottom.

$M_{Rd,b}^{-} = M_{Rd,b}^{+} = 452 \times 0.9 \times 0.36 \times 434.8/10^3 = 63.7$ kNm

Beams B3:

$d = 0.5 - 0.04 = 0.46$ m. $A_{s,min} = 0.0032 \times 300 \times 460 = 442$ mm^2.

Maximum diameter of bars, d_{bL}, at the support on the interior column, per Eq. (5.2a):

$\min N = 6.5 \times 3 \times 10 = 195$ kN

For $\nu_d = \min N/(0.35 \times 0.6 \times 23333) = 0.04$ and $\rho' = 0.5\rho_{max}$:

$d_{bL}/h_c \leq 7.5 \times 1.032 \times 3.2/(1.2 \times 1.375 \times 434.8) = 0.0345$, $d_{bL} \leq 0.0345 \times 600 = 20$ mm

$A_{s,min}$: 3Φ16 (603mm^2) at top and bottom.

The seismic moment is one-half the moment at the top of the column for seismic action in direction Y (with $V = 225/5 = 45$ kN per column):

$M_{B3,E} = \pm 0.5 \times 3.15 \times 45 = \pm 70.9$ kNm.

The moment due to the quasi-permanent gravity loads of 6.5×3.0= 19.5 kN/m is:

$M_{B3,G} = -19.5 \times 10^2/12 = -162.5$ kNm

and due to the factored gravity loads, 1.35G:

at the supports: $M_{B3,1.35G} = -1.35 \times 162.5 = -219.5$ kNm, at midspan: 219.5/2= 109.7 kNm.

The design hogging moment at the supports is: max(219.5; 162.5+70.9=233.4) = 233.4 kNm and the sagging one: 70.9-162.5 < 0 (there is no sagging moment).

Bottom reinforcement at mid-span (for 109.7 kNm): A_s = 582 mm^2 : 3Φ16 (603 mm^2)

Top reinforcement (for 233.4 kNm): A_s = 1353 mm^2: 3Φ16 ($A_{s,min}$) + 4Φ16 additional top bars at the supports (total: 1407 mm^2).

Bottom reinforcement at the supports at least 50% of top reinforcement area (704 mm^2); we place 3Φ16 ($A_{s,min}$) + 1Φ16 in addition, over the critical region at the beam end supports; total: 804 mm^2;

ρ = 1407/(300×460) = 0.0102, ρ' = 804/(300×460) = 0.00583,

$\rho_{max}=\rho'+0.0018 f_{cd}/(\mu_{\varphi}\varepsilon_{yd}f_{yd})$= 0.00583 + 0.0018×23.33/(8.9×0.00217×434.8)= 0.01083 > ρ = 0.0102.

$M_{Rd,b}^{-}$ = 1407×0.9×0.46×434.8/10^3= 253.3 kNm, $M_{Rd,b}^{+}$= 804×0.9×0.46×434.8/10^3= 144.7 kNm

7) There is no capacity design of the columns around joints, as the building has just one storey in one direction and two in the other; so, the columns are dimensioned on the basis of the analysis results alone.

Distance of centre of vertical bars from nearest concrete surface: $d_1 = c+d_{bh}+d_{bL}/2$ ~ 0.04m.

Minimum number of bars, to respect the maximum spacing of 150 mm between bars engaged at a stirrup corner or cross-tie: 3 on the short sides, 5 on the long ones (total 12); $A_{s,min}$ = 0.01×350×600 = 2100 mm^2.

Minimum reinforcement: 4Φ18 (at the corners) + 8Φ14 (intermediate bars along the sides): 2250 mm^2; ρ = 2250/(350×600)=0.0107 > 0.01.

Direction Y:

Maximum M_d = 3.85×45 = 173.5 kNm (at the base), μ_d = 173.5/(0.35×0.56^2×23333)=0.06765

Direction X:

Maximum M_d = 2.04×40 = 81.6 kNm (at the base), μ_d = 81.6/(0.6×0.31^2×23333)=0.06065.

As the design moments are low, we will put the minimum reinforcement and we will check whether the moment resistance it provides meets the demands from the analysis.

In order to have the "web" reinforcement uniformly spread between ω_{1d} and ω_{2d}, half the area of

1Φ14 is subtracted from each corner bar and counts in ω_{vd}

Direction Y:

d = 560 mm, d_1 = 40 mm, $\delta_1 = d_1/d$=40/560=0.0715, v_d= 195/(0.35×0.56×23333)=0.0426.

