The materials' σ-ε behavior and models

Overview: Behavior of concrete materials & their interaction in cyclic loading

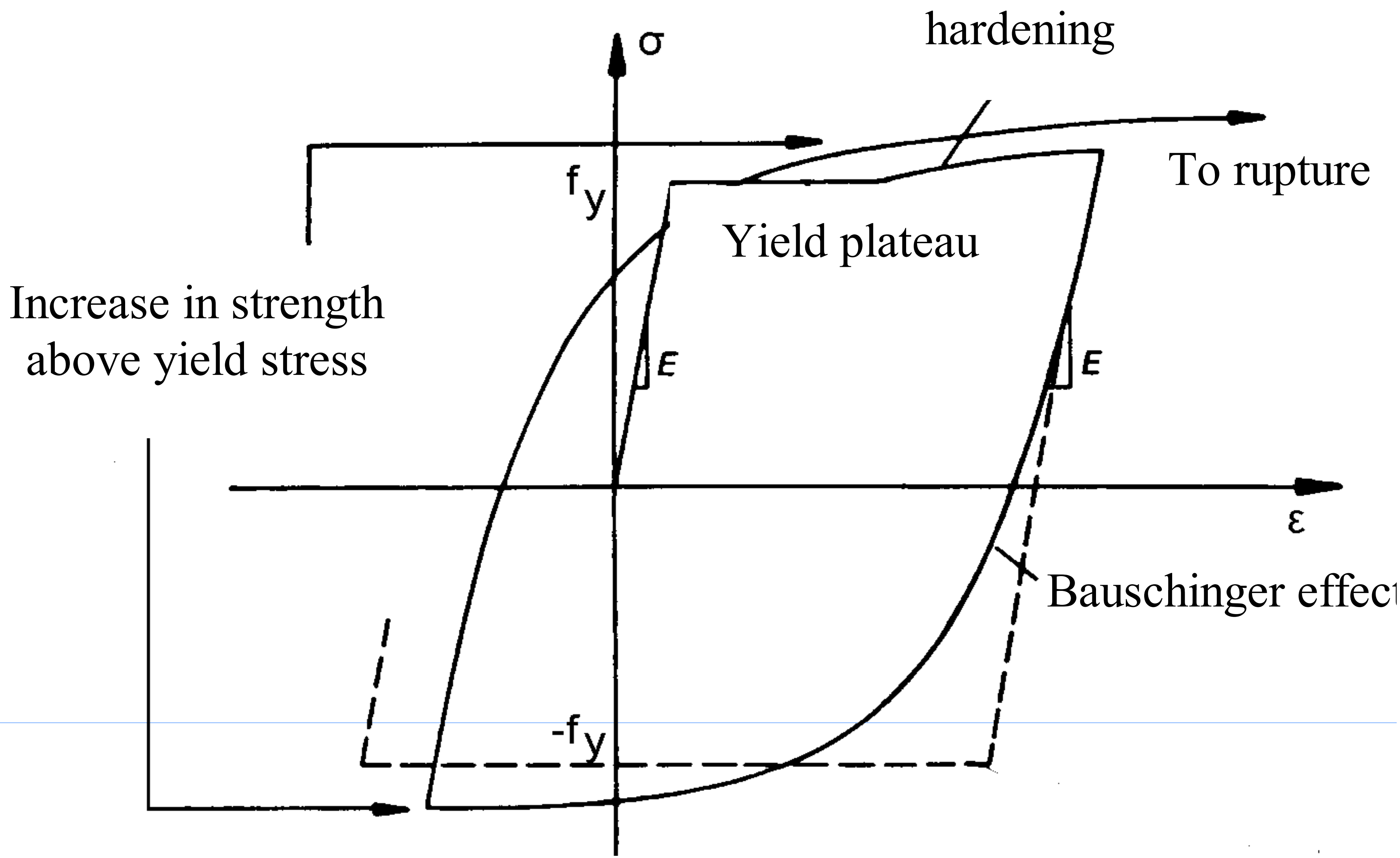
- Inherently ductile (stable hysteresis loops, considerable energy dissipation up to large deformations): only steel bars in tension (they buckle in compression).
- Concrete: fairly brittle. If well confined, it sustains cycles of large compressive strains w/o drop in resistance (but cannot dissipate energy).
- The only way to dissipate significant energy in large amplitude deformation cycles, is by combining:
  - reinforcing steel in the direction of tensile internal forces/stresses;
  - concrete & reinforcement in the direction of compressive internal forces/stresses, with confinement of concrete & restraint of bar buckling by closely spaced transverse reinforcement.

This is feasible where inelastic stresses/strains are always in directions where reinforcement can be conveniently placed:

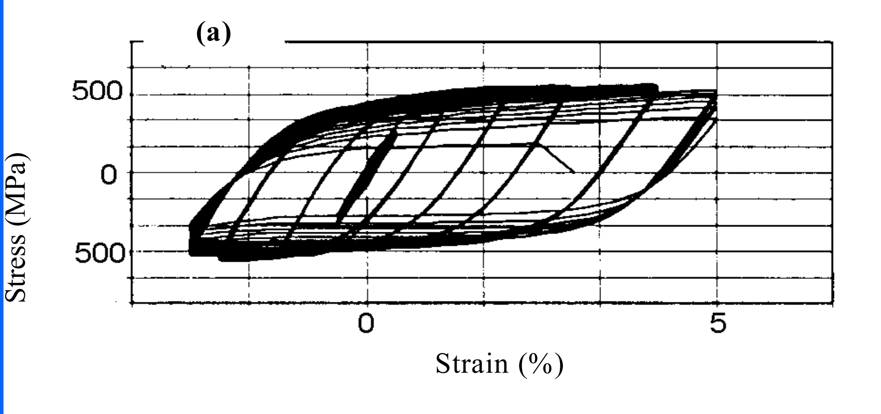
- in beams, columns, slender walls: in the longitudinal direction.

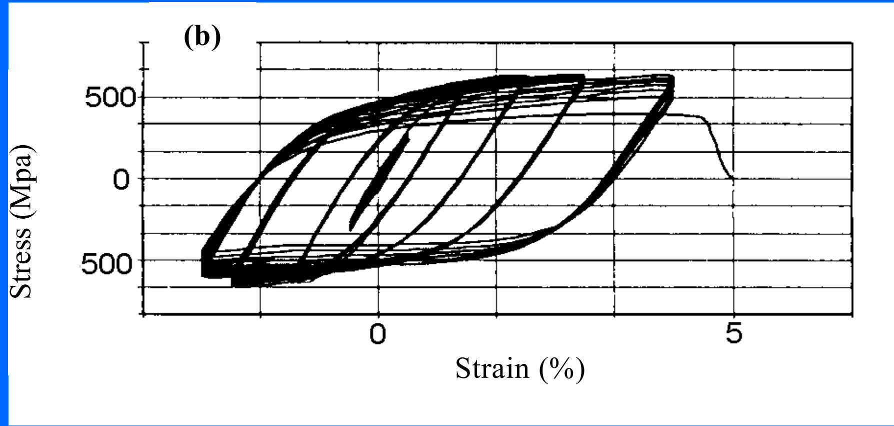
Reinforcing steel

Uniaxial σ-ε behavior in cyclic loading

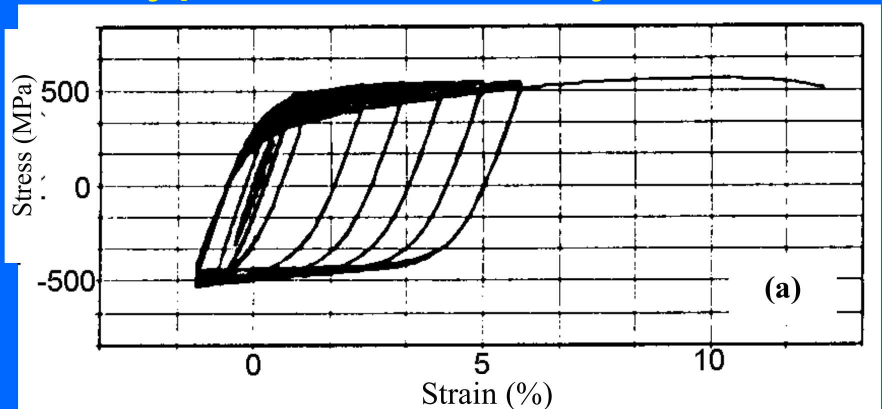


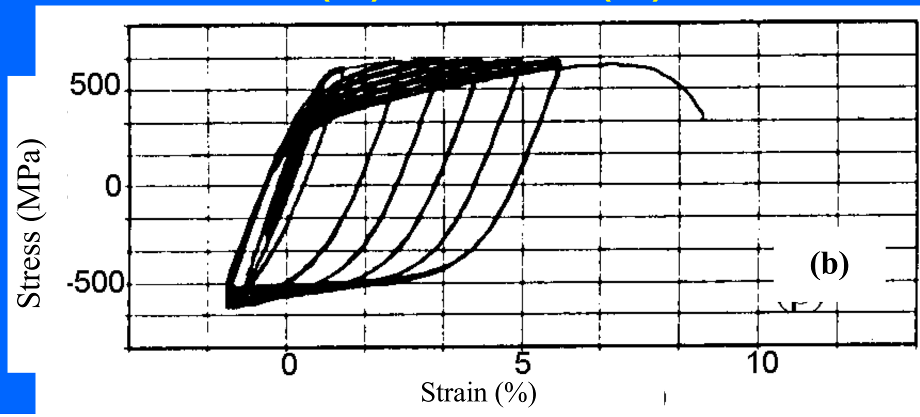
Typical σ-ε history in column bars (a) S400; (b) S500:



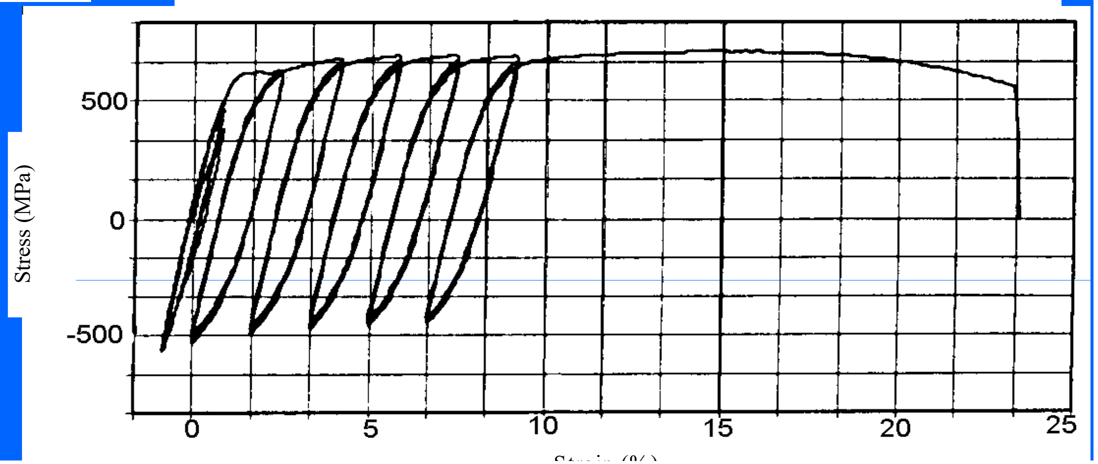


Typical σ-ε history in beam bottom bars (a) S400; (b) S500:

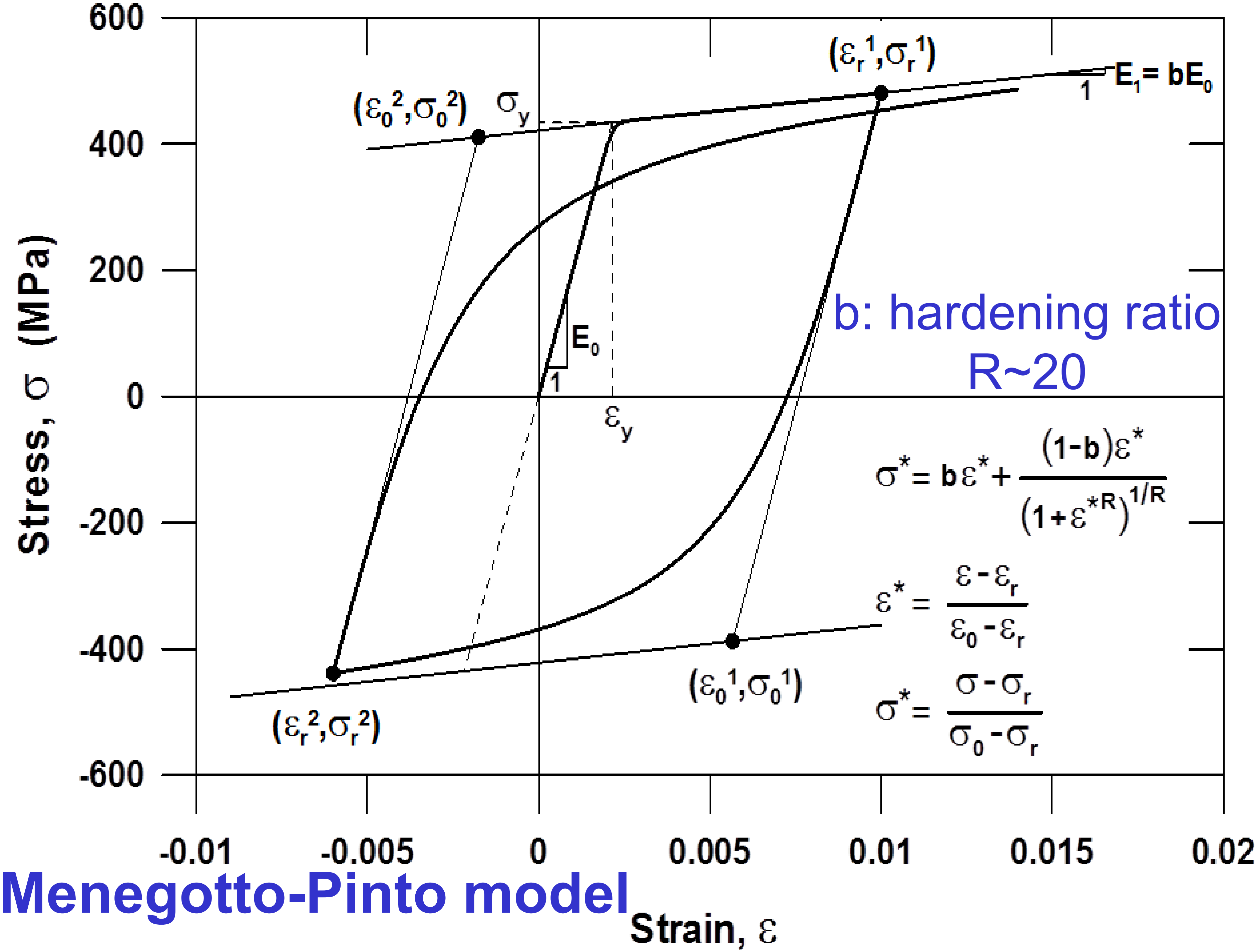




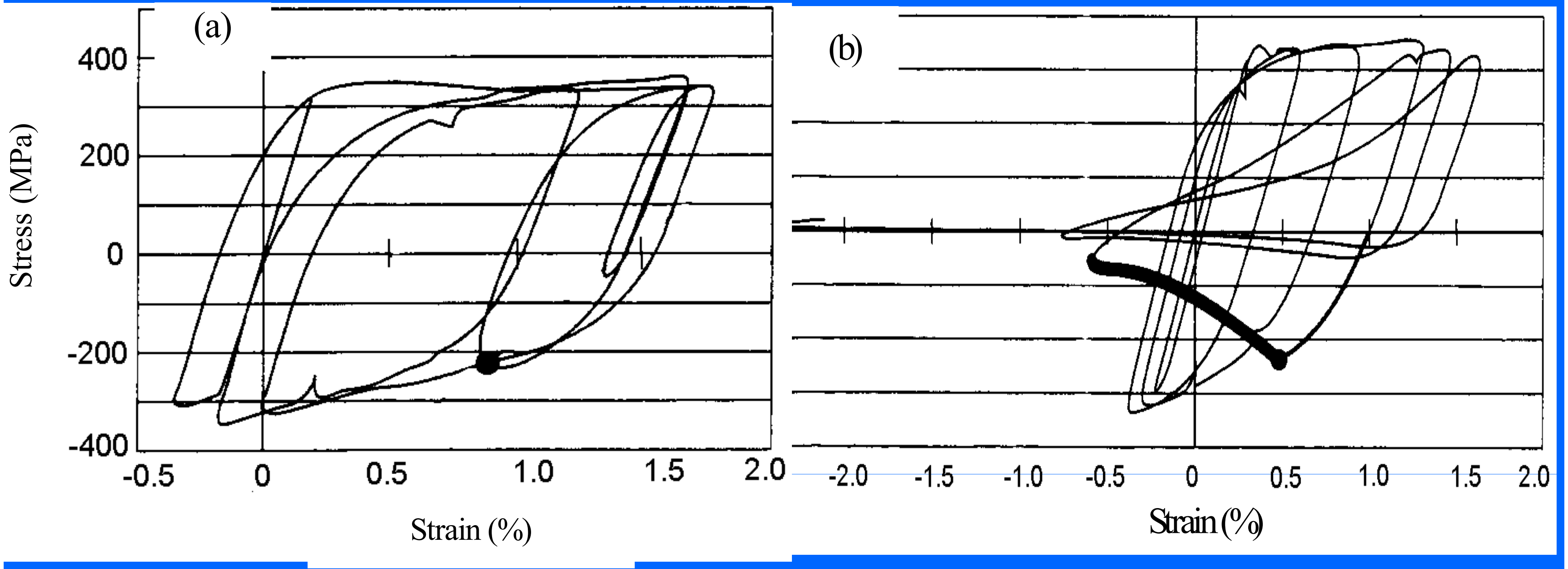
Typical σ-ε history in beam top bars; S500:



Menegotto-Pinto model



Buckling of reinforcing bars



σ-ε loops of bar that buckles in a cyclically loaded concrete member:

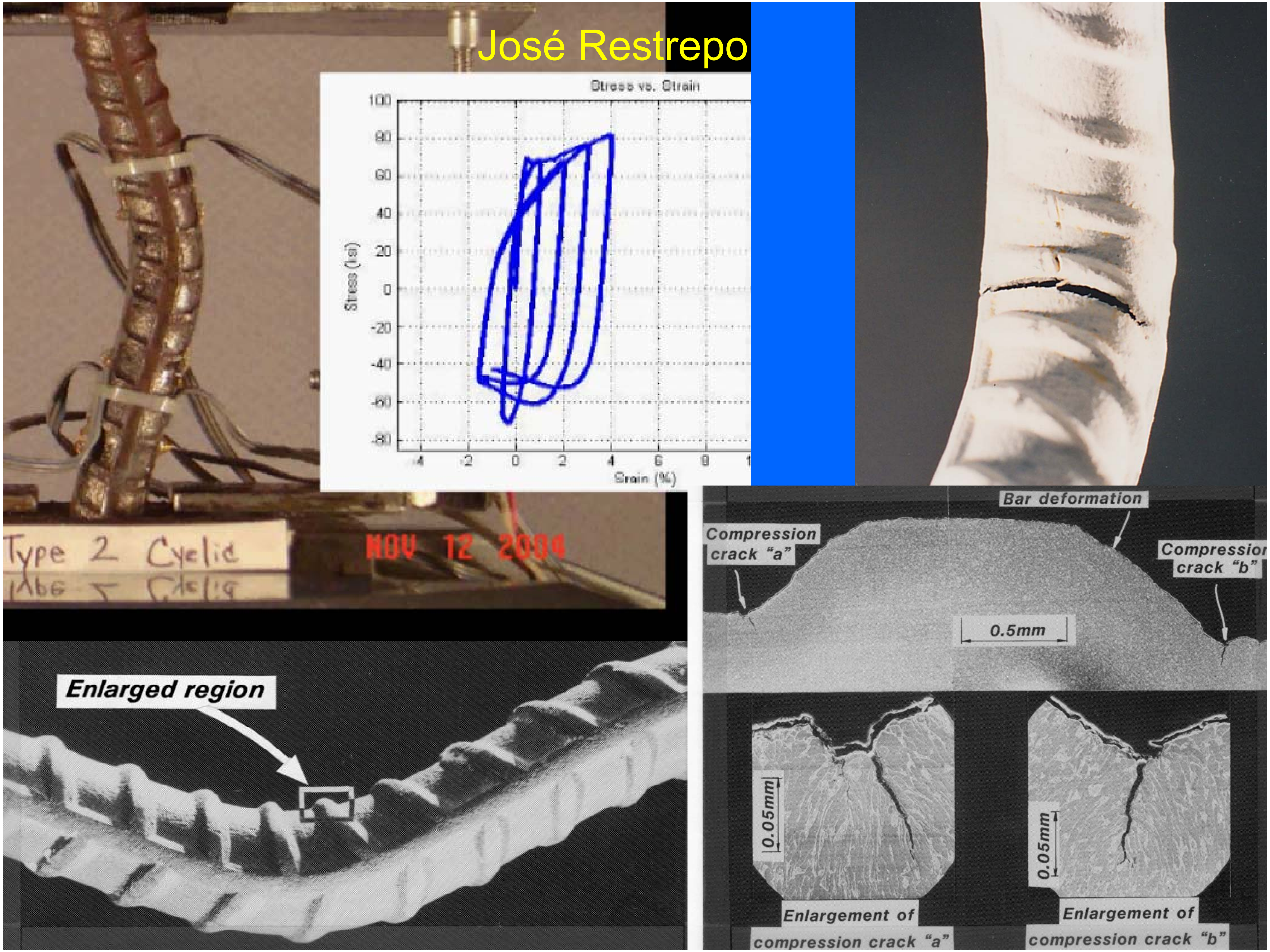
(a) diagram of stress *σ* vs real strain *ε* along axis of the buckled bar;

(b) diagram of stress vs apparent strain in the original direction of the bar.

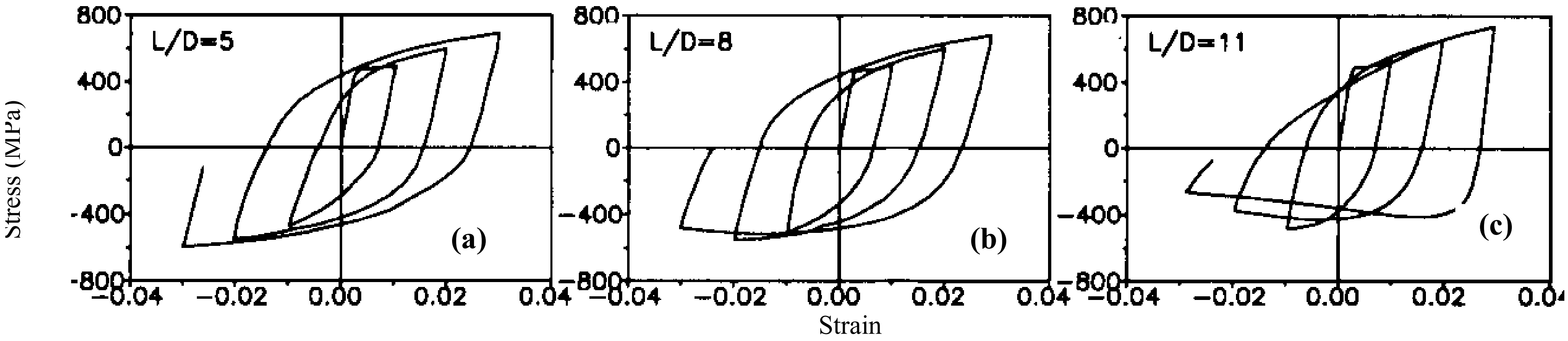
Buckling of the bar shown as: •.

Bar buckling has consequences that normally precipitate member failure:

– If concrete cover has not spalled off, it does so upon bar buckling (early bar buckling occurs only outwards)
– Surrounding concrete has to carry compressive force released from buckled ba
– Buckled bars: not effective for confinement of concrete (especially if buckling extends beyond two consecutive stirrups, causing their permanent extension).
– Buckled bar may rupture in tension immediately afterwards:
  - Buckling induces additional flexural strains in the bar, tensile on one side, compressive on the opposite, superimposed on mean axial strain of the bar (which is tensile due to prior yielding of bar in tension & its permanent extension).
  - The shorter the length, *L*, over which bar buckling occurs, the larger are the additional flexural strains.
  - The total (mean axial plus flexural) strain of extreme fibres of the bar may approach or exceed the steel rupture strain at the root of rib (especially in Tempcore steels) → crack at the surface of the buckled bar.
  - After reversal of loading, a bar that has buckled straightens up and – depending on magnitude of the new half-cycle – may go into the inelastic range in tension → previously formed crack may extend through the entire bar cross-section → complete loss.


José Restrepo
Stress vs. Strain
Stress (ksi)
Strain (%)
Type 2 Cyclic
NOV 12 2004
Enlarged region
Bar deformation
Compression crack "a"
Compression crack "b"
0.5mm
0.05mm
0.05mm
Enlargement of compression crack "a"
Enlargement of compression crack "b"

Effect of bar length which is unrestrained by stirrups



Loops of stress vs apparent strain of bar subjected to cyclic loading with reversals at equal and opposite strain values:

(a) no bar buckling;

(b), (c) bar buckling occurs.

L/D : ratio of free bar length to diameter.

Buckling of bars in concrete members

- Bars in the compression zone follow the member curvature, which is concave outwards.
  - Inwards buckling is prevented by the concrete.
  - Outwards buckling (the cover already spalled off, or will spall upon buckling) has to overcome & reverse any bar pre-curvature → ~impossible.
- Corner bars buckle first & outwards, but ~normal to the plane of bending.
- Intermediate bars buckle after the concrete core next to them has disintegrated.

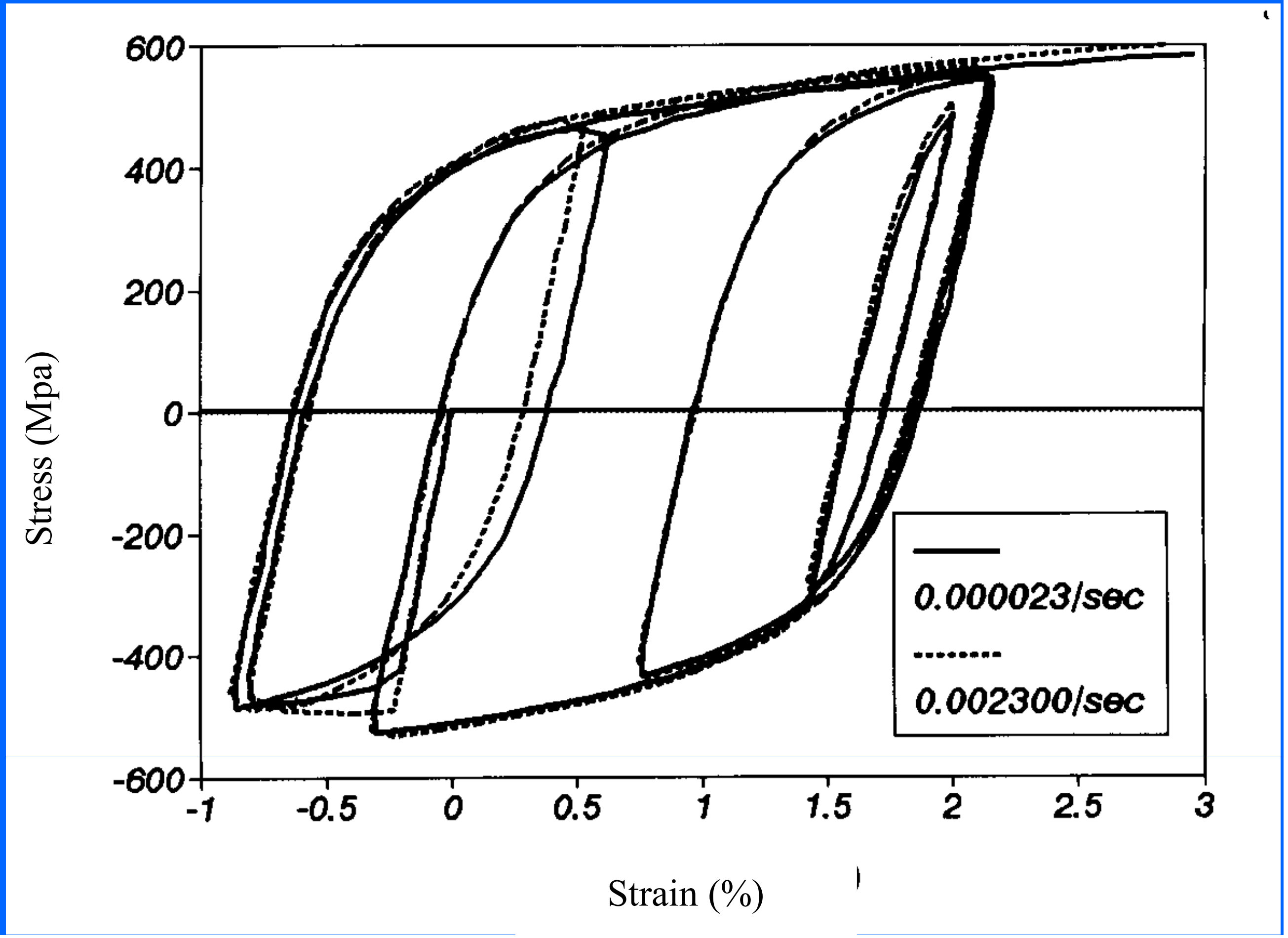
Strain-rate effects on mechanical behavior of bars

Effect of strain rate, $\dot{\varepsilon}$, on σ-ε parameters in monotonic loading:

Increase with strain rate , relative to those measured in the lab for quasi-static loading $\dot{\varepsilon}$ under $\dot{\varepsilon}$=5x10^{-5} sec^{-1} by:

cln($\dot{\varepsilon}$/5x10^{-5}), where:

- c=6MPa for f_y,
- c=7MPa for f_t and
- c=0.3% for ε_{su}



Reinforcing steel - Important *σ-ε* parameters:

- Modulus of elasticity: E_s=200GPa
- Yield stress: f_y
- Tensile strength in monotonic loading: f_t
- Strain at maximum stress (: at tensile strength) in monotonic loading = uniform elongation at rupture, ε_{su}.