$\omega_{1d}=\omega_{2d}$=509/(350×560)×434.8/23.33 = 0.0484, ω_{vd}=1232/(350×560)×434.8/23.33 = 0.1171.

The limit in Eqs. (5.37a), (5.37b) is:

v_2 = 0.117×[0.0715×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.0715)+ 0.0715×(0.0035-0.002/3)/(0.0035-0.00217) = 0.1523-0.0876=0.0647 > v_d= 0.0415.

Eq. (5.39b) applies and takes the form:

[1-0.002/(3×0.0035)+ 0.117×(0.0035+0.00217)^2/(2×0.0035×0.00217×(1-0.0715)]ξ^2

-[0.0415+0.0484×(1-0.0035/0.00217)+ 0.117×(1+0.0715×0.0035/0.00217)/(1-0.0715)]ξ-

[0.0484-0.5×0.117×0.0715/(1-0.0715)]×0.0715×0.0035/0.00217=0 → 1.0762ξ^2-0.1524ξ-0.00506=0

→ ξ=0.1695

Eq. (5.38b) gives:

$$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \begin{array}{l}\xi[0.40476-0.33676\xi]+0.5\cdot(1-0.0715)\cdot 0.0484\left(1+1.613\dfrac{\xi-0.0715}{\xi}\right)+\\ 0.25\cdot 0.117[1.62\xi-0.0715]\left[1+1.613\left(\dfrac{\xi-0.0715}{\xi}\right)\right][1-0.0715/3-1.08\xi]/(1-0.0715)\end{array} = 0.112,$$

from which $M_{Rd,c}$ = 287 kNm.

The minimum reinforcement provides the resistance required in direction Y.

Direction X:

d = 310 mm, d_1 = 40 mm, $\delta_1 = d_1/d$=40/310=0.129, v_d= 195/(0.31×0.6×23333)=0.0449;

$\omega_{1d}=\omega_{2d}$=817/(310×600)×434.8/23.33 =0.08185, ω_{vd}=616/(310×600)×434.8/23.33 = 0.06171.

The limit in Eqs. (5.37a), (5.37b) is:

v_2 = 0.06171×[0.129×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.129)+ 0.129×(0.0035-0.002/3)/(0.0035-0.00217) = 0.247> v_d= 0.0449.

Eq. (5.39b) applies and takes the form:

[1-0.002/(3×0.0035)+ 0.06171×(0.0035+0.00217)^2/(2×0.0035×0.00217×(1-0.129)]ξ^2

-[0.0449+0.08185×(1-0.0035/0.00217)+ 0.06171×(1+0.129×0.0035/0.00217)/(1-0.129)]ξ-

[0.08185-0.5×0.06171×0.129/(1-0.129)]×0.129×0.0035/0.00217=0 → $0.9595\xi^2$-0.08032ξ-0.01608=0

→ ξ=0.178

Eq. (5.38b) gives: $$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi\left[0.40476-0.33676\xi\right]+0.5\cdot(1-0.129)\cdot 0.08185\left(1+1.613\frac{\xi-0.129}{\xi}\right)+ 0.25\cdot 0.06171\left[1.62\xi-0.129\right]\left[1+1.613\left(\frac{\xi-0.129}{\xi}\right)\right]\left[1-0.129/3-1.08\xi\right]/(1-0.129) = 0.116,$$

from which $M_{Rd,c}$ = 156 kNm > 81.6 kNm. The minimum reinforcement suffices.

8) Capacity design shears

Beams B1:

At the joints: $\sum M_{Rd,c}$ = 156 kNm, $\sum M_{Rd,b}$ = 2×47.8 = 95.6 kNm

max$V_{CD,B1}$ = 1.2×2×47.8/2.65 = 43.3 kN, min$V_{CD,B1}$ = -43.3 kN, ζ = -1 (full reversal of shear).

Beams B2:

At the joints: $\sum M_{Rd,c}$ = 2×156 = 312 kNm, $\sum M_{Rd,b}$ = 2×63.7 = 127.4 kNm.

max$V_{CD,B2}$ = 1.2×2×63.7/2.65 = 57.7 kN, min$V_{CD,B2}$ = -57.7 kN, ζ = -1 (full reversal of shear).

Beams B3:

At the joints: $\sum M_{Rd,c}$ = 287 kNm, $\sum M_{Rd,b}$ = 253.3+144.7 = 398 kNm.

max$V_{CD,B3}$=1.2×[253.3×287/398+144.7×287/398]/9.4 +19.5×9.4/2 = 36.6+91.65 = 128.25 kN

min$V_{CD,B3}$ = -36.6+91.65 = 55.05 kN, ζ = 55.05/128.25 > 0 (no reversal of shears).

Interior column, direction Y

At the top joint: $\sum M_{Rd,c}$ = 287 kNm < $\sum M_{Rd,b}$ = 253.3+144.7 = 398 kNm.

$V_{CD,CY}$ = 1.3×[287+287]/6.5 = 114.8 kN.

Interior column, direction X:

Lower storey: at the joint: $\sum M_{Rd,c}$ = 2×156 kNm = 312 kNm > $\sum M_{Rd,b}$ = 2×63.7 = 127.4 kNm.

$V_{CD,CX1}$ = 1.3×[156+156×127.4/312]/3.1 = 92.1 kN.

Upper storey:

at the roof joint: $\sum M_{Rd,c}$ = 156 kNm > $\sum M_{Rd,b}$ = 2×47.8 = 95.6 kNm;

at the intermediate storey: $\sum M_{Rd,c}$ = 2×156 kNm = 312 kNm > $\sum M_{Rd,b}$ = 2×63.7 = 127.4 kNm.

$V_{CD,CX2}$ = 1.3×[156×95.6/156+156×127.4/312]/3.1 = 66.8 kN.

9) Design of beam shear reinforcement

Beams B1:

max V_{Ed} = 43.3 kN, ζ = -1 (full reversal of shear).

In Eq. (5.49): $(2+\zeta)f_{ctd}b_w d$ =(2-1)×(0.7×3200/1.5)×0.3×0.36 = 161.3 > max V_{Ed} = 43.3 kN → no need for diagonal or inclined shear reinforcement.

Minimum shear reinforcement ratio per Eurocode 2: minρ_w=0.08$\sqrt{f_{ck}}$(MPa)/f_{yk}(MPa)= 0.000947.

In critical regions of length 0.6 m:

cotθ = 1; maximum stirrup spacing < 6d_{bL}= 6×12 = 72 mm; < h/4 = 100 mm; < 24d_{bw} = 24×6= 144 mm. Φ6/70 (808 mm^2/m, ρ_w = 808/(300×1000)=0.002693 > minρ_w).

$V_{Rd,s}$=$b_w z \rho_w f_{ywd}$cotθ = 0.3×0.9×0.36×0.002693×434800×1.0 = 113.8 kN > max V_{Ed} = 43.3 kN.

Outside the critical regions:

cot$\theta \leq$ 2.5; maximum stirrup spacing = 0.75d= 0.75×360 = 270 mm.

Φ6/200 (283 mm^2/m, ρ_w = 283/(300×1000)=0.000943 ~ minρ_w).

$V_{Rd,s}$=$b_w z \rho_w f_{ywd}$cotθ = 0.3×0.9×0.36×0.000943×434800×2.5 = 99.6 kN > max V_{Ed} = 43.3 kN.

Beams B2:

max $V_{CD,B2}$ = 57.7 kN, ζ = -1.

On the resistance side, whatever has been said above for B1 applies; the minimum reinforcement of Φ6/70 in critical regions of length 0.6 m and Φ6/200 outside, suffices.

Beams B3:

In critical regions of length 0.75 m:

cotθ = 1; maximum stirrup spacing < 6d_{bL}= 6×16 = 96 mm; < h/4 = 125 mm; < 24d_{bw} = 24×6= 144 mm.

At a distance of d = 0.46 m from the face of the support: V_{Ed} (0.46 m) = 36.6+19.5×(9.4/2-0.46)= 119.3 kN

$V_{Rd,s}=b_w z\rho_w f_{ywd}\cot\theta = 0.3\times0.9\times0.46\times434800\times1.0\rho_w = 54000\rho_w > V_{Ed} = 119.3$ kN $\rightarrow \rho_w > 0.00221$.

Φ6/85 (665 mm^2/m, $\rho_w = 665/(300\times1000)=0.002217 > \min\rho_w$).