Important σ-ε parameters for earthquake resistance:

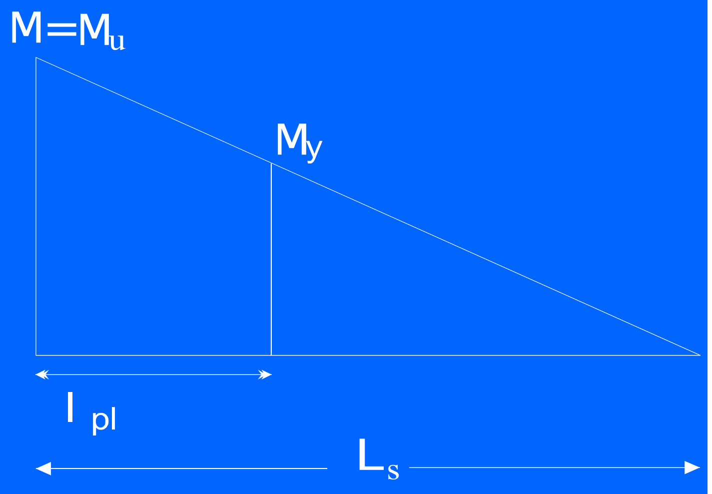
- Hardening ratio: f_t/f_y
- (Uniform) Elongation at maximum stress, ε_{su}.

Important σ-ε parameters & requirements on reinforcing steel for earthquake resistance:

- Uniform strain at maximum stress, ε_{su}.
- Ultimate curvature of member section, as controlled by rupture of tension reinforcement: $$\varphi_{su} = \frac{\varepsilon_{su}}{(1 - \xi_{su})d}$$
- Hardening ratio, f_t/f_y:
- Large f_t/f_y: Longer member plastification, when the end section reaches the ultimate moment

$$l_{pl} = L_s\left(1 - \frac{M_y}{M_u}\right) = L_s\left(1 - \frac{f_y}{f_t}\right)$$

$$\theta_u = \phi_y \frac{L_s}{3} + \frac{l_{pl}}{3}\left[\phi_u\left(1 + \frac{f_y}{2f_t}\right) - \phi_y\left(\frac{1}{2} + \frac{f_t}{f_y}\right)\right]$$



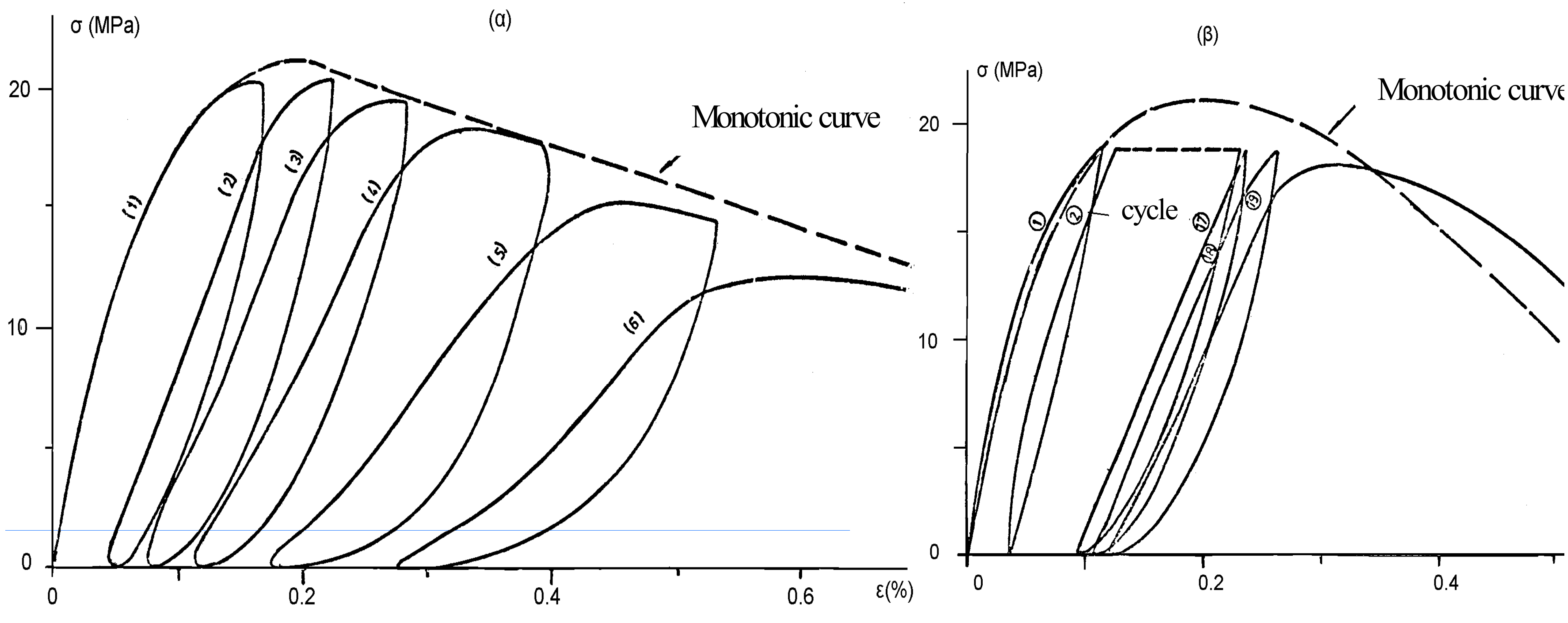
- Large f_t/f_y or $f_{y,actual}/f_y$: Flexural overstrength (in beams) may trigger plastic hinges in the columns, or shear failures in general

Eurocode 8 requirements for reinforcing steel in earthquake-resistant buildings:

Ductility Class	**DC L or M**	**DC H**
10%-fractile yield strength, f_{yk}	400 to 600	
10%-fractile hardening ratio, $(f_t/f_y)_{k,0.10}$	≥1.08	≥1.15 <1.35
10%-fractile strain at maximum stress, $\varepsilon_{su,k,0.10}$	≥5.0%	≥7.5%
95%-fractile actual yield strength, $f_{yk,0.95}/f_{yk}$	-	≤1.25

The Concrete

σ-ε behavior in cyclic uniaxial compression



Effect of confinement by $\sigma_2=\sigma_3=p<\sigma_1$ on σ_1-ε_1 ultimate strength parameters (in compression)

- Effect on ultimate strength: $f_c^* = f_c(1+K)$
- Effect on strain at ultimate strength: $\varepsilon_{c1}^* \approx \varepsilon_{c1}(1+5K)$

– Newman, Newman '71 (adopted in EC8-Part 3): $K \approx 3.7\left(\frac{p}{f_c}\right)^{0.86}$