Outside the critical regions:

$\cot\theta \leq 2.5$; maximum stirrup spacing = $0.75d = 0.75\times460 = 345$ mm.

At a distance of $z\cot\theta = 0.9\times0.46\times2.5 = 1.035$ m from the end of the critical region:

$V_{Ed} = 36.6+19.5\times(9.4/2-0.75-1.035) = 93.4$ kN

Φ6/200 (283 mm^2/m, $\rho_w = 283/(300\times1000)=0.000943 \sim \min\rho_w = 0.000947$).

$V_{Rd,s}=b_w z\rho_w f_{ywd}\cot\theta = 0.3\times0.9\times0.46\times0.000943\times434800\times2.5 = 127.3$ kN $> V_{Ed} = 93.4$ kN.

10) Transverse reinforcement of interior column

Over a distance from the end sections 1.5 times the critical region length:

- stirrup diameter, $d_{bw} \geq 6$ mm, $0.4\sqrt{(f_{yd}/f_{ywd})}d_{bL} = 0.4\times18 = 7.2$ mm;
- spacing $\leq 6d_{bL} = 6\times14 = 84$ mm; $b_o/3 = (350-2\times25-8)/3 = 97.5$ mm

Critical region length:

– In direction Y: $\geq 1.5h_c = 0.9$ m and $\geq H_{cl}/5 = 1.3$ m, i.e. 1.3 m

– In direction X: $\geq 1.5h_c = 0.9$ m and $\geq H_{cl}/5 = 0.62$ m, i.e. 0.9 m; this applies to the intermediate level too.

So, 1.5 times the critical region length for direction Y covers a distance of up to 1.95 m from the base or from the soffit of the roof beam, leaving free 3.1-1.95 = 1.15 m from the top and the soffit of Beam B2, which should accommodate 1.5 times the critical region length for direction X, i.e., 1.5×0.9=1.35 m. Therefore, in the end, the minimum transverse reinforcement of critical regions applies throughout the column height.

Stirrups chosen are Φ8/80 (628 mm^2/m per stirrup leg); apart from the perimeter tie, they comprise a diamond engaging the four mid-side bars (with legs at an inclination to direction Y of $\tan\delta = (300/2-8-18/2)/(550/2-8-18/2)=0.515$ ($\delta = 0.476$ rad) and a rectangular stirrup engaging the two other intermediate bars of the long sides; that latter tie contributes to shear strength and confinement only in direction Y. The shear resistance provided by these stirrups is:

- In direction Y:

  $V_{Rd,s}$ = 195×0.9×0.56/6.5+0.9×0.56×628×0.4348×(2+2cosδ) cotθ= 15+520cotθ (where the first term is the contribution of the axial force to shear resistance and the paranthesis at the end is the effective no. of stirrup legs) and

  $V_{Rd,max}$ = 0.3×(1-35/250)×0.35×0.9×0.56×23333×(1+195/(0.35×0.6×23333))sin2θ = 1104sin2θ,

  for cotθ = 1.783: $V_{Rd,s}$ = $V_{Rd,max}$ = 942 kN >> $V_{Ed,Y}$ = 114.8 kN

- In direction X:

  $V_{Rd,s}$ = 195×0.9×0.31/3.1+0.9×0.31×628×0.4348×(4+2sinδ)cotθ = 17.5 +374.5cotθ and

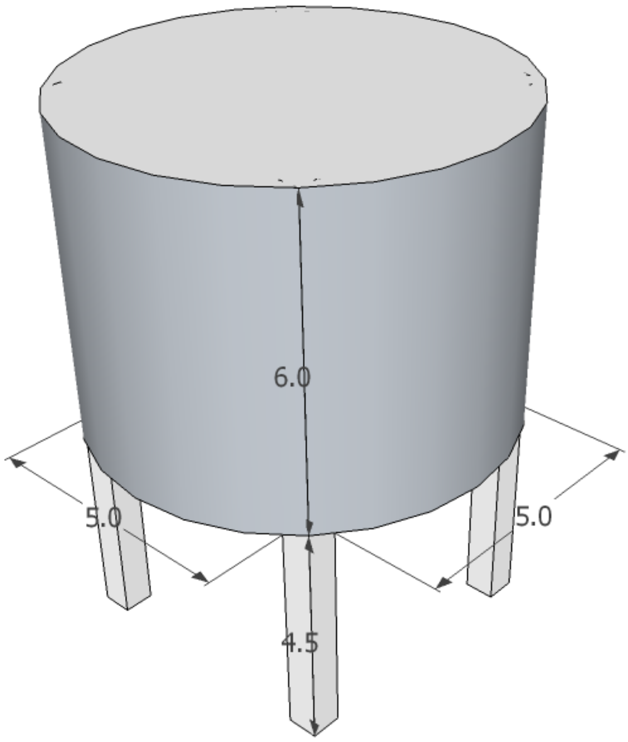
  $V_{Rd,max}$ = 0.3×(1-35/250)×0.60×0.9×0.31×23333×(1+195/(0.35×0.6×23333))sin2θ = 1040sin2θ;

  for cotθ = 2.105: $V_{Rd,s}$ = $V_{Rd,max}$ = 806 kN >> $V_{Ed,X}$ = 92.1 kN.

In the critical regions at the base, as well as at the top of the column where in the Y direction $\sum M_{Rd,c}$ <1.3$\sum M_{Rd,b}$, the transverse reinforcement should provide confinement through an effective mechanical ratio $a\omega_{wd} > 30\mu_\phi v_d \varepsilon_{yd} b_c/b_o$ -0.035, where ε_{yd} = 0.00217, v_d= 195/(0.35×0.6×23333)=0.04

- In direction Y: μ_ϕ = 8.9, b_c= 0.35, b_o = 0.266 m, $a\omega_{wd} > 30\mu_\phi v_d \varepsilon_{yd} b_c/b_o$ -0.035= -0.0046.
- In direction X: μ_ϕ = 10.7, b_c= 0.6, b_o = 0.516 m, $a\omega_{wd} > 30\mu_\phi v_d \varepsilon_{yd} b_c/b_o$ -0.035= -0.0026.

Thanks to the low axial load ratio, the target values of μ_ϕ can be achieved without confinement.



Question 5.6

An elevated concrete silo, 8 m in diameter, is supported on four concrete columns at a 5 m square arrangement (Fig. 5.23). The columns are 0.6 m square and have a clear height of 4.5 m, with double fixity at top and bottom. The silo may be considered as rigid, with a centre of mass 3 m above the top of the supporting columns. According to Part 4 of Eurocode 8 ("Silos, tanks and pipelines"), the seismic design of the columns and their foundation follows Part 1 of Eurocode 8, except that the *q*-factor is reduced by 30% owing to the irregularity in elevation. The design peak ground acceleration (on rock) is 0.3g and the Eurocode 8 spectrum for ground type B applies. Ductility Class Medium (DC M) is chosen. The total weight of the silo and its contents is 3000 kN and may be taken as permanent load. Concrete grade is C35/45 and steel is of Class C with 500 MPa nominal yield stress; concrete cover to reinforcement is c =30 mm. Importance Class is II (ordinary).

1) Considering the structure as a SDOF system, calculate its period and compute its design base shear and the horizontal displacements under the design seismic action. Calculate the sensitivity coefficient to second order effects. Compute the correlation coefficient of the two natural modes in horizontal seismic action components X and Y and consider the implications for the CQC rule. Consider the case of a horizontal seismic action component acting along the diagonal of the column

section (including the implications for the column axial forces, as calculated from the overall overturning moment at column midheight).

2) Calculate the accidental eccentricity per Part 1 of Eurocode 8 and its effects on column internal forces, for concurrent horizontal seismic action components X and Y. Discuss the implications of the correlation of the modes in the context of accidental eccentricity.

3) Dimension the vertical reinforcement of the columns.

4) Calculate the capacity design shears of the columns.

Answer of Question 5.6:

f_{cd}=35/1.5=23.33 MPa; E_c=34000000 kPa; f_{yd}=500/1.15=434.8 MPa.

1) For a single column: $EI = 0.5\times34000000\times0.6^4/12 = 183600$ kNm^2, $K = 12EI/h^3 = 12\times183600/4.5^3$ = 24178 kN/m. For all four columns: K = 96710 kN/m. Mass: 3000/9.81 = 305.8 t.

$T = 2\pi\sqrt{(305.8/96710)} = 0.35$ s.

q-factor: DC M frame, with system redundancy $\alpha_u/\alpha_1 = 1.1$, reduced by 30% for irregularity in elevation:

$q = 0.7\times3\times1.1= 2.31$ (54% larger than the value of 1.5 for inverted pendulum systems of DC M).