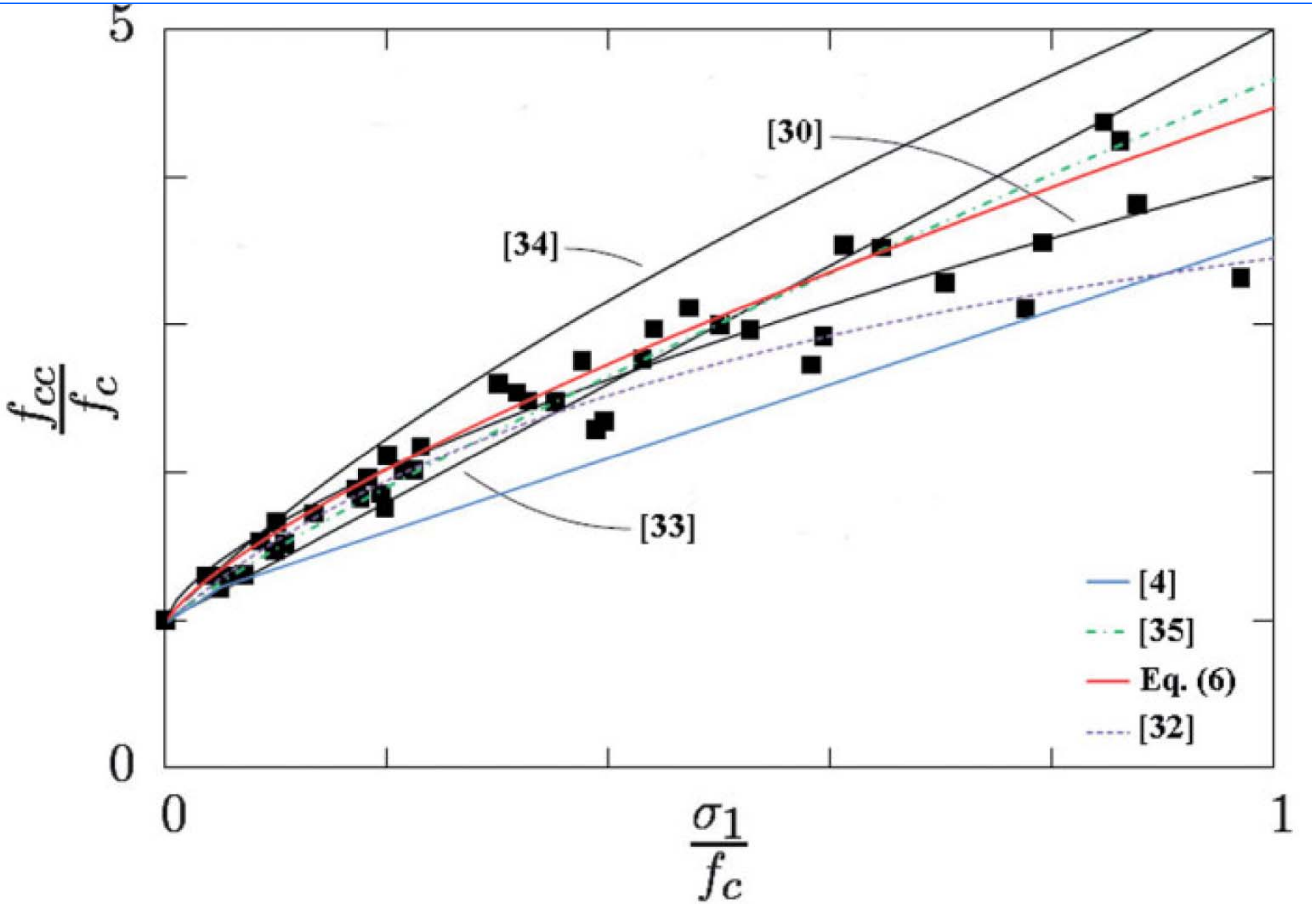
– *fib* Model Code 2010 and new EC2: $K \approx 3.5\left(\frac{p}{f_c}\right)^{0.75}$

– Elwi & Murray '79 (adopted by Mander et al):

$$K = 2.254\left[\sqrt{1+7.94\frac{p}{f_c}}-1\right]-\frac{2p}{f_c}$$

– CEB/FIP Model Code 90 (adopted in EC2 & EC8-Part 1):

$$f^* = \beta f_c = \min\left(1+5\frac{p}{f_c};1.25+2.5\frac{p}{f_c}\right)f_c \qquad \varepsilon_{co}^* = \varepsilon_{co}\beta^2$$



[30] Guidotti
[32] Mander et al
[33] Richart et al.
[34] Ottosen
[35] Newman & Newman
[4] CEB MC90, EC2
Eq.(6): fib MC2010, New EC2

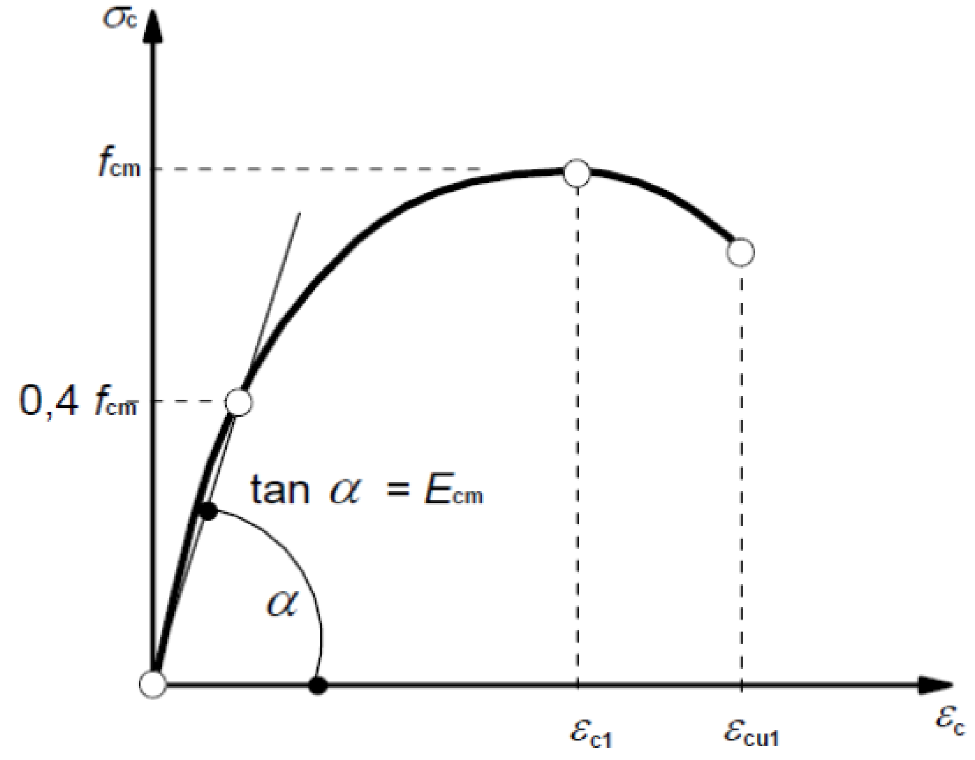
Test results compared to confinement models for ultimate strength

σ_1-ε_1 law of (confined) concrete up to & beyond ultimate strength

EC2 (& CEB/FIP MC90) law, with $k=1.05E_{cm}\varepsilon_{c1}^{*}/f_c^{*}$

$E_{cm}(\text{MPa}) = 10000\,(f_c^{*}(\text{MPa}))^{1/3}$

$$\frac{\sigma}{f_c^{*}} = \frac{\dfrac{\varepsilon}{\varepsilon^{*}_{c1}}\left(k - \dfrac{\varepsilon}{\varepsilon^{*}_{c1}}\right)}{1 + (k-2)\dfrac{\varepsilon}{\varepsilon^{*}_{c1}}}$$



Practical confinement of concrete members by transverse reinforcement

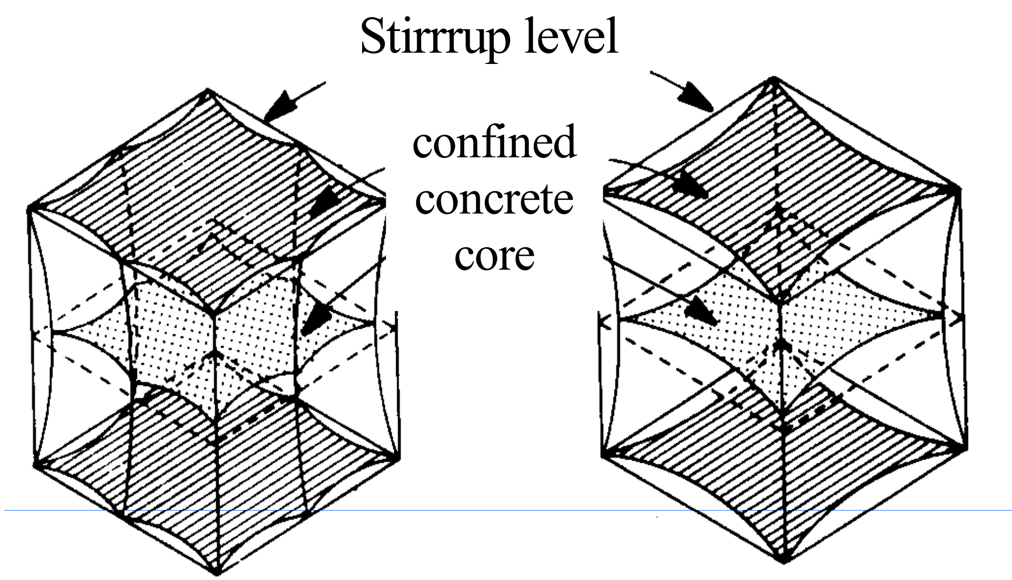
- rectangular section & ties: $p / f_c \approx a \min(\rho_x, \rho_y) f_{yw} / f_c = 0.5 a \omega_w$

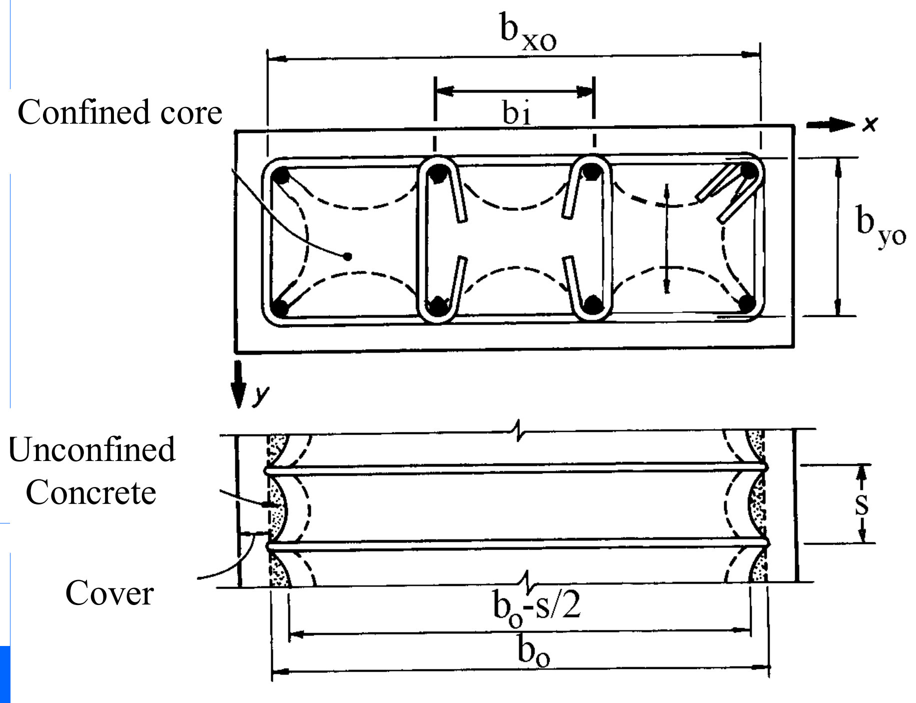
$$\rho_w = 2\rho_s = 2\min(\rho_x, \rho_y) = 2\min(\Sigma \mathrm{A}_{swx} / b_{yo}, \Sigma \mathrm{A}_{swy} / b_{xo}) / s$$