$V_b = (0.3\times1.2)\times2.5/2.31\times3000 = 1170$ kN.

Computed displacement: $u_d=V_b/K$=1170/96710=0.0121 m. Seismic displacement $u_e=qu_d$ =2.31×0.0121= 0.028 m

$\theta = 0.028\times3000/(1170\times4.5) = 0.016 < 0.10$.

The correlation coefficient of the two natural modes in orthogonal directions is obtained from Eq. (3.89) for $\beta_{nr} = 1$; it is $\rho_{nr} = 1$. This introduces a full cross-term in Eq. (3.88); however, the participation factor of each one of the two vibration modes is zero in the orthogonal direction. So, there is no practical implication in this respect of the full correlation of these two modes. The case of the seismic axial force in the column, considered next, is an illustration.

At column mid-height the bending moments in the columns are zero. If we consider a horizontal section across the columns at that level, the overturning moment produced by the horizontal seismic force acting at silo mid-height is equilibrated by a couple of axial forces in the columns: tensile on the windward side, compressive on the leeward one. The overturning moment with respect to that level is $(3+4.5/2)V_b = 5.25V_b = 6142$ kNm, and is equilibrated by forces of $6142/(2\times5) = 614$ kN in the four columns, tensile in the two windward ones, compressive in the other two.

- If the effects of the two horizontal components are combined via the linear approximation, Eq. (3.100), in the combination of 100% of EX with 30% of EY the column shear force in the direction of EX is equal to $V_b/4$, while in the orthogonal direction each column is subjected to a shear force of $0.3V_b/4 = 0.075V_b$. These biaxial shears, and the biaxial bending they produce, are combined with a column axial force equal to $\pm(1+0.3)\times614 = \pm798$ kN.
- If the effects of the two horizontal components are combined with the SRSS rule, Eq. (3.99), the peak moments and shears in the columns do not change. The peak axial force becomes $\sqrt{2}\times614 =$ 869 kN. As a matter of fact (Fardis 2009), the biaxial moments and shears which are concurrent with the peak axial force are equal to $1/\sqrt{2}$ of their unidirectional peak values. This conclusion is independent of the correlation of the two modes, because each mode has zero participation factor in the orthogonal direction. So, two combinations should be considered: a) uniaxial bending, due to a column unidirectional shear of $V_b/4$, acting together with a column axial force of ±614 kN; b) equal biaxial moments and shears, corresponding to column shears of $V_b/(4\sqrt{2})$ in each direction, alongside a column axial force of $\pm614\sqrt{2} = 868$ kN. Physically, the second combination is produced by a single component seismic action along the diagonal of the columns.

2) The accidental eccentricity is $e_{aX} = 0.05\times8 = 0.4$ m along X, $e_{aY} =0.4$ m along Y. The accidental eccentricity of a unidirectional base shear, V_b, induces a shear force of $0.4V_b\times2.5/(8\times2.5^2) = 0.02V_b$ in each direction of the column: it increases the design shear force in a column from $V_b/4$ to $0.27V_b$ and introduces a force of $0.02V_b$ in the orthogonal direction.

- If the effects of the two horizontal components are combined via the linear approximation, Eq. (3.100), the torque about the vertical axis increases by 30%. So, in the combination of 100% of EX and 30% of EY, the column shear forces in the direction of EX increase to $V_b/4 + 1.3\times0.02V_b$ = 0.276×1170 = 323 kN, while in the orthogonal direction EY each column is subjected to a shear force of $0.3\times0.25V_b$, plus a contribution $1.3\times0.02V_b$ of the accidental eccentricities, i.e. to a total of $0.101V_b$= 0.101×1170 = 118 kN. According to part 2 of the answer to this question, these biaxial shears, and the biaxial bending they produce, are combined with a column axial force of ±(1+0.3)×614 = ±798 kN.

- If the effects of the two horizontal components are combined with the SRSS rule, Eq. (3.99), the torsional moments due to the two eccentricities are combined into one, which induces in each direction of a column an additional shear force of $\sqrt{2}\times0.02V_b = 0.0282V_b$. Since the silo has perfect symmetry and independent unidirectional response in each horizontal direction, the application of Eq. (3.99) does not increase the maximum unidirectional shears and moments in the columns. So, combination a) per the last paragraph of part 2 of this answer gives a shear force of $0.25V_b + 0.0282V_b$ = 0.2782×1170 = 325.5 kN in the direction of EX and a shear of $0.0282V_b$ = 33 kN contributed by the accidental eccentricities to the concurrent shear in the orthogonal direction EY (with this latter shear being zero according to the last paragraph of part 2). These biaxial shears and moments act together with a column axial force of ±614 kN. Combination b) in the last paragraph of part 2 gives a shear force of $0.25V_b/\sqrt{2}+ 0.0282V_b = 0.205V_b$ = 240 kN in both directions of the column, acting together with a column axial force of ±614√2 = ±868 kN.

Note that, in all cases above, the tensile axial force overcomes the column compression due to gravity, which is equal to 750 kN at the top and 790.5 kN at the base of a column.

The perfect correlation of the two modes, i.e. in the X and the Y direction, would have a significant impact on the column internal forces due to simultaneous seismic action components in X and Y, if the accidental eccentricities had been addressed not per the simplified static approach of Eurocode 8, but by shifting laterally the centre of mass, producing a system with three coupled degrees of freedom. The three modes of that system would be strongly correlated.

The column shear forces produce moments at the column end sections equal to the shear times half the column length (i.e., 2.25 m).

Combination EX, EY		V_{Ex}, kN	M_{Ex}, kNm	V_{Ey}, kN	M_{Ey}, kNm	N_E, kN	max$N$, kN top/bottom	min$N$, kN top/bottom
Eq. (3.100) 100%-30%		323	727	118	266	±798	1548/1588	-48*/-8*
Eq. (3.99) SRSS	Case (a)	325.5	732	33	74	±614	1364/1404	136/176
	Case (b)	240	540	240	540	±868	1618/1658	-118*/-78*

* Net tension

The use of Eq. (3.99), with its four cases (including the ± sign in the axial force) is more sound and rational; so, it is used here as the basis of dimensioning.

3) Minimum reinforcement 0.01×600×600=3600 mm^2. At least 4 laterally engaged bars per side, e.g., 12Φ20 (3770 mm^2), ρ=3770/(600×600)=0.0105 > ρ_{min}=0.01.

As the section is large, we calculate the moment resistance provided by the minimum reinforcement and then check whether it is sufficient.

d_1 = 50 mm, $\delta_1 = d_1/d$=50/550=0.091.

Half of the corner bars count in ω_{vd}, in order to have the "web" reinforcement uniformly spread between ω_{1d} and ω_{2d}:

$\omega_{1d}=\omega_{2d}$=0.25×3770/(600×550)×434.8/23.33 = 0.05322, $\omega_{vd}=2\omega_{1d}$=0.10644

The limit ν_2 of Eq. (5.37a) is:

ν_2 = 0.10644×[0.091×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.091)+0.091×(0.0035-0.002/3)/(0.0035-0.00217) = 0.1939-0.0717=0.1222. N_2 = 0.1222×0.6×0.55×23333=941 kN

All cases with min$N$ are below ν_2; Eqs. (5.37b), (5.38b), (5.39b) apply. Those with max$N$ are above ν_2. Most critical are the min$N$ cases, in which Eq. (5.39b) applies and takes the form:

[1-0.002/(3×0.0035)+0.10644×(0.0035+0.00217)^2/(2×0.0035×0.00217×(1-0.091)]ξ^2

-[v_d+0.05322×(1-0.0035/0.00217)+0.10644×(1+0.091×0.0035/0.00217)/(1-0.091)]ξ-

[0.05322-0.5×0.10644×0.091/(1-0.091)]×0.091×0.0035/0.00217=0 → 1.05735ξ^2-[v_d+0.10166]ξ-

0.00703=0

Eq. (5.38b) gives: $$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi[0.40476-0.33676\xi]+0.024188\left(1+1.613\frac{\xi-0.091}{\xi}\right)+ 0.029274[1.62\xi-0.091]\left[1+1.613\left(\frac{\xi-0.091}{\xi}\right)\right][0.97-1.08\xi]$$

Case (a) of Eq. (3.99) has: v_d =136/(0.6×0.55×23333)=0.018, for which Eqs. (5.38b), (5.39b) give $M_{Rd,c}$= 460 kNm, only! The reinforcement needs to be drastically increased.