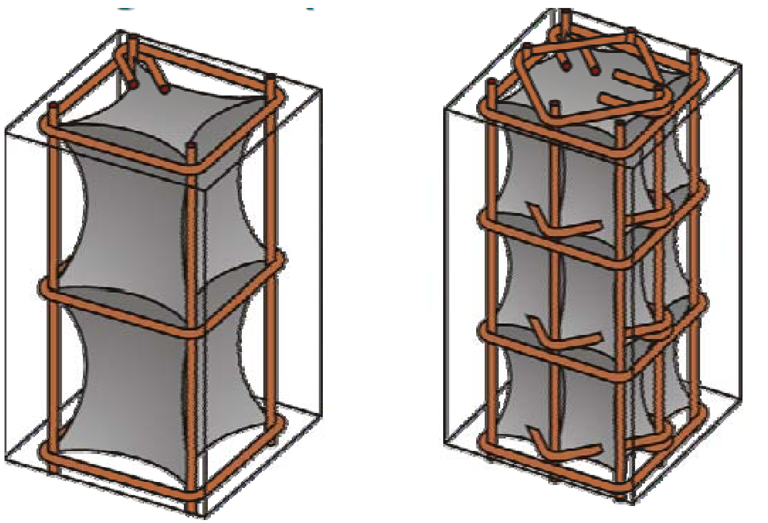
- circular section & ties: $$\frac{p}{f_c} = 0.5 a \rho_w \frac{f_{yw}}{f_c} = 0.5 a \omega_w$$

$$\rho_w = 2\rho_s = 2(2\mathrm{A}_{sw} / D_o s)$$

confinement effectiveness factor – rectangular section w/ stirrups: $\alpha = \alpha_n \alpha_s$





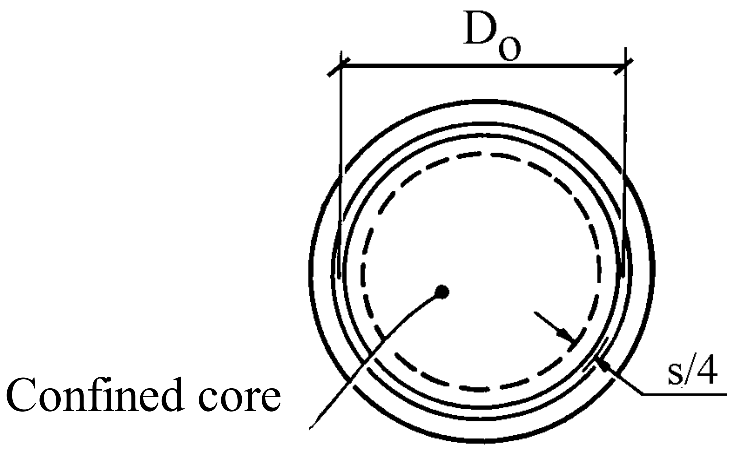


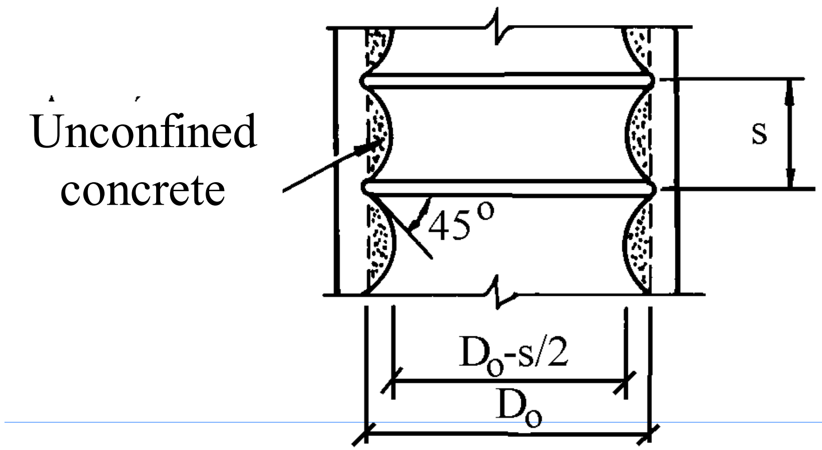
Fully confined or unconfined parts:
– at the cross-section level and
– along the length of the member, for rectangular ties

$$a_n = 1 - \frac{\sum b_i^2 / 6}{b_{xo} b_{yo}} \qquad a_s = \left(1 - \frac{s}{2b_{xo}}\right)\left(1 - \frac{s}{2b_{yo}}\right)$$

confinement effectiveness factor – circular section: $a_n = 1$

$$\alpha = \alpha_n \alpha_s$$





- w/ stirrups:

$$\alpha_s = \left(1 - \frac{s}{2D_o}\right)^2$$

- w/ spirals:

$$a_s = \left(1 - \frac{s}{4D_o}\right)^2 \approx 1 - \frac{s}{2D_o}$$

Fully confined or unconfined parts along the length of the member, for circular ties or spirals

Ultimate strain of concrete in concentric compression confined by $\sigma_2=\sigma_3=p<\sigma_1$

– CEB/FIP Model Code 90 (adopted in EC2 & EC8-Part 1)

$$\varepsilon_{cu}{}^{*}=0.0035+0.2p/f_c \qquad \frac{p}{f_c}=0.5\rho_w\frac{f_{yw}}{f_c}=0.5\omega_w$$

– Mander et al '88 (strain energy at fracture of ties):

$$\rho_w f_{yw}\varepsilon_{su,w}\approx f_c{}^{*}\left(\varepsilon_{cu}{}^{*}-\varepsilon_{cu}\right) \qquad \varepsilon_{cu}{}^{*}\approx\varepsilon_{cu}+\frac{\omega_w\varepsilon_{su,w}}{(1+K)}$$

– Paulay & Priestley '92:

$$\varepsilon_{cu}{}^{*}\approx 0.004+1.4\frac{\omega_w\varepsilon_{su,w}}{(1+K)}$$

Strain of extreme compression fibers at ultimate curvature of confined section in flexure (& axial load)

– Biskinis & Fardis 2007 (adopted in ***fib*** Model Code 2010)

Size-effect of section or core depth, *h*, or neutral axis depth, *x*:

- Monotonic loading:

$$\varepsilon_{cu}^{*} = 0.0035 + \left(\frac{10}{h_c(mm)}\right)^2 + 0.57\frac{p}{f_c^{*}}$$

- Cyclic loading:

$$\varepsilon_{cu}^{*} = 0.0035 + \left(\frac{10}{h_c(mm)}\right)^2 + 0.4\frac{p}{f_c^{*}}$$

– Grammatikou et al 2016 (adopted in new EC8)

- Unconfined concrete:

$$0.0035 \le \varepsilon_{cu} = \left(18.5/h(mm)\right)^2 \le 0.01$$

$$\varepsilon_{cu} = 0.0035 + \left(3.5/x(mm)\right)^2 \le 0.01$$

- Confined core after spalling of cover:

– rectangular compression zone (uniaxial bending):

$$\varepsilon_{cu,c} = \varepsilon_{cu} + 0.04\sqrt{p/f_c}$$

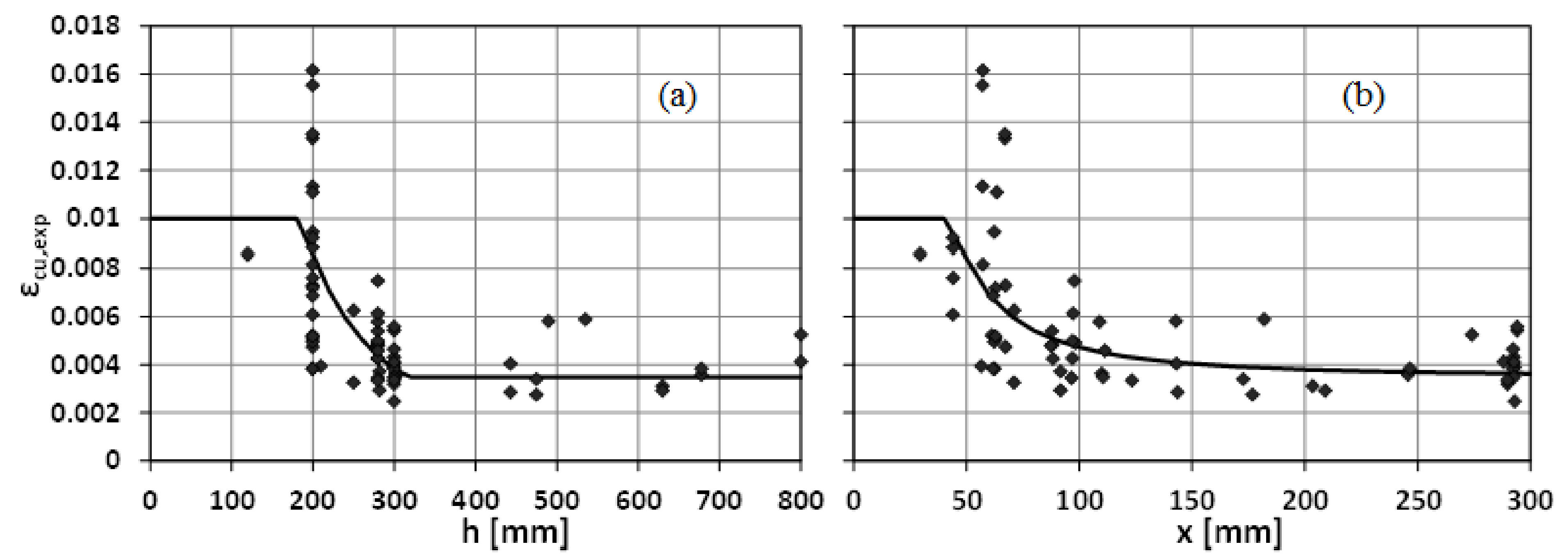
– circular sections:

$$\varepsilon_{cu,c} = \varepsilon_{cu} + 0.07\sqrt{p/f_c}$$

– triangular compression zone (biaxial bending):

$$\varepsilon_{cu,c} = \varepsilon_{cu} + 0.12\sqrt{p/f_c}$$

Size-effect of depth, *h*, or neutral axis depth, *x*, of the full section or of the confined core