As engaged bars are needed at third-points of each side anyway, we should either maintain the number of bars to 12 and considerably increase their diameter, or place an extra bar at mid-point between the present 12 bars. This second option gives 24 bars. We opt for 24Φ20 (7540 mm^2), ρ=7540/(600×600)=0.021 >0.01.

ω_{1d}=ω_{2d}=0.25×7540/(600×550)×434.8/23.33 = 0.10644, ω_{vd}=2ω_{1d}=0.21288.

v_2= 0.21288×[0.091×(0.0035+0.00217)/(0.0035-0.00217)-1]/(1-0.091)+0.091×(0.0035-0.002/3)/(0.0035-0.00217) = 0.1939-0.1434=0.0505. All cases with min$N$ are below v_2.

Therefore, Eq. (5.39b) applies and takes the form:

[1-0.002/(3×0.0035)+0.21288×(0.0035+0.00217)^2/(2×0.0035×0.00217×(1-0.091)]ξ^2

-[v_d+0.10644×(1-0.0035/0.00217)+0.21288×(1+0.091×0.0035/0.00217)/(1-0.091)]ξ-

[0.10644-0.5×0.21288×0.091/(1-0.091)]×0.091×0.0035/0.00217=0 → 1.30518ξ^2-[v_d+0.20332]ξ-

0.01406=0

Eq. (5.38b) gives: $$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi[0.40476-0.33676\xi]+0.048376\left(1+1.613\frac{\xi-0.091}{\xi}\right)+ 0.058548[1.62\xi-0.091]\left[1+1.613\left(\frac{\xi-0.091}{\xi}\right)\right][0.97-1.08\xi]$$

Case (a) of Eq. (3.99) with v_d =0.018 gives, through the form of Eqs. (5.38b), (5.39b) above: $M_{Rd,c}$= 834 kNm, which is sufficient for case (a) of Eq. (3.99), and for Eq. (3.100) as well, leaving a margin for the concurrent, smaller component in the orthogonal direction. The only case that now needs to be checked explicitly is case (b), with the equal biaxial moments. However, that case cannot be addressed with the present analytical tools. So, recourse is sought to the safe-sided option given by Eurocode 8 to check under uniaxial bending for a moment equal to one of the components of the biaxial case divided by 0.7. That option gives a uniaxial moment of 540/0.7 = 771 kNm, under an axial tension of ν_d = 118/(0.6×0.55×23333) = -0.015 (tension). For this value Eqs. (5.38b), (5.39b) give $M_{Rd,c}$= 784 kNm, which exceeds the uniaxial value of 771 kNm. An additional confirmation on the basis of biaxial bending interaction diagrams (CEB/FIP Manual for bending and compression, 1982) gives a value of 580 kNm for the moment resistance of the section under equal biaxial moments. This value exceeds by 7.5% the equal biaxial moment demands of 540 kNm for case (b). Therefore, all things considered, the verification of case (b) is deemed to be met.

4) Capacity design shears of the column

The most adverse situation is under max$N$. However, the cases with the higher max$N$ values in the table with the column internal forces are associated with strongly biaxial bending of the columns; case (b) in the application of Eq. (3.99) is characteristic in this respect. So, it is nonsensical to base the calculation of the capacity design shears on the value of max$N$ of one of these cases, using uniaxial values of $M_{Rd,c}$. The most rational of all uniaxial cases, namely case (a) in the application of Eq. (3.99), is considered instead.

For max$N$ = 1404 kN, ν_d =1404/(0.6×0.55×23333)=0.182 > ν_2, and Eqs. (5.37a), (5.38a), (5.39a) apply, giving: ξ = (0.909ν_d+1.091×0.21288)/(0.909×0.8095+2×0.21288)=0.342, and

$$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi[0.40476 - 0.33676\xi] + 0.096754 + 0.2342[(\xi - 0.091)(1-\xi) - 0.1281\xi^2] = 0.231,\ M_{Rd,c} = 978 \text{ kNm},$$

while, for max$N$ = 1364 kN at the top: ν_d =1364/(0.6×0.55×23333)=0.177> ν_2, ξ = (0.909ν_d+1.091×0.21288)/ (0.909×0.8095+2×0.21288) = 0.3385, $M_{Rd,c}$ = 974.4 kNm. So, the capacity design shear is:

V_{CD} = 1.1×(978+974.4)/4.5 = 477.3 kN.