Confinement by Fiber-Reinforced Polymer (FRP) jacket

Lam & Teng strength model:

$$\frac{f_{cc}}{f_c} = 1 + 3.3\left(\frac{b_x}{b_y}\right)^2 \frac{a_f \rho_f f_{u,f}}{f_c}$$

ρ_f: geometric ratio of FRP in direction of loading,

$b_y > b_x$

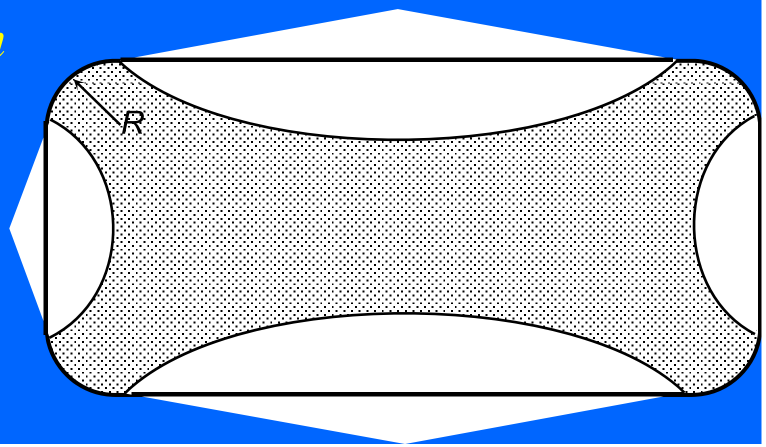
$f_{u,f}=0.6E_f\varepsilon_{u,f}$ (E_f: Elastic Modulus of FRP, $\varepsilon_{u,f}$ failure strain)

Confinement effectiveness factor:

- circular section: $\alpha_f = 1$
- rectangular section: $\alpha_f = \alpha_n$

$(\alpha_s = 1)$

$$a_n = 1 - \frac{(b_x - 2R)^2 + (b_y - 2R)^2}{3b_x b_y}$$



Strain of extreme compression fibers at ultimate curvature of FRP-jacketed section in flexure w axial load

– Grammatikou et al 2018 (adopted in new EC8)

$$\varepsilon_{cu,c,f} = \varepsilon_{cu} + a_f \beta_f \min\left(0.5;\ \frac{\rho_f f_{u,f}}{f_c^{'}}\right)\left(1 - \min\left(0.5;\ \frac{\rho_f f_{u,f}}{f_c^{'}}\right)\right)$$

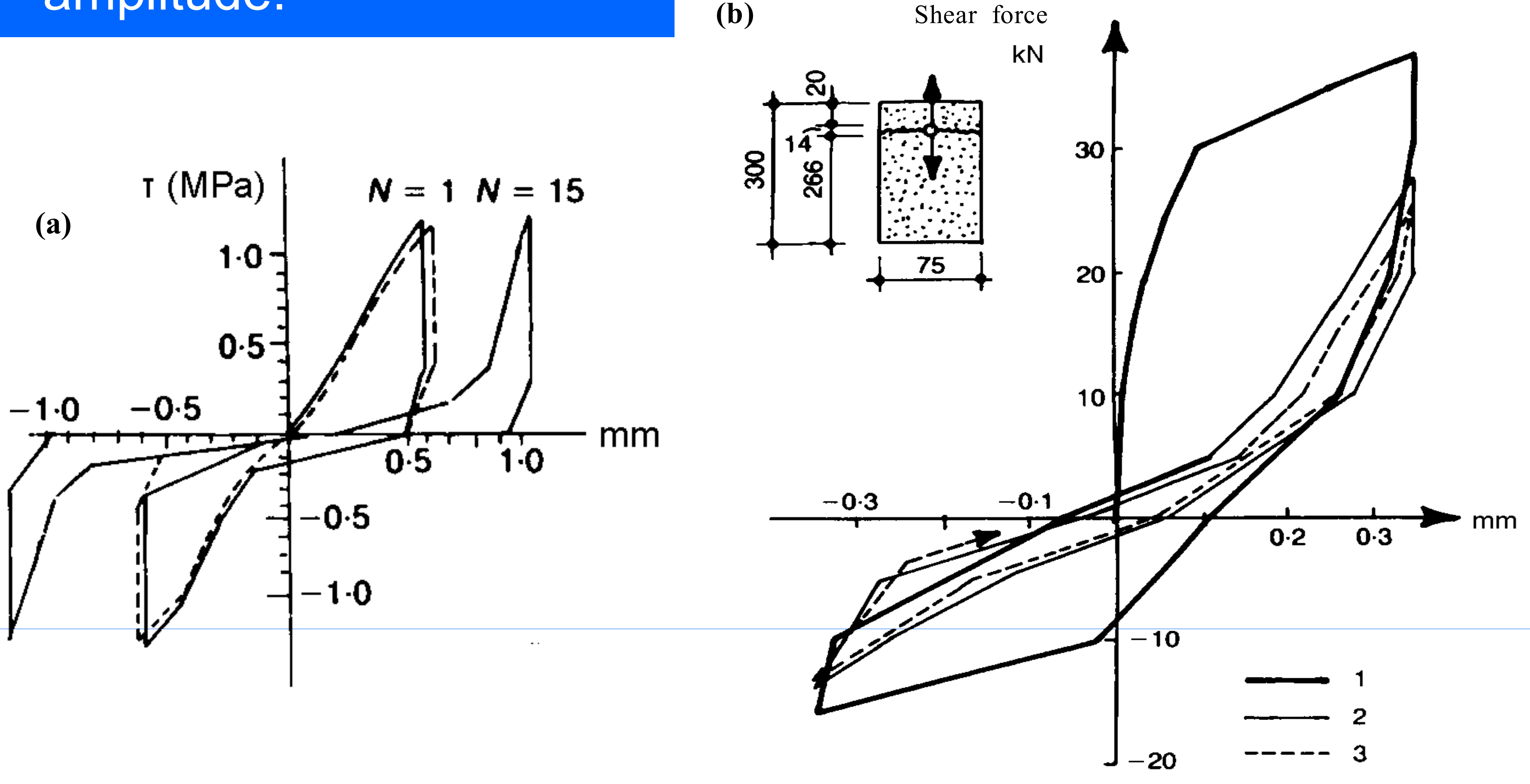
– β_f=0.115 for Carbon FRP (CFRP), Glass FRP (GFRP) and polyacetal fiber (PAF) sheets,

– β_f=0.1 for Aramid FRP (AFRP).

The interactions

Shear transfer in cracks: aggregate interlock & dowel action

(a) Increase of shear slip in cycles with constant shear stress amplitude, for shear transfer by aggregate interlock;
(b) reduction of dowel force in cycles with constant shear slip amplitude.



Bond strength of ribbed bars - monotonic loading

- Eurocode 2 (& 8):

  Design bond strength (: bond stress corresponding to 0.1mm slip):

  - $f_{bd}=2.25f_{ctd}=2.25\text{x}0.7f_{ctm}/\gamma_c=0.315f_{ck}^{2/3}$ (MPa & $\gamma_c=1.5$) in good bond conditions ("vertical" or "bottom horizontal" bars): 2-3MPa if f_{ck}=16-30MPa;
  - $f_{bd}=0.7\text{x}(2.25f_{ctd})=0.22f_{ck}^{2/3}$, in poor bond conditions ("top" "horizontal" bars) 1.5-2 MPa for f_{ck}=16-30 MPa.

- CEB/FIP Model Code 90:

  Ultimate bond strength in unconfined concrete (at ~0.6mm slip):

  - $f_b=2\sqrt{f_c}$ (MPa, w/o γ_c) in good bond conditions: 8-11MPa for f_{ck}=16-30MPa;
  - $f_b=\sqrt{f_c}$ in poor bond conditions: 4-5.5 MPa for f_{ck}=16-30 MPa.

- Huang '96:

  Ultimate bond strength in unconfined concrete (at ~1mm slip):

  - $f_b=0.45f_c$ (w/o γ_c) in good bond conditions: 7-13.5MPa for f_{ck}=16-30MPa;
  - $f_b=0.225f_c$ in poor bond conditions: 3.5-6.5 MPa for f_{ck}=16-30 MPa.

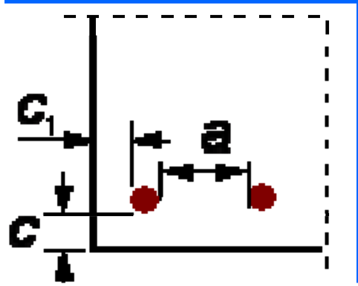
  But bond strength drops by 80% after steel yields!!

- Max. steel stress that may develop at ribbed bar anchorage or lap-splicing for good bond conditions (limited by splitting):
  - Eligehausen, Lettow (2007), ***fib*** Model Code 2010:

$$f_s(MPa) = 51.2\left(\frac{l_b}{d_b}\right)^{0.55}\left(\frac{f_c(MPa)}{20}\right)^{0.25}\left(\frac{20}{\max(d_b;20mm)}\right)^{0.2}\left[\left(\frac{c_d}{d_b}\right)^{1/3}\left(\frac{c_{max}}{c_d}\right)^{0.1} + kK_{tr}\right] \le f_y$$

  - ***fib*** Bulletin 72 (2014) & new Eurocode 2 (2018 draft):

$$f_s(MPa) = 54\left(\frac{l_b}{d_b}\right)^{0.55}\left(\frac{f_c(MPa)}{25}\right)^{0.25}\left(\frac{25}{d_b}\right)^{0.2}\left[\left(\frac{c_d}{d_b}\right)^{0.25}\left(\frac{c_{max}}{c_d}\right)^{0.1} + 1.2kK_{tr}\right] \le f_y$$

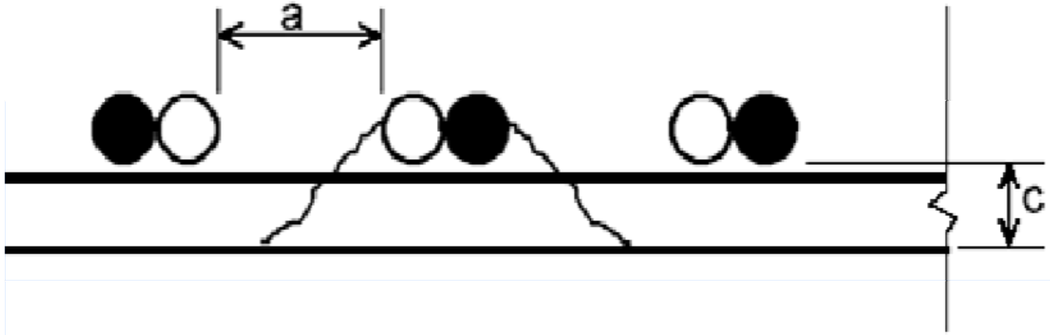


c_d = min [$a/2$; c_1; c] ≥ d_b , $c_d \le 3d_b$,

c_{max} = max [$a/2$; c_1; c] ≤ $5d_b$

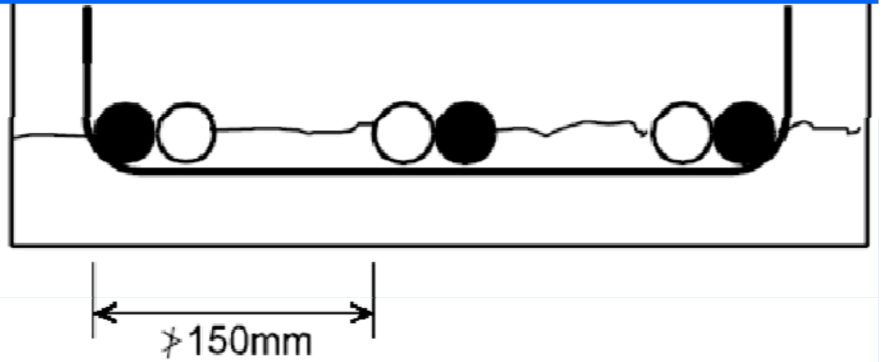
Pull-out stress: set $c_d = 3d_b$ $c_{max} = 5d_b$

$$kK_{tr} = k\frac{1}{n_b d_b}\left(\frac{n_l A_{sh}}{s_h}\right)$$



$a \not< 3.c \Rightarrow n_l=1,\ n_b=1,\ k=5$

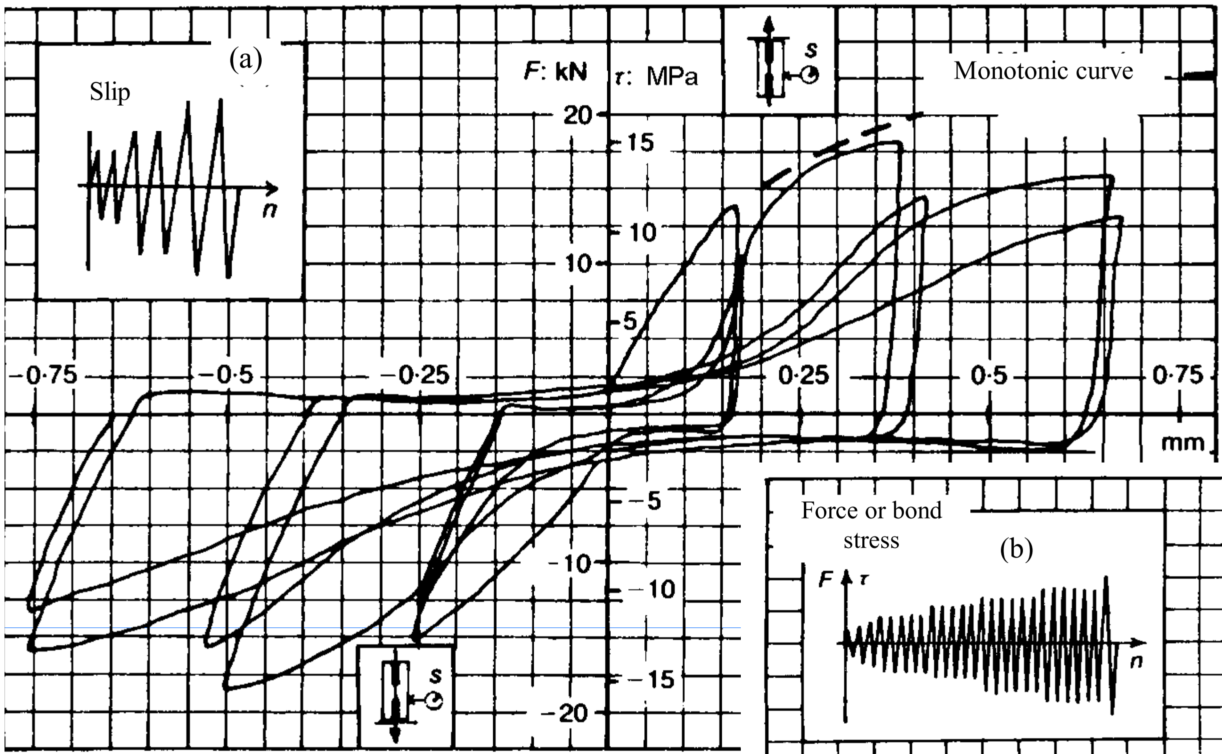
$a < 3.c \Rightarrow k=0$

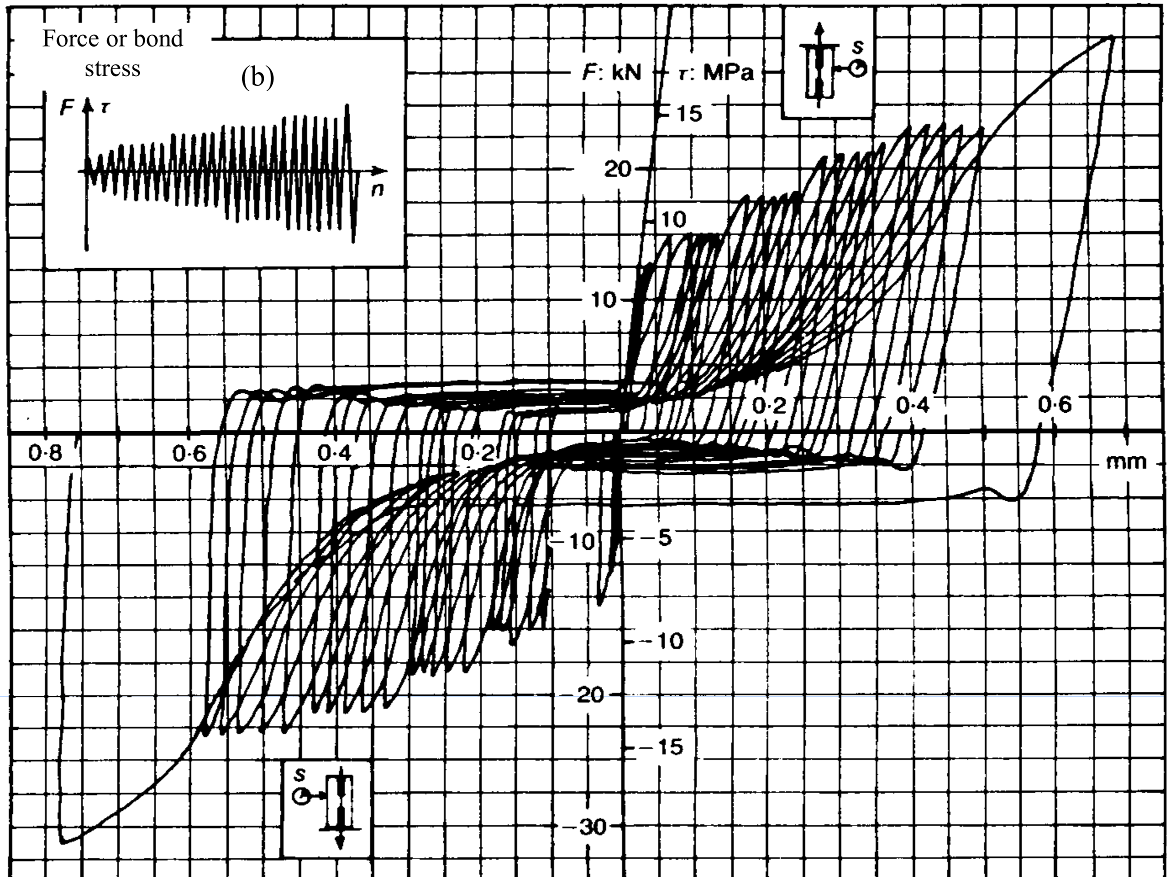


Example:

$n_l=2,\ n_b=3,\ k=10$

Bond stress vs slip of ribbed bars in cyclic loading





Splitting cracks along corner bars or loss of cover due to bond failure under seismic loading